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HIGH SPEED INTERNAL COMBUSTION ENGINES

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HIGH SPEED INTERNAL COMBUSTION ENGINES

By

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WITH NUMEROUS ILLUSTRATIONS AND TABLES

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PREFACE

The present volume is the outcome of an endeavour to collect and to classify, in as brief a form as possible, the more important information relating to the subject of high speed internal combustion engines, as viewed from the theoretical and experimental sides.

Although many excellent treatises upon the subject of internal combustion engines have appeared from time to time, and some apology would at first appear to be necessary in introducing yet another work upon the subject, yet it must be acknowledged that the majority of books hitherto written have been chiefly concerned with the heavier, and slow speed types of engine, and that there exists the necessity for a work of the present nature.

With one or two notable exceptions, the subject of high speed internal combustion engines does not appear to have received, in collective publication form, the attention that its more recent rapid development should merit. The writer has attempted, but with no pretence at completeness, to analyse the more important of the available theoretical and experimental results, and to present them to the reader in some kind of order and sequence.

The present volume is intended to form a companion to one upon the subject of high speed internal combustion engine design, so that the prominence given to certain branches of the subject may perhaps be appreciated. The importance and influence of experimental work upon design has been realized, and a chapter upon the methods of measuring the pressures within the cylinder of high speed internal combustion engines has therefore been included.

Although the subject matter in the present work has been carefully checked, it is possible that some slips or errors may have passed unnoticed, and the author would be glad if his attention were drawn to same.

In conclusion, the writer would like to take this opportunity of acknowledging the kind assistance rendered by Prof. W. Watson and Mr. W. E. Dommett in the preparation of this book.

ARTHUR W. JUDGE.

London, 1916.

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CHAPTER I

THE THERMODYNAMICS OF THE INTERNAL COMBUSTION ENGINE

In studying the various phenomena and changes which occur within the cylinder of the internal combustion engine, it is very important, not to say essential, to be fairly well acquainted with the theory underlying the behaviour of the working substances, which in the present instance are treated as being gaseous.

The fundamental properties which define the physical state of a gas from a thermodynamical point of view are its pressure, temperature and volume, for with a knowledge of these three factors the whole of the other properties can be deduced.

It will be necessary, before dealing with these properties and with the laws concerned, to define the units employed, commencing with those connected with temperature.

The *unit of heat,* which is used for expressing quantities of heat, is defined as the quantity of heat necessary to change the temperature of unit mass of water one unit. The unit now adopted by English engineers is the "British Thermal Unit," which represents the quantity of heat required to change the temperature of 1 lb. of water 1° Fah., and as this quantity slightly varies with the initial temperature of the water, it is general to specify 60° F. as the initial temperature.

In the metric system, the unit of heat is the Calorie, which represents the heat required to raise 1 kilogramme of water through 1° C. It might be mentioned in passing that one Calorie $= 3{\cdot}967$ B.T.U.s., or one B.T U. $= 0{\cdot}252$ Calories.

The unit of work is the *foot pound,* that is, the work done in lifting 1 lb. weight through a vertical distance of a foot; the metric unit of work is the *kilogramme-metre,* which is similarly explained.

As heat is convertible into mechanical work, it is necessary to know the relation between the two respective units, for which information we were first indebted to Dr. Joule, and subsequently to other experimenters. The actual relation now adopted as correct is that one B.T.U. of heat is equivalent to 778 foot pounds of work; * this latter quantity being denoted by the letter J.

---

* For the work of various experimenters, vide *Phil. Trans. Roy. Soc.,* 1893 and 1897.

The two principal laws governing the expansion and compression of gases are known as Boyle's and Charles's Laws respectively. Neither law is strictly true for permanent gases, although true in every sense for the ideally perfect gas.

Boyle's Law.

Boyle's Law states that for a given mass of gas, at constant temperature, the pressure varies inversely as the volume ; or, using the letters P and V to represent pressure and volume respectively,

$P \propto \frac{1}{V}$, that is, the product $P \times V = \text{constant}$. . . . . (1)

For air :—If V is the volume of 1 lb. of air in cub. feet and **P** is the pressure in lbs. per sq. foot, at the constant temperature of freezing water, 32° F.,

Then we have $P.V = 26220$.

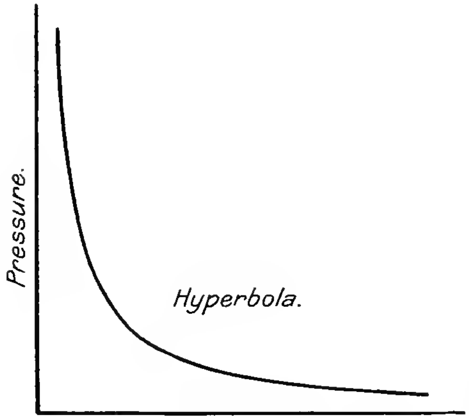

FIG. 1.

The relation between P and V can be graphically represented by a hyperbolic curve, as shown in Fig. 1.

Charles's Law.

Charles's Law states that when a given mass of gas expands under constant pressure, equal increments of temperature produce equal increments of volume, and it also states that all gases expand alike.

Thus if V_o is the volume at zero temperature of a given quantity of gas expanding at constant pressure, and if V is its volume at any other temperature T^1

Then $V = V_o(1 + aT^1)$, where a is a constant.

Thus when T^1 is in Centigrade units a is very nearly $\frac{1}{273}$, and when T^1 is in Fahrenheit units a is very nearly $\frac{1}{461}$.

Assuming Charles's Law to be correct for the temperatures below zero, in the above units, it will be seen that the temperature at which the volume V of the gas vanishes is given by

$$V = O = V_o\,(1 + aT^1)$$

$$\text{or } T^1 = -\frac{1}{a} = -273°\,\text{C, or } -461°\,\text{F.}$$

Actual experiments upon contracting gases, however, show that the above law fails at low temperatures, as most gases solidify before such a low temperature is attained, but the absolute zero of temperature is a useful conception, and in thermodynamical relations it is customary and useful to express temperatures on this *Absolute Scale.*

Thus in Centigrade units we have Absolute Temperature $T = T^1 + 273°$; in Fahrenheit units Absolute Temperature $T = T^1 + 461$.

Using this relation Charles's Law becomes $V = V_o.T.a.$. (2)

By combining equations (1) and (2), the relation is at once obtained where R is a constant,

$$P.V = R.T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Using the same units for P and V as before, and expressing T in Absolute Fahrenheit units, we get $P.V = 53{\cdot}18\ T$ for air.

The value of the constant R for any gas depends upon the specific heat of the gas under different conditions.

By the *specific heat* of a gas is meant the quantity of heat required to raise unit mass of the gas through 1° of temperature. The pound and Fahrenheit units are usually employed.

There are, however, two specific heats to which the above definition applies, namely, the specific heat at constant volume, usually denoted by C_v, and the specific heat at constant pressure or C_p.

Joule's Law.

This law states that heat and mechanical work are mutually convertible, a unit of heat being equivalent to a certain amount of mechanical work, called the *Mechanical Equivalent of Heat.* The relation between these two quantities has already been given.

This relation can be written as $w = J.H$ . . . . . (4)
where w = the amount of mechanical work in work units.
and H = the quantity of heat in heat units.
J = the Mechanical Equivalent of Heat = 778 in Fahrenheit units.

When a gas expands without actually doing external work and without taking in or giving out heat (*i.e., without changing its internal energy*), its temperature remains constant: a fact established by Joule. This definition clearly illustrates the meaning of the term Internal or Intrinsic Energy, for it shows that the internal energy of a given quantity of gas depends only upon its temperature.

When a quantity of heat H is supplied to a gas, and work is done by the gas, denoted by W (in heat units), and if E_0 is the initial and E the final internal energy of the gas, Joule's Law may be expressed as : Heat supplied = work done + increase in internal energy,

$$\text{or } H = W + (E - E_o) \qquad \ldots\ldots\ldots\ldots \quad (5)$$

For a small change we have $dH = dW + dE$.

When the gas is subjected to a complete cycle of operations, the final internal energy is the same as the initial energy,

$$\begin{aligned} \text{so that } dE &= 0 \\ \text{then } dH &= dW \\ \text{and } \int dH &= \int dW \end{aligned}$$

which, with the constant of integration zero, gives $H = W$ (that is, the work done during the cycle is equivalent to the heat supplied).

Here the quantities H and W are expressed in the same units.

Specific Heat at Constant Pressure, C_p.

If a given quantity of gas be heated at constant pressure P from temperature T_2 to T_1 and if V_2 and V_1 represent its initial and final volumes, we have :—

Heat taken in $= C_p. (T_1 - T_2)$

Work done $= P(V_1 - V_2) = R(T_1 - T_2)$ from Boyle's Law.

Also from the above relation (4), $H = W + (E - E_o)$

$$\text{then } C_p(T_1 - T_2) = \frac{P}{J}(V_1 - V_2) + \text{change in internal energy}$$

$$= \frac{R}{J}(T_1 - T_2) + C_v(T_1 - T_2)$$

$$\text{From which } C_p - C_v = \frac{R}{J} \qquad \ldots\ldots\ldots\ldots \quad (6)$$

This gives one relation between the specific heats of a gas in terms of the constant R (mechanical units).

Dividing both sides of Equation (6) by C_v and expressing R in heat units, we have $\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$

The ratio of the specific heats $\frac{C_p}{C_v}$ of a gas is generally denoted by the symbol γ, so that $\gamma = 1 + \frac{R}{C_v} = \frac{1}{1 - \frac{R}{C_p}}$ . . . . . (7)

In the above relations it is assumed that both C_p and C_v remain constant at all temperatures, but the experiments of Holborn and Hemming and others show that the specific heats of a gas at constant pressure and volume vary to a small degree with the temperature; this will be considered more fully later on.

We can now write the relation $P.V = R.T$

$$\text{as } P.V = (C_p - C_v).J.T \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The Expansion of Gases.

A gas may expand or contract in two principal ways, namely :—

(1) It may expand or contract at constant temperature, that is *Isothermally*.

(2) It may expand or contract so that it neither receives nor loses heat in any manner, that is *Adiabatically*.

When a gas expands isothermally, it follows Boyle's Law, according to the equation

$$P.V = R.T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and its curve of expansion is a rectangular hyperbola, as shown in Fig. 1.

When adiabatic expansion occurs, the work which the gas does in expansion is done at the expense of its stock of internal energy, no heat being exchanged between the gas and outside sources by the usual processes of conduction and radiation.

In actual internal combustion engines, the expansion and compression of the gases occupies only a very small period of time, so that the gases themselves have very little opportunity for losing or gaining heat, and hence the performances are sensibly adiabatic.

Equation of an Adiabatic Line.

In order to obtain the equation of an adiabatic expansion curve, in terms of pressure and volume, it will be necessary to consider the general equation giving the change in the condition of a gas by the addition of an infinitesimal amount of heat δH.

Since heat supplied = Change of internal energy
+ External work done

then for a very small change due to heat δH supplied :—

$$\delta H = C_v.\delta T + P.\delta V$$

$$\text{or } dH = C_v dT + P dV \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Since T is a function of pressure and volume, we may write

$$T = f(P.V) \text{ and } dT = \frac{\delta T}{\delta P}.dP + \frac{\delta T}{\delta V}dV.$$

Also we have $P.V = R.T$

$$\text{and so } dT = \frac{V.dP}{R.} + \frac{P.dV}{R.}$$

Hence Equation (9) becomes $dH = Cv\left\{\frac{V.dP}{R} + \frac{PdV}{R}\right\} + P.dV$

Knowing that $C_p - C_v = R$, or $C_v + R = C_p$, we now obtain by substitution $dH = \frac{1}{R}\left\{C_vV.dP + C_p.P.dV\right\}$ . . . . . (10)

This is a *general expression for the heat required for a given gaseous change.*

Now in the case of an adiabatic change, no heat is given to or lost by the gas in expanding, so that $dH = 0$.

Hence $\frac{1}{R}\left\{C_v.V.dP + C_p.P.dV\right\} = 0.$

$$\therefore \frac{dP}{P} + \frac{C_p}{C_v} \cdot \frac{dV}{V} = 0$$

Integrating $\log P + \gamma \log V = \text{constant}$

and putting $\frac{C_p}{C_v} = \gamma$ or $PV^\gamma = \text{constant}$ . . . . . (11)

This is the general equation for an adiabatic change, and the value of γ for air is about 1·41 ; the values for other gases are given in Table VI.

If we now consider the relation between the volume of a gas and its temperature due to an adiabatic change, and let P, V, T represent the initial, and P_o, V_o, T_o the final state of the gas, we have $PV^\gamma = P_oV_o^\gamma$.

$$\text{Also } P = \frac{R.T}{V} \text{ and } P_o = \frac{R.T_o}{V_o}$$

$$\text{Hence } \frac{P}{P_o} = \left(\frac{V_o}{V}\right)\left(\frac{T}{T_o}\right)$$

$$\text{and } \left(\frac{V_o}{V}\right)^\gamma = \left(\frac{V_o}{V}\right)\left(\frac{T}{T_o}\right)$$

$$\text{So that } \underline{\frac{T}{T_o} = \left(\frac{V_o}{V}\right)^{\gamma-1}} \quad . \quad . \quad . \quad . \quad (12)$$

Similarly it can be shown that the relation between the pressure and temperature in an adiabatic change is represented by

$$P^{\frac{1-\gamma}{\gamma}}. \; T = \text{constant}. \quad . \quad . \quad . \quad . \quad (12\text{A})$$

Here it must be remembered that temperatures are expressed on the absolute scale.

Example.—As an example of the use of these adiabatic formulae consider a quantity of dry air to be compressed adiabatically from an initial volume of 600 c.c. to a final volume of 150 c.c., its initial temperature being 60° F. (= 521° absolute). The value of γ for dry air is 1·408.

Here we have $T_o = 521$

$$\text{and } \frac{T}{521} = \left(\frac{600}{150}\right)^{1 \cdot 4 - 1} = 4^{0 \cdot 4} = 1 \cdot 741$$

Whence T = 907° absolute or 446° F.

In like manner it can be shown that the pressure, assumed to be originally atmospheric, rises to $4^{1 \cdot 4}$, or nearly seven atmospheres.

It will frequently be necessary to consider cycles of operations in which both isothermal and adiabatic changes are involved, so that it will be of value to compare the two changes graphically as shown in Fig. 2.

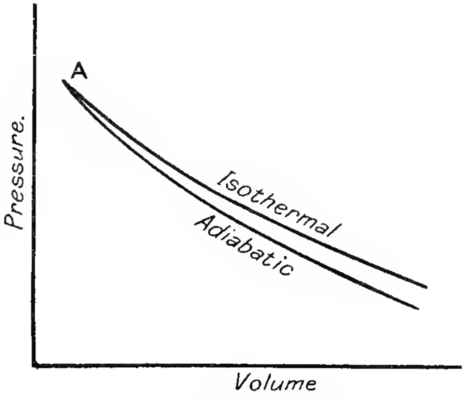

FIG. 2.

It will be seen that if a gas originally defined by the co-ordinates of the point A be expanded respectively in the above two manners, the adiabatic method gives a more rapid rate of decrease of pressure.

This can be proved by finding the slope (or tangent) of the two curves at A.

For the isothermal PV = constant

$$\text{and } \frac{dP}{dV} = \frac{P}{V}$$

For the adiabatic PV^γ = constant

$$\text{and } \frac{dP}{dV} = \gamma \frac{P}{V}$$

and as γ is always greater than unity, the adiabatic has the greater slope, and thus lies below the isothermal. Conversely, if a given volume of gas be compressed, adiabatically and isothermally, it will attain a higher final pressure in the former case, the final volumes being the same.

The compression of the charge of explosive mixture in such a case as that of a high-speed internal combustion engine is so rapid that it is nearly adiabatic, whereas in very slow-running engines, where time elapses for the charge to dissipate some of its heat, the compression approaches an isothermal one.

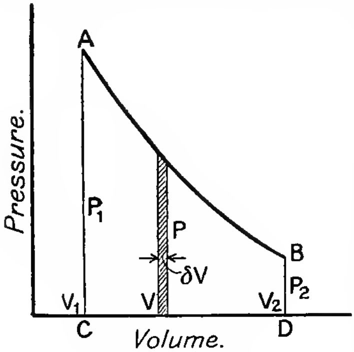

FIG. 3.

Work Done in Expansion.

(1) *Isothermal Expansion.* Consider a given quantity of gas as represented by A on the curve AB (Fig. 3) to expand isothermally to B, and let P_1V_1 represent its state at A, and P_2V_2 at B.

Now since the work done in a small expansion δV is $P.\delta V$ we have by integration

$$\text{Work done in expansion} = \int_{V_1}^{V_2} P.dV$$

since $P_1V_1 = P_2V_2 = PV = \text{constant} = K$

$$\text{Then work done} = \int_{V_1}^{V_2} .K.\frac{dV}{V}$$

$$= K.\log_e\left(\frac{V_2}{V_1}\right)$$

$$= \underline{P_1V_1 \log_e r} \quad . \ (13)$$

where r is the ratio of expansion $= \frac{V_2}{V_1}$

$$\text{That is} = \underline{R.T. \log_e r}$$

expressed in terms of the constant temperature T.

(2) *Adiabatic Expansion.* In the case of an adiabatic expansion our expression for the work done is the same, namely :—

$$\text{Work done} = \int_{V_1}^{V_2} P.dV$$

and since $P_1V_1{}^{\gamma} = P_2V_2{}^{\gamma} = P.V^{\gamma} = \text{constant} = K^1$

$$\text{we get } W = \int_{V_1}^{V_2} \frac{K^1}{V^{\gamma}}dV$$

$$W = \frac{K^1}{1-\gamma}\left[V_2{}^{1-\gamma} - V_1{}^{1-\gamma}\right]$$

Or we may write this as $\frac{P_1V_1}{1-\gamma}\left\{\left(\frac{V_2}{V_1}\right)^{1-\gamma} - 1\right\}$ . . . . (14)

Again if T_1 and T_2 be the initial and final temperatures

$$\text{then } \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2}$$

$$\text{and } W = \frac{P_1V_1}{1-\gamma}\left\{\frac{T_2}{T_1} - 1\right\} = \frac{RT_1}{1-\gamma}\left\{\frac{T_2 - T_1}{T_1}\right\}$$

or by substituting $C_v = \frac{R}{1-\gamma}$ $\underline{W = C_v(T_2 - T_1)}$. (15)

(3) *Expansion according to PV^n=constant.* Supposing the expansion occurs according to the law PV^n = constant, it follows from the above that the work done is given by

$$W = \frac{P_1V_1^n}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right]$$

$$= \frac{P_1V_1}{1-n}\left[\left(\frac{V_2}{V_1}\right)^{1-n} - \quad\right]$$

Since $r = \frac{V_2}{V_1}$ $W = \frac{P_1V_1}{1-n}\left(r^{1-n} - 1\right)$ . . . . . (16)

This result may also be written as

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

and because $P_1V_1^n = P_2V_2^n$ $W = \frac{R(T_1 - T_2)}{n-1}$ in terms of the initial and final temperatures.

The Heat Required for a Given Change.

It frequently happens that the initial and final volumes V_1 and V_2 of a gas are known, or at least are measurable from an indicator diagram, and also the corresponding pressures, and that from them we wish to deduce the quantity of heat required for the given change from P_1V_1 to P_2V_2.

As before, we have $dH = C_v.dT + P.dV$ . . . . . . (9)

$$\text{Integrating } \therefore H = C_v\int_{T_1}^{T_2} dT + \int_{V_1}^{V_2} PdV$$

$$\underline{= C_v(T_2 - T_1) + \text{External work done } (17)}$$

Now if the law of expansion is known to be of the form PV^n = constant, we can integrate $P.dV$ to get the external work, as in (17), done and hence H is determinate.

Rate of Heat Reception per Unit Change of Volume. [Data from the Indicator Diagram.]

If the working substance or gas expands according to any given law, we can find the rate at which it receives or rejects heat per unit change of volume, from an investigation of the indicator diagram. As in the above example we have

$$dH = C_v.dT + P.dV \quad \ldots\ldots\ldots\ldots \quad (9)$$

or

$$\frac{dH}{dV} = C_v.\frac{dT}{dV} + P$$

As P.V. = R.T

$$\frac{dV}{dT} = \frac{R}{P + V.\dfrac{dP}{dV}}$$

then by substitution

$$\frac{dH}{dV} = C_v\left[\frac{1}{R}\left(P + V.\frac{dP}{dV}\right) + \frac{P}{C_v}\right]$$

$$= \frac{1}{\gamma-1}\left[P + V.\frac{dP}{dV} + (\gamma-1)P\right] \text{for } C_v = \frac{R}{\gamma-1}$$

$$= \frac{1}{\gamma-1}\left(\gamma P + V.\frac{dP}{dV}\right) \quad \ldots\ldots \quad (18)$$

All of these quantities can be measured from the indicator diagram, although the slope of the expansion curve cannot be obtained very accurately, and it is then possible to plot a curve showing the rate of heat reception or rejection at all parts of the diagram.

In the case of the expansion law being of the form $P.V.^n$ = constant we have

$$dH = C_v dT + P.dV$$

and integrating

$$H = C_v\int_{T_1}^{T_2} dT + \int_{V_1}^{V_2} P.dV$$

$$= C_v\left[T_2 - T_1\right] + \frac{P_1V_1}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right)$$

$$= \frac{C_v}{R}\left(P_2V_2 - P_1V_1\right) + \frac{P_1V_1}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right)$$

$$= \frac{K}{\gamma-1}\left[V_2^{1-n} - V_1^{1-n}\right] + \frac{K}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right]$$

For PV = Constant = K and $C_v = \dfrac{R}{\gamma - 1}$

$$= \frac{K}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right] \times \frac{\gamma - n}{\gamma - 1}$$

From (15) H = *External work done* $\times \dfrac{\gamma - n}{\gamma - 1}$

So that in this case the heat required for a given change, as represented on the indicator diagram, can be readily obtained by measurements from the same.

In order to obtain the value of the index n, that is, to find the law of expansion, the pressures P_1, P_2 and corresponding volumes V_1 and V_2 at two points on the curve (Fig. 3) can be measured.

$$P_1 V_1^n = P_2 V_2^n = \text{constant.}$$

Taking logarithms, $\text{Log } P_1 + n \log V_1 = \log P_2 + n \log V_2$

$$\text{and so } n = \frac{\log P_2 - \log P_1}{\log V_1 - \log V_2}$$

General Equation for the Specific Heat of a Gas.

In all cases of gases possessing variable specific heats, if a small quantity of heat δH be given to a unit mass of a gas, thereby raising its temperature from T to $T + \delta T$

then the specific heat at temperature $T = \frac{dH}{dT}$ in the limit;

and the specific heat at constant pressure $C_p = \left(\frac{dH}{dT}\right)_p$

Similarly, the specific heat at constant volume $C_v = \left(\frac{dH}{dT}\right)_v$

that is to say, if the expression for the quantity of heat is known in terms of p, v and t, the specific heats are the partial differential coefficients, with p and v constant, respectively.

Again, for a small change in the gas we have

$dH = dE + dW$ where E = the intrinsic energy $= \phi(V.T)$

$$\text{Then } dH = \left(\frac{dE}{dT}\right)_v dT + \left(\frac{dE}{dV}\right)_T dV + PdV$$

$$\text{Hence the specific heat, or } \frac{dH}{dT} = \left(\frac{dE}{dT}\right)_v + \left(\frac{dE}{dV}\right)_T \frac{dV}{dT} + P.\frac{dV}{dT} \quad (20)$$

This is a general equation for the specific heat of a gas. It can be applied to the cases already considered.

Thus $C_v = \left(\frac{dH}{dT}\right)_v = \left(\frac{dE}{dT}\right)_v$ from (19) (dV being of course zero).

For the specific heat at constant pressure

$$\text{we have } C_p = \left(\frac{dH}{dT}\right)_p = \left(\frac{dE}{dT}\right)_v + \left\{\left(\frac{dE}{dV}\right)_T + P\right\}\left(\frac{dV}{dT}\right)_p$$

Hence the difference between the specific heats is :—

$$C_p - C_v = \left\{\left(\frac{dE}{dV}\right)_T + P\right\}\left(\frac{dV}{dT}\right)_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

For a perfect gas we know that $PV = RT$

$$\text{and that } P.\frac{dV}{dT} = R$$

Also, it is known from Joule's porous plug experiment, which will be found fully described in works upon Heat, that no external work is performed in free expansion, consequently

$$dH = dE + 0$$

and if $E = \phi(V.T)$, then $dE = \left(\frac{dE}{dT}\right)_v dT + \left(\frac{dE}{dV}\right)_T dV$

$$\therefore \left(\frac{dE}{dV}\right)_T = 0$$

Hence from (21) we have: $C_p - C_v = P.\left(\frac{dV}{dT}\right)_p = R$ in heat units, or if R is in mechanical units $C_p - C_v = \frac{R}{J}$.

Graphical Representation of Heat Quantities.

Consider a given quantity of gas to expand adiabatically and indefinitely from an initial condition as represented on the P.V. diagram by the co-ordinates of the point A (Fig. 4).

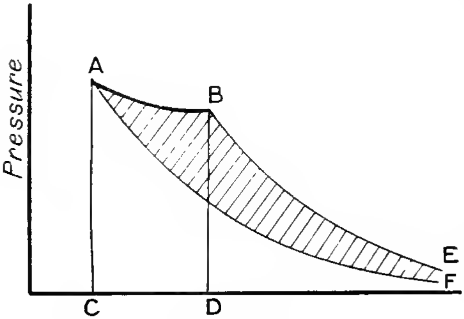

FIG. 4.

The general equation for a change of state is $dH = dE + P.dV$.

But $dH = 0$ for adiabatic expansion, for heat is neither given to nor subtracted from the gas.

Hence $-dE = dW$ and since the area ACF between the curve AF and the pressure datum line represents the work done in changing from A to F, it also represents the intrinsic energy of the gas.

Again, suppose the gas expands in any manner from A to B, then the heat added is equal to the difference between the intrinsic energies at B and A + external work done

$$\begin{aligned} \text{or } H &= E_A^B + W \\ &= \text{Area (BDE} - \text{ACF)} + \overset{\text{Area}}{\text{ABCD}} \\ &= \text{shaded area ABEF} \end{aligned}$$

Hence the quantity of heat required for expanding the gas from

A to B is represented by the area between the two adiabatics through A and B.

Heat Required for a Given Change in Terms of P and V.

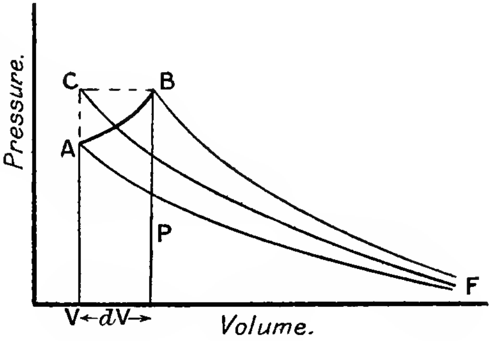

FIG. 5.

Consider again a small expansion from A to B (Fig. 5). If BC and AC be drawn parallel to the co-ordinate axes respectively then heat added from A to B = area ACF + area BCF − area ACB.

Now area BCF is equivalent to constant pressure change along CB and can be expressed as $\left(\frac{dH}{dV}\right)_p .dV$

Similarly ACF is equivalent to constant volume change along AC and can be expressed as $\left(\frac{dH}{dP}\right)_v dP$

Hence heat added from A to B $=\left(\frac{dH}{dV}\right)_p dV + \left(\frac{dH}{dP}\right)_v dP$ in the limit, for ACB is of the second order and negligible in the limit.

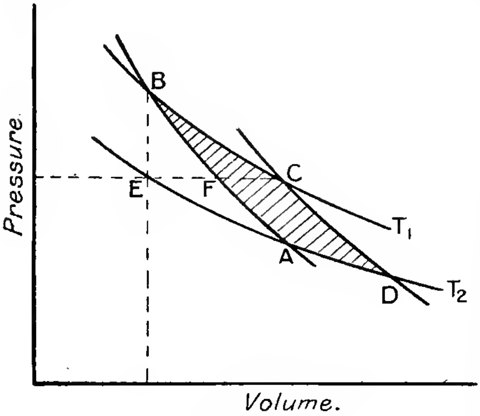

FIG. 6.

Other Expressions for the Rate of Heat Reception.

Before considering these relations it will be necessary to enunciate the *Second Law of Thermodynamics*, which states that :—For a completely reversible engine working upon a cycle represented by two adiabatics, BA and CD (Fig. 6), and two isothermals, BC and AD, the ratio of the work done to the heat received during the cycle is a function of the temperature only. This cycle is known as the Carnot cycle.

In other words, the efficiency of an engine working upon the Carnot cycle depends only upon the working temperatures.

Expressed in symbols $\frac{W}{H} = \psi(T)$

and by differentiation $\frac{dW}{H} = \phi(T).dT$

If the temperatures of working are " absolute " then $\phi(T) = \frac{1}{T}$

$$\text{and } \therefore \frac{dW}{H} = \frac{dT}{T} \text{ or integrating } \frac{W}{T} = \frac{T_1 - T_2}{T_1}$$

where T^1 and T^{11} are the working temperatures of the isothermals.

We may then write the second law of thermodynamics as

$$\frac{dW}{H} = \frac{H_1 - H_2}{H_1} \text{ or } \frac{T_1 - T_2}{T_1}$$

or $\frac{H_1}{T_1} = \frac{H_2}{T_2}$ That is $\int \frac{dH}{T} = 0$ for a cycle.

Heat Reception Changes, with Pressure and Volume Changes, at Constant Temperature.

Next, the area of the parallelogram ABCD (Fig. 6) = twice area BAC

$$= \text{twice area BEC}$$

$$\text{or } W = (P_1 - P_2)(V_1 - V_2)$$

Now $\frac{W}{H} = \frac{T_1 - T_2}{T_1}$, so that $H = \frac{T_1}{T_2 - T_1} (P_1 - P_2)(V_1 - V_2)$

If now the cycle of operations represented by the two adiabatics BA and CD, and the isothermals BC and AC, be infinitesimally small, we can write the above expression as

$$-\left(\frac{\delta H}{\delta P}\right)_T = T \left(\frac{\delta V}{\delta T}\right)_p \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The negative sign is given because H is decreasing as P increases, or in other words, an absorption of heat corresponds with a fall in pressure.

Equation (23) gives the *rate of change of heat reception with the pressure change* at constant temperature, and may be re-written in the form

$$\underline{\left(\frac{\delta H}{\delta V}\right)_T = T\left(\frac{\delta P}{\delta T}\right)_v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Equation (23) gives an expression for the rate of change of heat reception with relation to volume change at constant temperature.

We can, alternatively to methods already dealt with, obtain general expressions for the difference in the two specific heats as follows :—

Consider a unit mass of gas at B (Fig. 6), and let it be cooled down to E (at constant volume).

The heat given up is then $h = C_v(T_1 - T_2)$.

Next suppose the state of the gas be altered (at constant pressure) to F. Since F is on an adiabatic through B, no net heat is added or subtracted along the paths BE and EF.

Hence heat added in change BE $= -$ heat added along EF $= h$.

Now restore the given mass of the gas to C along EC (constant pressure). Then heat required for change EC = heat for EF + heat for FC

$$= h + H$$

But this is the heat required at constant pressure $= C_p(T_1 - T_2)$

Then we have $h = C_v\,(T_1 - T_2)$

$$H + h = C_p\,(T_1 - T_2)$$

$$\therefore\ C_p - C_v = \frac{H}{T_1 - T_2} = \frac{T_1}{(T_1 - T_2)^2}(P_1 - P_2)(V_1 - V_2)$$

In the limit this becomes $C_p - C_v = T.\left(\frac{\delta P}{\delta T}\right)_v\left(\frac{\delta V}{\delta T}\right)_p$ . . (24)

Which is *the general relation between the specific heats of a gas in terms of temperature, pressure and volume.*

Example.—For a perfect gas. $PV = RT$

$$\text{Then } \left(\frac{\delta P}{\delta T}\right)_v = \frac{R}{V} \text{ and } \left(\frac{\delta V}{\delta T}\right)_p = \frac{R}{P}$$

$$\text{Hence } C_p - C_v = T.\frac{R}{V}\ \frac{R}{P} = R$$

Thus in the case of an ideal gas the difference between the specific heats is always constant, and is independent of any variations C_p or C_v separately.

Variable Specific Heats ; Some Experimental Results.

It has previously been pointed out that, although for purposes of calculation the specific heats of a gas are assumed to be constant, yet in actual gases which deviate from the ideal state, expressed by Boyle's and Charles's Law, the specific heats are not constant.

The experiments of Mallard and Le Chatelier,* for instance, show that in the case of Nitrogen $C_v = 0{\cdot}171 + 0{\cdot}0000215\ T$
and of Oxygen $C_v = 0{\cdot}150 + 0{\cdot}0000188\ T$

Later values, however, published by the same experimenters in 1887 give rather different values for the constants,† but agree in the general variation of C_v with temperature.

Again, the experiments of Holborn and Henning ‡ on specific heats of gases at constant pressure show that for nitrogen

$$C^1_p = {\cdot}2350 + {\cdot}000019\ T$$

C^1_p being the mean specific heat at constant pressure between O^o and T^o

$$\text{and for } CO_2\ C^1_p = {\cdot}2010 + {\cdot}0000742\ T - {\cdot}000000018\ T^2$$

The results of measurements of the mean specific heats at constant pressure by Holborn and Austin are given in Table I, for the cases of steam, carbon dioxide and nitrogen, and are referred to the specific heat of water which is taken as unity.

In Table II are given the values for the specific heat at constant pressure, for carbon dioxide at different temperatures, obtained by different experimenters, and in the last column is given the mean of these values.

TABLE I

VARIATION OF MEAN SPECIFIC HEATS AT CONSTANT PRESSURE WITH TEMPERATURE (Holborn and Austin)

Temperature ° C.	Value of Mean C_p		
	Steam (H_2O)	Carbon Dioxide CO_2	Nitrogen N
110 to 280 . . .	0·465	—	—
110 to 200 .	—	0·217	0·240
110 to 400	0·467	0·229	0·242
110 to 600 . . .	0·473	0·240	0·247
110 to 800 . . .	0·482	0·250	0·251
110 to 1,000 . . .	0·494	0·258	0·254
110 to 1,200 . . .	0·510	0·264	0·258
110 to 1,400 . . .	0·532	0·272	0·262

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* *Recherche's sur la Combustion des Mélanges gazeux explosifs*, Paris, 1883.

† Vide *Engineering*, June 27, 1902.

‡ *Ann d. Phys.* 23, 1907, page 809.

TABLE II

VARIATION OF C_p WITH TEMPERATURE FOR CO_2

Temperature ° C.	Values of C_p			
	Zangen.	Mallard and Le Chatelier.	Holborn and Austin.	Mean Value.
0 . . .	0·1980	0·1880	0·2028	0·1963
100 . . .	0·2100	0·2140	0·2161	0·2133
200 . . .	0·2220	0·2390	0·2285	0·2298
400 . .	0·2450	0·2840	0·2502	0·2597
600 . .	0·2690	0·3230	0·2678	0·2866
800 . . .	0·2920	0·3550	0·2815	0·3095

The general form of variation of C_p with T takes the form

$$C_p = a + bT + cT^2$$

The variation of the specific heats with temperature have also been investigated for different gases by Regnault, Wiedemann, Holborn and Austin,* and others. Experiments were also made by Dugald Clerk upon the products of explosion in a gas engine. The gases consisted of a mixture of CO_2, water vapour and air, and the engine was used as a pump, indicator diagrams being taken at the same time. The results are given in full in Clerk's own book.†

Much valuable information upon the subject of specific heat variation can be obtained from the Gaseous Explosions Committee's Reports, 1908–1910, of the British Association for the Advancement of Science, Burlington House, W.—the first Report, dealing with this part of the subject.

The curves shown in Fig. 7 illustrate the variation of the specific heats, as determined by Holborn and Henning, for the gases mentioned thereon. It will be observed that, instead of plotting specific heat, the energy per gramme molecule has been substituted, that is, the internal energy per unit volume. This is preferable, as in nearly all experiments of this nature the increase of internal energy in unit volume is measured.

The "true" or instantaneous specific heat C is the rate of change of the internal energy E per unit volume with the temperature T. This quantity is often termed the "Thermal Capacity."

Expressed in symbols $C = \frac{dE}{dT}$. Where the gramme molecule is taken as the unit volume, E is expressed in calories, and T in Centigrade units. C is often termed the "Volumetric Heat."

The volume of the gramme molecule, which is sensibly constant for all gases, is 22·25 litres at 0° C. and 760 mm. pressure, and

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* Wiss, *Abhandlungen der Phys. Techn. Reichsanstalt,* 1905.

† *The Gas, Petrol and Oil Engine* (Longmans, Green & Co.).

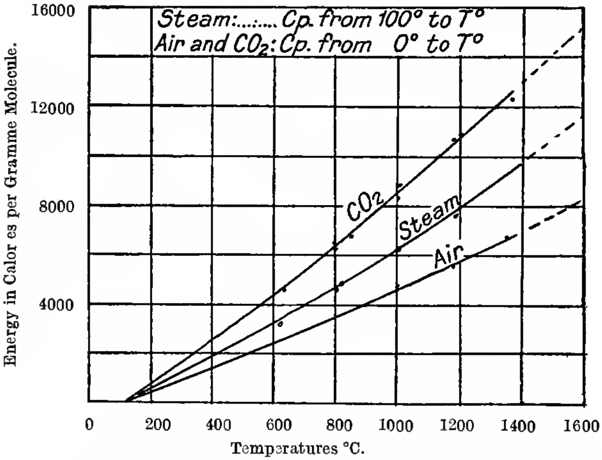

FIG. 7.—VARIATION OF SPECIFIC HEATS AT CONSTANT PRESSURE. (Holborn and Henning.)

1 calorie per gramme molecule is equivalent to 3·96 ft. lbs. per cub. foot.

Referring to the curves given in Fig. 7, it will be at once seen that since $C = \frac{dE}{dT}$ the specific heat at any temperature is given by the slope of the curve.

The internal energy curve, representing the mean values of the results of the more reliable experimenters, for a gas engine mixture consisting of 12 per cent. H_2O, 5 per cent. of CO_2 and 83 per cent. of N and O, is shown in Fig. 7A. This curve will be found very useful in making temperature and pressure calculations in connexion with internal combustion engines, and as a fairly simple

TABLE III

SHOWING THE VARIATIONS OF THE VOLUMETRIC HEATS AND γ WITH TEMPERATURE

	100° C.		600° C.		1,100° C.	
	Volumetric Heat.	γ	Volumetric Heat.	γ	Volumetric Heat.	γ
Air . . .	4·9	1·404	5·2	1·38	5·75	1·345
Steam . .	6·6	1·30	6·85	1·29	8·5	1·24
CO_2. . .	7·5	1·26	9·95	1·20	11·1	1·18

means for taking into account the variation of the specific heats with the temperatures. Thus if the total internal energy of a gaseous mixture be known, due, say, to the addition by means of

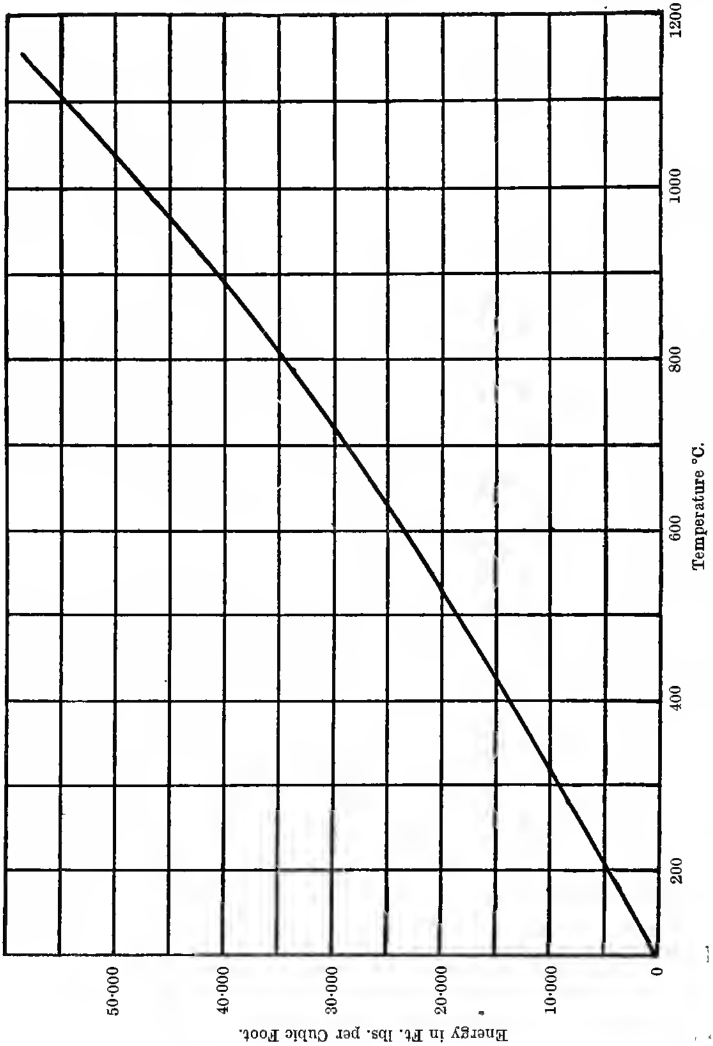

Fig. 7a.—The Internal Energy Curve for Gas Engine Mixture.

the heat of combustion, or otherwise, of a known quantity of heat, the final temperature can be obtained from the curve.

In Table III are given some of the values for the ratio of the specific heats at different temperatures for the gases mentioned therein.

It will be observed that γ decreases with rise of temperature in each case.

Apparently the rate of increase in the specific heat of air is constant, but in the case of steam this quantity increases as the temperature increases.

It should, however, be mentioned that the results of modern experiments upon the variation of specific heats are not altogether concordant.

It is now the usual practice to define the specific heat of a gas as the quantity of heat (expressed in foot pounds) necessary to raise 1 cubic foot of the gas, initially at 0° C. and 760 mm. pressure, through 1° C.

The Volumetric Heat mentioned previously is convertible into the new units by the relation :—

Specific heat in foot pounds per cub. foot $= 0{\cdot}252$ "volumetric heat."

If the specific heat of a gas in lb. ° C. units be represented by the letter s and 1 cubic foot weighs w lbs.

then *new specific heat* in terms of foot pounds per cub. foot

$$= w.s.\ 1400.$$

Effect of Variability of Specific Heat upon Calculations.

In several of the equations already obtained, and in which the term γ (for the ratio of the specific heats) appears, it was assumed that γ was constant ; but as the remarks in the preceding paragraph show, neither of the specific heats is really constant, but is a function of the temperature.

$$\text{Thus } C_v = \varphi(T)$$
$$\text{and } C_p = f(T)$$

And these expressions must be considered in calculations involving C_v and C_p.

However, for all ordinary purposes the specific heats may be taken as being linear functions of the temperature, and can be written as :—

$$C_p = \alpha + kT$$
$$\text{and } C_v = \beta + kT$$
$$\text{So that } C_p - C_v = \alpha - \beta = R$$
$$\text{and } \gamma = \frac{C_p}{C_v} = \frac{\alpha + kT}{\beta + kT}$$

Also when $T = O$ $C_p = \alpha$ and $C_v = \beta$, so that $\gamma_o = \dfrac{d}{\beta}$

As an illustration of the effect of variation in the specific heats, consider the results obtained in Equation (18).

Here we have $dH = C_v.dT + P.dV$

$$\text{and } \frac{dH}{dV} = \frac{C_v}{R}.\left(P + V.\frac{dP}{dV}\right) + P$$

But $C_v = \beta + kT$ and $R = \alpha - \beta$

$$\text{Hence } \frac{dH}{dV} = p\left\{1 + \frac{\beta + kT}{\alpha - \beta}\right\} + V.\frac{dP}{dV}\left(\frac{\beta + sT}{\alpha - \beta}\right)$$

$$= \frac{1}{\alpha - \beta}\left\{\alpha P + kT.p + kT\frac{dP}{dV} + \beta V.\frac{dP}{dV}\right\}$$

$$\underline{= \frac{kT}{\alpha - \beta}\left(P + V.\frac{dP}{dV}\right) + \frac{1}{\gamma_o - 1}\left\{\gamma_o P + V.\frac{dP}{dV}\right\}} \quad . (25)$$

This expression should be compared with the result of Equation (18), namely :—

$$\frac{dH}{dV} = \frac{1}{\gamma - 1}\left\{\gamma P + V.\frac{dP}{dV}\right\}$$

When the expansion follows the law PV^n = constant, we have $V.\frac{dP}{dV} = - nP$, and we can substitute this value in the expression for the rate of heat reception per unit volume.

$$\text{We then have } \frac{dH}{dV} = \frac{\gamma_o - n}{\gamma_o - 1}P + \frac{kT}{\alpha - \beta}P(1 - n)$$

$$= \left\{\frac{\gamma_o - n}{\gamma_o - 1} - \frac{n - 1}{\alpha - \beta}kT\right\}P$$

And integrating this expression in terms of V we have

$$H = \frac{\gamma_o - n}{\gamma_o - 1}\left\{\frac{K}{1 - n}(V_2^{1-n} - V_1^{1-n}) - \frac{n - 1}{\alpha - \beta}.k\int_{V_1}^{V_2} T.P.dV\right.$$

Hence, knowing T in terms of P or V, the second expression can be integrated and a definite result obtained.

As a further example of the application of these principles, consider the new adiabatic equation with variable specific heats.

$$\text{From Equation (9) we have } dH = \frac{1}{R}\left\{C_v.V.dP + C_p.P.dV\right\}$$

Since along an adiabatic $H = O$,

$$\text{then } C_v.V.dP = - C_p.P.dV$$

$$\text{or } (\beta + kT).VdP = - (\alpha + kT).P.dV,$$

$$\text{which can be written } V.\frac{dP}{dV} + \frac{\alpha + kT}{\beta + kT}.P = O$$

The solution of this differential equation is

$$P\beta_o V\alpha_o . \varepsilon^{k.T} = \text{constant}$$

where $C_p = \alpha_o + kT^1$ and $C_v = \beta_o + kT^1$.

Now when $T^1 = 273$ we have $\alpha_o = \alpha - 273k$

$$\beta_o = \beta - 273k$$

So that the adiabatic law becomes

$P^{(\beta - 273k)}\ V^{(\alpha 273k)}\ \epsilon^{kT} =$ constant, where $\varepsilon = 2{\cdot}71828$ the Napierian base.

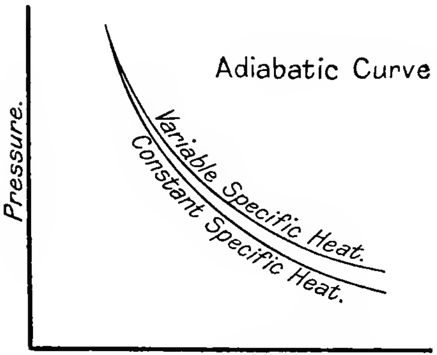

FIG. 8.

The effect of the variation of specific heats upon the adiabatic curve is to make it less steep than when the law $PV^\gamma =$ constant is followed, as shown in Fig. 8.

Other adaptations of formulae, given for cases involving constant specific heats to the cases in which the specific heats vary, will no doubt suggest themselves to the reader.

Variation of Specific Heats with Pressure and Volume.

To those wishing to study the subject of specific heat variation with pressure and volume, the following results should prove useful.

The total heat F of a gas is represented by the sum of its internal energy E and the work done in expansion P.V, that is :

$$F = (E + P.V)$$

Upon differentiating

$$\begin{aligned} dF &= d(E + PV) \\ &= dE + P.dV + VdP \\ &= dH + V.dP \\ &= C_p.dT - \left\{T.\left(\frac{dV}{dT}\right) - V.\right\}dP \quad . \quad . \ (26) \end{aligned}$$

This is an expression for *the change of total heat,* and in the above form all the terms are commensurate.

Again, when a quantity of gas changes its state from A to B, as represented on the PV diagram, the total heat F is independent of the form of the path from A to B, which, put in other words, means that $d(E + pv)$ is an exact differential.

On investigation of Equation (25) we have $\left(\frac{dF}{dT}\right)_p = C_p$ . . . (*a*)

$$\text{and } \left(\frac{dF}{dP}\right)_T = V - T.\left(\frac{dV}{dT}\right)_p \quad (b)$$

$$\text{For } dF = \left(\frac{dF}{dT}\right)_p dT + \left(\frac{dF}{dP}\right)_T dP$$

Also, since the order of differentiation is indifferent

$$\text{From } (a) \left(\frac{dC_p}{dP}\right)_T = \left(\frac{d^2F}{dT.dP}\right) = \left(\frac{dV}{dT}\right)_p - \left(\frac{dV}{dT}\right)_p - \left(\frac{d^2V}{dT^2}\right)_p T$$

$$\text{That is } \left(\frac{dC_p}{dP}\right)_T = -T.\left(\frac{d^2V}{dT^2}\right)_p \quad . \quad . \quad . \quad . \quad (27)$$

This is an expression for the variation of C_p with the pressure, at constant temperature.

We have similarly $\left(\frac{dC_v}{dV}\right) = T.\left(\frac{d^2P)}{dT^2}\right)_V$ for the specific heat at constant volume.

The Characteristic Equation of a Gas.

Experiments* made upon gases under widely varying conditions of temperature, pressure and volume, conclusively prove that no gas obeys either Boyle's or Charles's law exactly.

The deviations of the physical properties of the real gas (or substance) from the ideal state may be conveniently expressed by means of an equation, in terms of the pressure, volume and temperature.

The equation expressing the relation between these quantities for the actual gas or substance is known as the "Characteristic Equation" of the gas or substance.

From the characteristic equation of any gas the expressions for the quantities considered in the preceding pages can be readily obtained.

As an example of this, it would be an instructive exercise to assume the modified form of Boyle's and Charles's law to be

$$V - b = \frac{RT}{P} - c \text{ where } C = C_o' \left(\frac{T_o}{T}\right)^n \text{ and } C_p.Q = (n + 1)c - b.$$

This form takes into account the discrepancies existing between the actual and the ideal states of a gas; the relation $C_p = C_{po} + \frac{n(n + 1)cP}{T}$ is then readily obtained by the preceding methods.

---

* *Vide* researches of Regnault, Amagat, Andrews and Dewar.

Many other interesting results can be obtained by the use of the above characteristic equation, such as the ratio of the specific heats, the intrinsic energy, etc.

Calculation of the Gas Constant.

In the equation $P.V = R.T$ it is frequently necessary to know the value of the constant R for calculation purposes, and hence it will be interesting here to consider the meaning of this constant.

From the above equation we have

$$P.dV = R.dT \text{ at constant pressure}$$

$$\text{and } R = \frac{P.dV}{dT} = \frac{\text{Work done}}{\text{Temperature change}}$$

from which it follows that we can define R as being the work done when unit mass of gas is heated through 1° change of temperature at constant pressure.

Now $PV = P_0V_0(1 + aT^1)$ from Boyle's and Charles's laws for any scale of temperature; on the absolute scale,

$$\text{where } T = \frac{1}{a} + T^1, \text{ and also since } P.V = R.T$$

$$\text{then } \underline{R = P_0V_0a}$$

Next consider two gases with densities ϱ and ϱ^1 and let m, m^1 be their respective molecular weights, then we know from *Avagadro's law* that $\varrho^1 m = \varrho m^1$ or $\frac{m}{\varrho} = \frac{m^1}{\varrho}$ (for the product of the molecular weight by the volume is constant for all gases).

Hence $P.V = P_0.V_0.a.T = a.P_0(mV_0).\frac{T}{m} = X.\frac{T}{m}$ where $X = P_0(V_0\,m)a$. This value X is called the "*absolute gas constant.*"

Knowing X and the molecular weight for any gas we can calculate R. Thus $R = \frac{P.V}{T} = \frac{X.T}{T\,m} = \frac{X}{m}$

As an example, the calculation of R for hydrogen works out as follows :—

$$X = a.P_0V_0.m = a.P_0.\frac{m}{\varrho_0}$$

For hydrogen the density of 0° C. is ·0000899 and P_0 or the atmospheric pressure at 0° C. = 1·0333 kilograms per sq. cm.

= 10·13600 dynes per sq. cm.

multiplying by $g = 981$ in C.G.S. units.

Substituting in $X = a.P_o.V_o.m = a.P_o \frac{m}{\varrho_o}$

$$X = \frac{1}{273}.\frac{1013600}{\cdot 0000899} = 826 \times 10^5 \text{ dynes per sq. cm.}$$

This result can be expressed in various ways, but generally for practical purposes we require to know X in lbs. per square foot ° C. So that $X = 8{\cdot}42 \times 10^5$ kilog. per sq. cm.

= 29800 lbs. per sq. inch (° C. and cubic inch units)
= 2772 lbs. per sq. foot (° C. and cubic feet units).

For the purposes of calculation it is convenient to remember that the pressure 1 atmosphere = 2116 lbs. per sq. foot, and that 1 cub. foot of air weighs ·0807 lbs. at 0° C. and ordinary atmospheric pressure.

Calculation of Gas Constant for Mixtures of Gases.

In calculations referring to internal combustion engines, we have to deal with mixtures of gases instead of a single gas, and it is therefore necessary to know the value of the gas constant in such cases.

Let masses M_1 and M_2 of gases be at respective pressures P_1 and P_2 when each occupy volume V.

$$\left.\begin{array}{r}\text{Then } P_1V = M_1R_1T\\ \text{and } P_2V = M_2R_2T\end{array}\right\}\text{the temperature being assumed constant.}$$

Let the gases mix, and occupy volume V at a final pressure P

$$\text{Then } V(P_1 + P_2) = T(M_1R_1 + M_2R_2) = V.P.$$

Now if v = the volume of unit mass of the mixture or "the specific volume of the mixture"

$$v = \frac{V}{M_1 + M_2} = \frac{M_1R_1 + M_2R_2}{M_1 + M_2} \cdot \frac{T}{P}$$

$$\text{Hence } P.v = \frac{M_1R_1 + M_2R_2}{M_1 + M_2}.T = R^1.T$$

So that R^1 the new gas constant $= \frac{M_1R_1 + M_2R_2}{M_1 + M_2}$

For a mixture of several gases,

$$\text{New gas constant } R = \frac{\Sigma M.R.}{\Sigma M}$$

Example.—To find the value of the gas constant for air. Air consists of 76·44 per cent. by weight of nitrogen and 23·56 per cent. of oxygen and $R_{\text{oxygen}} = 48{\cdot}6$

$$R_{\text{nitrogen}} = 55{\cdot}1$$

Hence $R_{\text{air}} = \frac{76{\cdot}44 \times 55{\cdot}1 + 23{\cdot}56 \times 48{\cdot}6}{100} = 53{\cdot}57$ foot-pounds per 1° Fah.

Specific Heats of a Gaseous Mixture.

It occasionally happens that the specific heats of a gaseous mixture is required for calculation purposes. If the percentage composition of the mixture be known, and the specific heats of the constituents, then it is a simple matter to find the required specific heats.

If M_1, M_2, M_3, etc., be the weights of the constituent gases, and ${}_1C_p$, ${}_2C_p$, ${}_3C_p$, etc., be their respective specific heats at constant pressure, then the specific heat C_p of the mixture is given by

$$C_p = \frac{m_1 . {}_1C_p + m_2 . {}_2C_{p\,2} + m_3 . {}_3C_p + \text{etc.}}{m_1 + m_2 + m_3 + \text{etc.}}$$

$$= \frac{\Sigma m . C_p}{\Sigma m}$$

Similarly $C_v = \dfrac{\Sigma m C_v}{\Sigma m}$ for the specific heat of the mixture at constant volume.

If the composition of the gases in an internal combustion engine be known, at any moment, and the specific heats at the given temperature of the gases, the above principle may be applied to find the specific heat of the mixture at the given moment and temperature. It must be remembered that the specific heats will vary with the temperature and composition of the gases.

As an example, consider the products of combustion in an internal combustion engine to consist as follows :—

$CO_2 = 9{\cdot}4$ volumes.
$N = 75{\cdot}0$,,
(steam) $H_2O = 15{\cdot}6$,,

Multiplying by the densities, in order to obtain the weights :—

$CO_2 = 9{\cdot}4 \times {\cdot}1238 = 1{\cdot}162$
$N = 75 \times {\cdot}0786 = 5{\cdot}900$
$H_2O = 15{\cdot}6 \times {\cdot}0502 = {\cdot}785$

The specific heats at constant volume expressed in foot-pounds per pound ° C. are : $CO_2 = 239$, $N = 242$ and steam$=511$.

Hence the specific heat of the products of combustion will be given by

$$C_v = \frac{1{\cdot}162 \times 239 + 5{\cdot}900 \times 242 + {\cdot}785 \times 511}{1{\cdot}162 + 5{\cdot}900 + {\cdot}785}$$

$$= \frac{2106}{7{\cdot}847} = 256 \text{ foot-pounds per lb. } ^\circ \text{ C.}$$

Similarly the value of C_p for the products can be estimated.

The values of the specific heats must be taken *at* the actual temperatures of the gases for correct calculation purposes.

Gas Constants.

In the application of the preceding thermodynamical results the proper units and symbols must be utilized in order to obtain the correct results in practice, so a few remarks will here be made in regard to some of the physical constants of the gases.

It not infrequently happens that in thermodynamic equations both heat and work units occur, so that for an intelligent interpretation of the results either the heat units must be multiplied by Joule's Equivalent (J = 778 Fah. units), or the work units divided by this quantity.

Thus in the general equation already treated

$$dH = C_v dT + P.dV \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This, expressed throughout in heat units, must be written

$$dH = C_v dT + \frac{1}{J}.P.dV$$

and so with other applications.

Again, in the treatment of the variation of specific heats it may have been noticed that several units were employed in which to express the specific heat, so that some further explanation may assist in rendering the meaning of these units clearer.

We know that $C_p - C_v = R$

$$\text{and } \frac{C_p}{C_v} = \gamma$$

$$\text{so that } C_p - C_v = C_v(\gamma - 1) = R = \frac{ap_o}{\varrho_o}$$

Where ϱ_o = density of the gas, p_o = initial pressure,

$$\text{that is } \varrho_o C_v(\gamma - 1) = ap_o$$

$$\text{or } C_v = \frac{ap_o}{\varrho_o(\gamma - 1)} \text{ and } C_p = \frac{\gamma.ap_o}{\varrho(\gamma - 1)}$$

Hence C_p and C_v can be calculated in any units by choosing a, p_o and ϱ in the same units.

As an example, suppose we wish to find the specific heat of the gases used in a gas engine in foot-pounds per cub. foot ° C. units.

In this case the ratio of specific heats $\gamma = 1{\cdot}2$.

Then C_v per unit volume $= \varrho(C_v$ per unit mass)

$$\text{And } \varrho.C_v = \frac{ap_o}{\gamma - 1} = \frac{1}{273} \times \frac{2116 \text{ lbs. per sq. foot}}{1{\cdot}2 - 1}$$

$$= \underline{38{\cdot}7 \text{ foot-pounds per cub. foot.}}$$

It often happens in gas calculations that we require the specific heats of unit volumes of gases. Let ϱ = density of the gas, or the weight per unit volume.

Then specific heat per unit volume $= \varrho.$(specific heat per unit mass).

$$\text{Hence } C_v{}^1 = \varrho C_v$$
$$C_p{}^1 = \varrho C_v$$

where $C_v{}^1$ and $C_p{}^1$ are the specific heats of unit volumes.

Now if the molecular weight of a gas be *m*, the volume of *m* grammes of the gas is sometimes taken as the unit. This unit is called the gramme molecule, and is very nearly constant for all gases, as would be expected from Avagadro's law.

In Table IV the value of the gramme molecule for different gases is given.

TABLE IV

Gas.	Volume in Litres of the Gramme Molecule.
H_2	22·24
N_2	22·28
O_2	22·22
CO	22·21
CO_2	22·08

From this table it will be seen that the volume of the gramme molecule is nearly constant.

As was mentioned earlier, when the specific heat of the gramme molecule is taken, it is called the "Volumetric Heat."

If the specific heat in foot-pounds per cub. foot be the unit, and it is required to convert into calories per gramme molecule, we have the relation :—

Calories per gramme molecule = 3·96 specific heat in foot-pounds per cub. foot.

The gramme-molecule units have been suggested as the ideal units in the Gaseous Explosions Committee's Report, 1908.

As an example of the use of these units, the value of the gas constant R will be found in terms of the gramme-molecule units.

$$R = C_v(\gamma - 1) = \frac{ap_0V_0}{J}. \quad \text{[See page 24.]}$$
$$= \frac{1013600 \times 22{\cdot}25 \times 1000}{273 \times 4{\cdot}19 \times 10^7}$$
$$= 1{\cdot}98 \text{ C.G.S. units in gramme molecules}$$

where $J = 4{\cdot}19 \times 10^7$ in C.G.S. units and $V_o = \dfrac{1}{\varrho_o}$

In foot-pounds per cub. foot units

$R = 1{\cdot}98 \times 3{\cdot}96 = 7{\cdot}75$, a value derived from

$$R = \frac{ap_o}{\varrho} = \frac{2116}{273} \text{ lbs. per sq. foot}$$
$$= 7{\cdot}75$$

For calculation purposes the following tables of Physical Constants of Gases have been compiled, and will be found useful in quickly expressing results in any of the usual units.

TABLE V

Name of Gas.	Density at 0° C. and 760 mm.						Specific Heat at Constant Volume.						
	Specific Gravity		Grams per c.c.	Cub. ft. per lb.	Lbs. per cub. ft.	Water =1	Calories per gm.-molecule =22·24 litres	Ergs. per gramme at 0° C.	Foot lbs. per lb.		Foot lbs. per cub. ft.		Ratio of Specific Heats γ
	Air=1	H=2							1° F.	1° C.	1° F.	1° C.	
Hydrogen. . .	0·0693	2	$\cdot 0_4 896$	178·3	$\cdot 0_2 559$	2·406	4·81	× 106 100·8	1,875	3,368	10·5	18·9	1·42
Oxygen . .	1·106	31·93	$\cdot 0_2 143$	11·17	·0895	0·155	4·95	6·49	121	217	10·8	19·4	1·40
Nitrogen . . .	0·971	28·05	$\cdot 0_2 126$	12·71	·0786	0·177	4·85	7·25	135	242	10·6	19·1	1·41
CO	0·955	27·93	$\cdot 0_2 125$	12·94	·0773	0·173	4·83	7·25	135	242	10·4	18·7	1·42
CO_2	1·529	43·90	$\cdot 0_2 197$	8·08	·1238	0·171*	7·51*	7·16*	133*	239*	16·5*	29·6*	1·27
Water (steam) .	0·622	17·96	$\cdot 0_2 805$	19·91	·0502	0·365	6·56	15·29	284	511	14·3	25·7	1·30
Air	1·000	28·88	$\cdot 0_2 129$	12·35	·0810	0·169	4·88	7·08	182	237	10·7	19·2	1·404

* At 100° C.

C_p can be deduced from right-hand table by the relation $C_p = \gamma C_v$.

TABLE VI

The Specific Heats of Gases

In the table below the specific heats of the ordinary gases are given, and are expressed in lb. ° C. units at normal pressure and temperature.

Name of Gas.	Specific Heat Water = 1.		R (= C_p—C_v)	$\frac{C_p}{C_v} = \gamma$
	At Constant Pressure.	At Constant Volume.		
Hydrogen	3·4090	2·4060	1·0030	1·417
Oxygen	0·2175	0·155	·062	1·401
Nitrogen	0·2438	0·173	·071	1·407
CO	0·245	0·173	·072	1·418
CO_2	0·217	0·171	·046	1·270
Air	0·2374	0·169	·068	1·405
Steam gas	0·475	0·365	·110	1·300
Marsh gas	0·593	0·467	·126	1·270
Ethylene	0·404	0·332	·072	1·217

Ideal Thermodynamical Cycles applicable to Internal Combustion Engines.

In practical heat engine cycles of operation for the purpose of comparing the actual working of the engine with that of other engines, it is necessary to have some ideal standard of comparison. In steam engine practice, the two usual cycles of operation used for comparison purposes are the Rankine (or Clausius) and the Carnot cycles.

It is possible to compare the working of a steam engine with the theoretical working on the above ideal cycles by knowing the temperature limits, pressures, etc., of the actual case, and to thus express the relative efficiency of the actual and the ideal performances.

In internal combustion engine practice a similar mode of comparison is adopted, but the ideal cycles are not usually the same as for steam-engine work.

There are three possible cycles of operation of the working gas in combustion engines, namely :—

(1) The Constant Temperature or the Carnot Cycle.

(2) The Constant Volume or the Otto Cycle.

(3) The Constant Pressure or " Diesel " Cycle.

These ideal cycles will be dealt with in order.

(1) The Carnot, or Constant Temperature Cycle.

This cycle of operations, performed upon a given mass of working substance, initially in a condition represented by the coordinates of the point D (Fig. 9) consist as follows :—

(*a*) Isothermal compression at constant temperature T_2 along DA according to the relation $P_D V_D = P_A . V_A = R . T_2$.

(*b*) Adiabatic compression along AB from volume V_A to volume V_B according to the relation $P_A V_A^\gamma = P_B V_B^\gamma$, the temperature increasing to T_1.

(*c*) Isothermal expansion along BC at constant temperature T_1 according to the relation $P_B V_B = P_C V_C = R . T_1$.

(*d*) Adiabatic expansion along CD to the initial temperature T_2 according to the relation $P_C V_C^\gamma = P_D V_D^\gamma$.

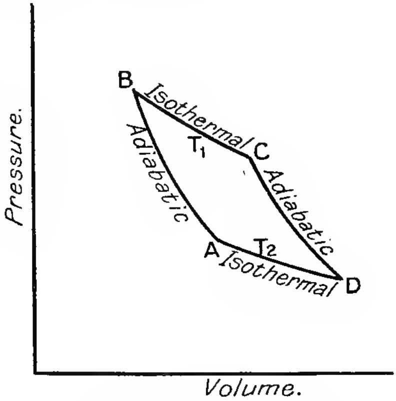

FIG. 9.—THE CARNOT CYCLE.

As an application of the principles of thermodynamics, and as a test of the suitability of the Carnot cycle to practical purposes, it will be interesting to study it in some detail.

Let V_A, V_B, etc., represent the volumes of the gaseous mixtures at A, B, etc., P_A, P_B, etc., the corresponding pressures, and T_A, T_B, etc., the corresponding temperatures.

Then for the adiabatic compression AB we have from Equation 12 :—

$$\frac{T_1}{T_2} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = \left(\frac{V_D}{V_C}\right)^{\gamma-1}$$

$$\text{so that } \frac{V_A}{V_B} = \frac{V_D}{V_C}$$

Now during this cycle, heat is only absorbed along the isothermal BC, and rejected during the isothermal compression line DA.

$$\text{That is, heat absorbed along BC} = H_1 = R . T_1 \log \frac{V_C}{V_B}$$

$$\text{And heat rejected along DA} = H_2 = R . T_2 \log \frac{V_D}{V_A}$$

Hence the Efficiency of the Cycle which is given by:—

$$\frac{\text{Heat absorbed} - \text{heat rejected}}{\text{Heat absorbed}} \text{ or } \frac{H_1 - H_2}{H_1}$$

$$= \frac{R.T_1 \log \frac{V_C}{V_B} - R.T_2 \log \frac{V_D}{V_A}}{R.T_1 \log \frac{V_C}{V_B}} = \frac{T_1 - T_2}{T^1} = 1 - \frac{T_2}{T_1} = 1 - \left(\frac{V_A}{V_B}\right)^{\gamma-1}$$

Let the "ratio of expansion" $r = \frac{V_D}{V_A} = \frac{V_C}{V_B}$

Then *the Efficiency of the Carnot Cycle* $= 1 - \left(\frac{1}{r}\right)^{\gamma-1}$. (28)

From these results it follows that the efficiency depends upon the initial and final temperatures (T_2 and T_1) of the gas, and further that the greater the expansion ratio $\frac{V_C}{V_B} = r$ the more efficient will the cycle become. If the gas could be expanded and compressed isothermally and indefinitely, the efficiency would be unity, for the expression $1 - \left(\frac{1}{r}\right)^{\gamma-1}$ becomes unity where $r = \infty$

The temperatures and pressures at the points A, B, C, and D can be calculated by methods already given, for example:—

the work done during the cycle $= H_1 - H_2 = R(T_1 - T_2) \log_e \left(\frac{1}{r}\right)$

$$\text{Also } \frac{T_1}{T_2} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = \left(\frac{P_B}{P_A}\right)^{\frac{1-\gamma}{\gamma}} = \left(\frac{P_D}{P_C}\right)^{\frac{1-\gamma}{\gamma}}$$

The practical employment of the Carnot Cycle is, however, impossible, as reference to Fig. 10 will show. This diagram, which is drawn to scale, represents an actual example worked out by Dugald Clerk * for the case of air working upon the Carnot or Constant Temperature Cycle, with the following data:—

Initial temperature before adiabatic compression	17° C. or 290° absolute
Initial pressure before adiabatic compression	14·7 lbs. per sq. inch
Isothermal temperature during expansion	806° absolute
Ratio of expansion r	12·24

---

* *The Gas, Petrol and Oil Engine,* Dugald Clerk.

It was found that the maximum pressure reached during adiabatic compression was 500 lbs. per sq. inch absolute, and the mean pressure was 6 lbs. per sq. inch.

$$\text{The efficiency E} = 1 - \left(\frac{1}{12{\cdot}24}\right)^{1-1{\cdot}404} = 0{\cdot}64$$

These results clearly indicate the impracticability of this type of cycle, for with a compression pressure of 500 lbs. per sq. inch, and a ratio of expansion of 12·24, the engine would have to be made exceedingly heavy and unduly long, whilst the diminutive power which would theoretically be developed is represented by the M.E.P. of 6 lbs. per sq. inch.

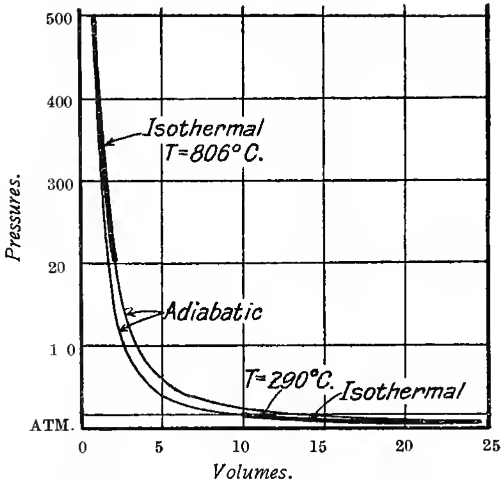

FIG. 10.—THE CARNOT CYCLE FOR AN ACTUAL CASE.

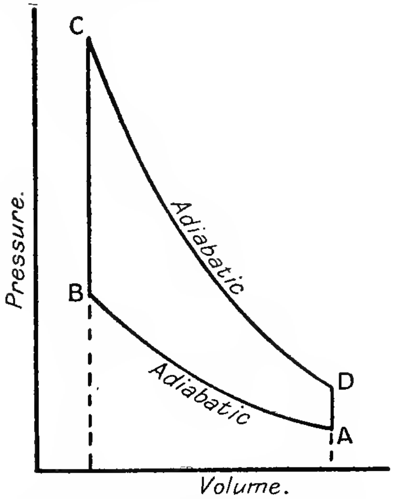

FIG. 11.—THE OTTO OR CONSTANT VOLUME CYCLE.

(2) The "Otto" or Constant Volume Cycle.

This cycle of working changes is represented in Fig. 11, and consists of:—

(*a*) Adiabatic compression of gas initially at A to B from volume V_A to V_B, according to the relation $P_A V_A^{\gamma} = P_B V_B^{\gamma}$.

(*b*) Addition of heat at constant volume V_B, and with rising temperature and pressure from B to C.

(*c*) Adiabatic expansion along CD to D, where $P_C V_C^{\gamma} = P_D V_D^{\gamma}$.

(*d*) Rejection of heat at constant volume from D to A, with falling temperature and pressure.

This is sensibly the cycle of operation employed in ordinary petrol, oil and gas engines, and hence a complete study of the changes occurring is important.

$$\text{We have here } \frac{V_A}{V_B} = \frac{V_D}{V_C} = r,$$

$$\text{Also for the adiabatic } \frac{P_B}{P_A} = \left(\frac{V_A}{V_B}\right)^{\gamma} = r^{\gamma}$$

$$\text{So that } P_B = P_A . r^{\gamma}$$

$$\text{Again we have for the adiabatic AB, } \frac{T_B}{T_A} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = r^{\gamma-1}$$

(from Equation 11).

$$\text{That is, } T_B = T_A . r^{\gamma-1}$$

Now the only heat added during the cycle is during the change BC, and the only heat rejected is along DA.

$$\text{Hence heat absorbed along BC} = H_1 = C_v(T_C - T_B)$$

$$\text{Hence heat rejected along DA} = H_2 = C_v(T_D - T_A)$$

So that the Efficiency

$$= \frac{H_1 - H_2}{H_1} = \frac{T_C - T_B - T_D + T_A}{T_C - T_B} = 1 - \frac{T_D - T_A}{T_C - T_B}$$

$$\text{But since } \frac{T_B}{T_A} = r^{\gamma-1} = \frac{T_C}{T_D}$$

$$\text{Then } \frac{T_C}{T_B} = \frac{T_D}{T_A} \text{ or } \frac{T_C - T_B}{T_D - T_A} = \frac{T_B}{T_A}$$

$$\text{So that the efficiency} = 1 - \frac{T_B}{T_A} = 1 - \underline{\left(\frac{1}{r}\right)^{\gamma-1}} \quad . \quad . \quad (29)$$

This expression is similar to that obtained in the case of the Carnot cycle.

Upon investigation it will be seen that the efficiency depends upon the temperature at the commencement and end of the compression strokes; expressed in another way, the efficiency depends upon the compression ratio r.

The results of tests upon internal combustion engines are now generally compared with the ideal performance of a quantity of air working upon the same Otto cycle, and under the same conditions of compression, etc.

The *Air Standard Efficiency*, as it is called, is given by

$E_{AIR} = 1 - \left(\frac{1}{r}\right)^{8}$, [since $(\gamma - 1)$ for air $= 0{\cdot}408$] and the results obtained by the use of this formula have been plotted in Fig. 12, showing the ideal efficiency E_{AIR} at different compression ratios.

A theoretical example of an Otto cycle, due to Dugald Clerk, shown in Fig. 13, represents an internal combustion engine working under the following physical conditions :—

Initial temperature before adiabatic compression. } = 17° C. or 290° absolute
Final temperature after adiabatic compression. } = 559° C. absolute

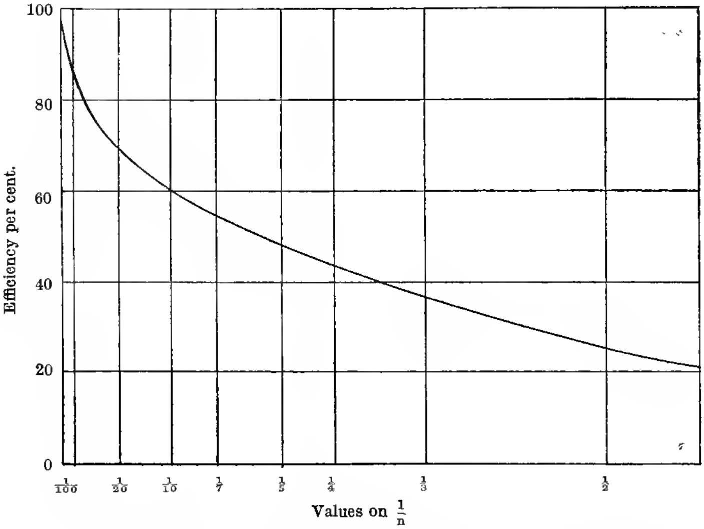

FIG. 12.—CURVE OF AIR STANDARD EFFICIENCIES FOR DIFFERENT COMPRESSIONS.

Temperature after explosion = 1973° C. absolute
Temperature at commencement of exhaust . . } = 1023° C.
The pressure of explosion = 500 lbs. per sq. inch absolute.
The value of r is 5.

The results of this cycle of operations are :—

Mean pressure	105 lbs. per sq. inch
Compression pressure	141·8 ,, ,,
Pressure at exhaust	51·8 ,, ,,

$$\text{Efficiency} = 1 - \left(\frac{1}{5}\right)^{0·408} = 0·48$$

In practical cases the expansion from the maximum pressure is not continued down to atmospheric as the small gain in the effective pressure (as represented by the increased area due to the "toe" of the diagram) is more than counteracted by the increased size and weight of the engine.

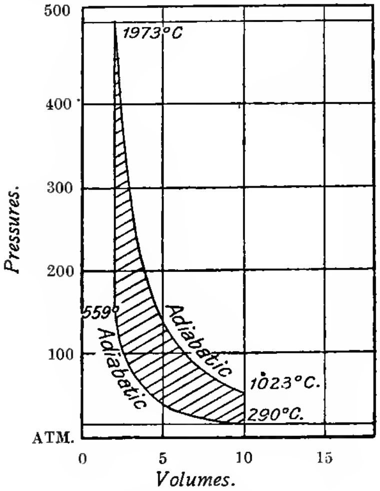

FIG. 13.—CONSTANT VOLUME CYCLE FOR AIR.

In a later chapter further reference will be made to actual utility of the Otto cycle of operations in practice, and also to its limitations.

(3) The Constant Pressure Cycle.

This cycle, which is approximately followed in Diesel engines, and which may eventually be applied to lighter motor-car engine practice, consists of two adiabatic operations and two constant pressure ones. The sequence of operations, as shown in Fig. 14, is :—

(*a*) Adiabatic compression along AB, here $P_A.V_A^\gamma = P_B.V_B^\gamma$.

(*b*) Expansion at constant pressure, with increasing temperature along BC.

(*c*) Adiabatic expansion along AC where $P_C.V_C^\gamma = P_D.V_D^\gamma$.

(*d*) Compression at constant pressure, with decreasing temperature along DA to initial volume at A.

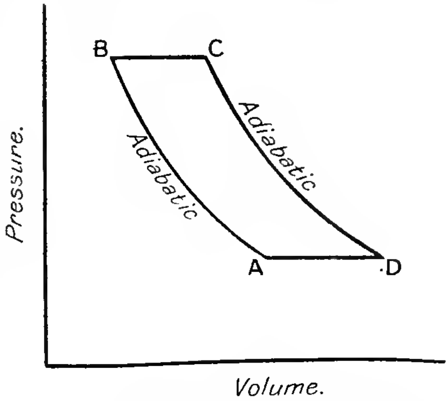

FIG. 14.—THE CONSTANT PRESSURE CYCLE.

In this case, as in the Otto cycle, we have :—

Heat absorbed along BC $= H_1 = C_P(T_C - T_B)$

Heat rejected along DA $= H_2 = C_P(T_D - T_A)$

$$\text{Whence Efficiency} = \frac{H_1 - H_2}{H_2} = \frac{T_C - T_B - T_D + T_A}{T_C - T_B} = 1 - \frac{T_A}{T_B}$$

$$= 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

$$\text{This follows since } \frac{T_C}{T_D} = \frac{T_A}{T_B} \text{ as before}$$

$$\text{and } \frac{T_A}{T_B} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = r^{\gamma-1}$$

It will be noticed that, as in the previous cases, the efficiency depends upon the temperatures before and after compression, that is, upon the compression ratio.

An example of this cycle, also due to Dugald Clerk, is shown in Fig. 15, with the following data and results :—

Temperature before adiabatic compression = 290° C. absolute
Temperature after adiabatic compression = 806° C. „
Temperature at end of expansion at constant pressure . . . . = 1973° C. „
Temperature at end of adiabatic expansion = 1023° C. „

The maximum pressure of adiabatic compression = 500 lbs. per sq. inch.

The pressure at end of adiabatic expansion = 51·8 lbs. per sq. inch.

Mean pressure during cycle = 117 lbs. per sq. inch.

The ratio of expansion = 5

The efficiency of cycle = 0·56.

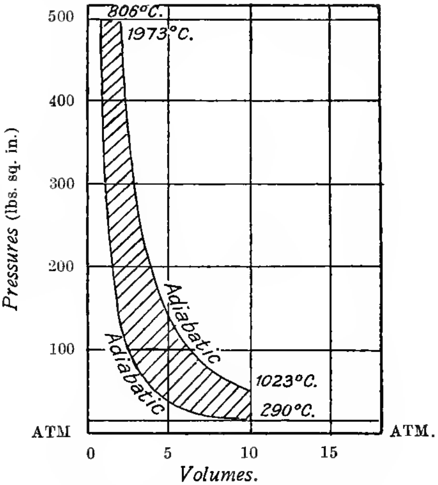

FIG. 15.—THE CONSTANT PRESSURE CYCLE FOR AIR.

In the cycle illustrated the expansion has not been continued down to atmospheric pressure, for the reasons assigned in the case of the Otto cycle ; the theoretical efficiency of the cycle would be 0·64 if the expansion were so continued.

The effect of cutting off the " toe " of the diagram is to give a greater mean effective pressure, and so more power for an equal engine weight. The actual M.E.P. when expansion takes place down to atmospheric pressure is 56 lbs. per sq. inch, as compared with the 117 lbs. given above.

This type of cycle is followed closely in practice, and the above results represent a possible case, with the exception, of course, that although the pressures and temperatures realized are approximately as stated, yet the efficiency is not so high. Actually, however, engines working upon this cycle show a much better efficiency, and therefore greater fuel economy, than when working upon any of the other cycles mentioned.

It is not outside the bounds of possibility to suppose that some more efficient cycle may be evolved in the future; but whatever form may be suggested, an analysis can be made on the foregoing lines and comparisons made.

Comparison of the Theoretical Cycles.

If we consider the three cycles previously dealt with in detail, the constant temperature, constant volume, and constant pressure cycles, it is evident from the similarity of the expressions for the efficiencies, namely $E = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$ that each of these cycles is capable of yielding efficiencies depending entirely upon the degree of adiabatic compression.

The constant temperature cycle, as already demonstrated, is practically impossible, as its application would necessitate exceedingly great expansion and very high compression pressure, and at the best would only yield low effective pressures for useful work.

Hence the two remaining cycles are those of chief interest to internal combustion engineers; and in regard to these, it must be remembered, as will be pointed out later in the practical application of these cycles, that the compression pressure employable is limited by the available maximum temperature. It is quite possible to attain the predetermined (and fixed) maximum temperature by adiabatic compression alone, in which case the conditions will be limited accordingly.

Remembering the limitations, the constant pressure cycle would be the appropriate one for internal combustion engines, but extended reference to this will be made later.

As a point of interest, a comparison of the constant volume and constant pressure cycles is given in Fig. 16.

The two cycles of operation as applied to the case of pure air obeying the ideal gas laws of Boyle and Charles, and for which

$\gamma = 1{\cdot}404$ and $E = 1 - \left(\frac{1}{r}\right)^{\cdot 404}$ were calculated for cases of equal compressions and for the same maximum temperatures.

The constant pressure cycle is shown shaded.

In the case of the "Otto" or constant volume cycle the compression ratio $r = 5$, and for the other cycle the compression ratio is the same.

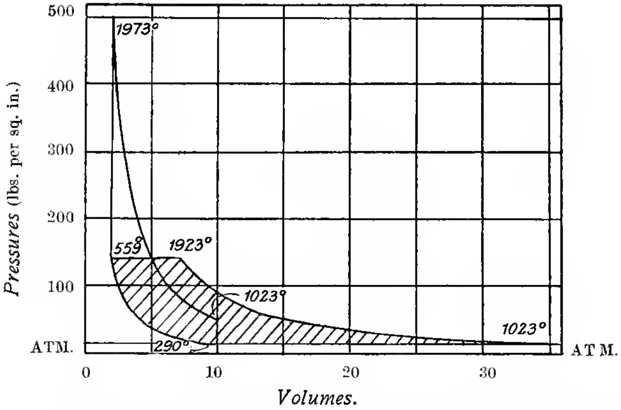

FIG. 16.—COMPARISON OF CONSTANT VOLUME AND CONSTANT PRESSURE CYCLES FOR AIR.

Table VII illustrates the results obtained.

TABLE VII

	Constant Pressure.	Constant Volume.
Compression pressure	142	142
Maximum pressure	142	500
Mean pressure	37	105
Terminal pressure	14·7	51·8
Efficiency	0·48	0·48
Maximum temperature	1,970° abs. C.	1,970° abs. C.
Initial temperature before adiabatic compression	290° ,,	290° ,,

It will be seen that in both cases $E = 1 - \left(\frac{1}{5}\right)^{0{\cdot}404} = 0{\cdot}48$ and that the compression pressures are the same.

In the constant volume cycle the ratio of maximum to mean pressure is 4·6, whereas in the constant pressure cycle this ratio is 3·8, so that in the latter case the range of stresses is smaller ;

at the same time it will be noticed that it requires a volume for its operation equal to three-and-a-half times the initial volume required in the case of the constant volume cycle.

The efficiencies being equal, the question as to which is the more suitable type of cycle resolves itself into one of practical convenience.

These two cycles are of paramount importance in practice, and each is adapted to its own peculiar uses ; for example, in one case the relatively greater fuel economy of the constant pressure cycle may be of primary importance, whereas weight and space may be of a secondary interest, and conversely for the other case.

In the foregoing treatment of the three possible cycles adaptable to internal combustion engines, it is necessary to emphasize the fact that the working gas in all cases has been pure dry air obeying the ideal conditions, and that the ratio of its specific heats has been taken as being of constant value 1·404.

Effect of Variable Specific Heats upon Standard Cycle Efficiencies.

The effect of the variability of the specific heats has been disregarded for the purposes of the comparison of the cycles, with a view to avoiding unnecessary complications in the treatment.

It is now of some interest to study the effect of the specific heat variation by comparing the ideal air standard cycle, for which the specific heats are assumed invariable, with the case in which the working gas (or gases) have specific heats which increase as the temperature increases, similar in fact to what actually occurs in practice in internal combustion engines.

In the ideal air standard cycle the efficiency, and therefore the work done, as represented by the area of the P.V. diagram, will depend upon the compression ratio, and this efficiency will not vary with the heat supplied per stroke.

In the variable specific heat case the efficiency will also vary with the compression ratio in nearly the same way, and in addition will vary with the heat supplied per stroke, but will be always less than the air standard efficiency.

Let H = heat supplied to an ideal engine (constant volume cycle)
C_v = mean specific heat (constant volume) between t and T_m
M = mass of gas
T = temperature at end of compression stroke
and T_m = maximum temperature attained

$$\text{Then } T_m = \frac{H}{M.C_v} + T \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Next consider the case when the specific heat follows a linear law of increase, with temperature, such as $C_v = C_o + b.T$.

For this purpose it is necessary to know the mean value of C_v between the initial temperature t and some other final temperature

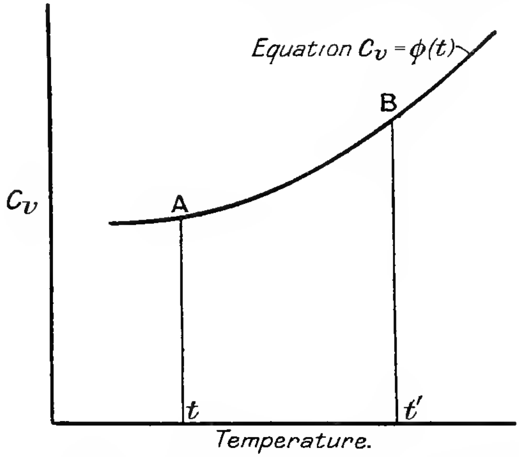

FIG. 17.—METHOD OF DETERMINING MEAN SPECIFIC HEATS.

t^1; this is done by taking the mean height of the curve (Fig. 17), the method employed being a general one.

$$\text{The mean height} = \frac{\Big[\text{area}\Big]_T^{T^1}}{T^1 - T}$$

$$= \frac{1}{T^1 - T}\int_T^{T^1} C_v dT$$

Substituting $C_o + bT$ for C_v we have :—

$$\text{Mean value } C_v = \frac{1}{T^1 - T}\int_T^{T^1} (C_o + bT)dT$$

$$= C_o + \frac{b}{2}(T^1 + T)$$

$$\left.\begin{array}{l}\text{Hence maximum temperature}\\ \text{attained during cycle}\end{array}\right\} = \frac{H}{M\left[C_o + \frac{b}{2}(T^1+T)\right]} + T \quad (31)$$

This temperature is evidently less than that attained under the conditions represented by Equation 30.

It will be seen, that since the maximum temperature attained before adiabatic expansion is lower in the second case, the work done during the cycle for a given heat supply H will be less, and hence the efficiency will be lower than in the ideal case.

Fig. 18 has been drawn to illustrate the difference in the theoretical diagrams between the air standard cycle and a variable specific heat cycle, working with the same compression ratios, volumes and masses of gas.

It will be noticed that the second cycle only attains a temperature of 2,000° C., whereas the ideal one reaches 3,070° C.

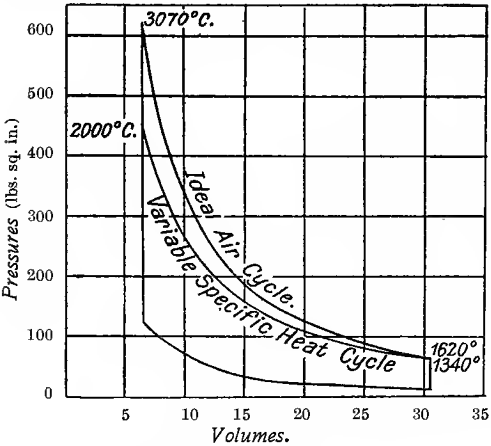

FIG. 18.—COMPARISON OF IDEAL CYCLES.

The following table also shows the difference between the two cycles and is based upon the results of actual calculations made upon the above assumptions.

TABLE VIII

Cycle.	Efficiency.	Mean Pressure.	Maximum Pressure.	Maximum Temperature.
Ideal air cycle	·463	153	620	3,070° C.
Ideal variable specific heat cycle	·356	118	447	2,000° C.

The above results can be arrived at mathematically by the method employed for deducing the expression for the efficiency of the constant volume cycle (p. 33).

Here we had (*a*) heat absorbed along BC, $H_1 = C_v(T_C - T_B)$
and (*b*) heat rejected along DA, $H_2 = C_v(T_D - T_A)$.

Instead of C_v, we must now write $C_o + \frac{b}{2}(T_C + T_B)$ in (*a*), and $C_o + \frac{b}{2}(T_D + T_B)$ in (*b*).

In terms of temperatures the new efficiency E_s will be

$$E_s = \frac{(T_C - T_B)\left[C_o + \frac{b}{2}(T_C + T_B)\right] - (T_D - T_A)\left[C_o + \frac{b}{2}(T_D + T_A)\right]}{(T_C - T_B)\left[C_o + \frac{b}{2}(T_C + T_B)\right]}$$

By combining this result with the new adiabatic law given on p. 5, the efficiency can be expressed in terms of T_C and T_B only, and becomes

$$E_s = E\left[1 - (1 - E).K.\frac{T_C + T_B}{2}\right]$$

where E is the efficiency of the ideal air cycle with constant specific heat, and K a constant depending upon the manner of variation of the specific heat with the temperature, that is upon the constants in the relation $C_v = C_o + b.T$.

In actual cases worked out upon the above assumptions it will be found that roughly speaking the efficiency decreases by about 1 per cent. when the specific heat increases by 2 per cent.

Entropy.

It is often useful and convenient to denote isothermal and adiabatic changes, not upon the pressure-volume diagram, (a method adopted in the present book up to this stage), but upon what is called a $T-\varphi$ or Temperature-Entropy* diagram. For pressure volume changes, however, the P.V diagram is generally the more convenient.

The Entropy* of a given quantity of any substance is name given to the ratio of the total heat of the substance divided by its absolute temperature.

Entropy is usually denoted by the symbol φ.†

$$\text{Thus } \varphi = \frac{H}{T}$$

If the substance receives or rejects heat it is said to alter its entropy ; thus if a small quantity of heat dH be given to or taken from a given quantity of a substance at an absolute temperature T, the small change of entropy is denoted by $d\varphi = \frac{dH}{T}$

The total entropy change for any thermodynamic operation is given by the integral between the previously determined temperature limits,

$$\text{Thus } \varphi = \int\frac{dH}{T}$$

---

* First instituted by Clausius.

† Rankine in *The Steam Engine* terms ϕ the "Thermodynamic Function."

It should be remembered in all entropy calculations that H represents the total heat of the substance ; thus in the case of a vapour, $H = h + l$, where h is the heat required to raise the temperature of the liquid to the evaporation point, and l is the latent heat of evaporation for the liquid.

In calculations it is convenient to have a datum from which to calculate the entropy, such as the freezing point of water or the absolute zero of temperature (– 273° C. or – 461° Fah.), and further it is usual to calculate the entropy per unit mass of the gas (or working substance).

The application of the notion of entropy to problems connected with the steam engine has proved very successful in solving in a very simple manner otherwise complicated problems, and students of steam engineering have become quite familiar with conceptions of entropy—and realize the importance thereof—but in internal combustion engine work, beyond a mere reference to entropy in the case of air-standard ideal cycles, its employment has hitherto been restricted.

In the case of an adiabatic change, heat is neither added to nor subtracted from the gas, so that $\delta H = 0$, and hence the entropy does not change during an adiabatic process.

The second law of thermodynamics (p. 13) can be re-written as $\int\frac{dH}{T} = 0$ for a completely reversible cycle, that is, the entropy as well as the pressure, volume and temperature of the working substance returns to its initial value after the completion of a cycle of changes.

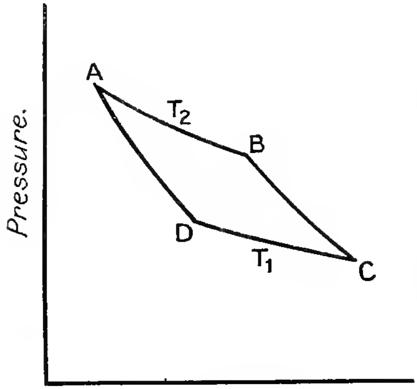

FIG. 19.

An example of a P.V diagram for the Carnot type of cycle ABCD, consisting of two adiabatics and two isothermals at T_1 and T_2 respectively, is given in Fig. 19. The conversion to a T.φ diagram requires the following process.

Let heat H_1 be the heat taken in during the isothermal expansion AB and H_2 the heat given out along CD.

Then $\frac{H_1}{T_1}$ = gain of entropy along AB and $\frac{H_2}{T_2}$ = loss of entropy along CD.

But from the second law of thermodynamics $\frac{H_1}{T_1} = \frac{H_2}{T_2}$, so that in passing from one adiabatic AD to another BC the entropy changes by the same amount.

Further, by making the steps small enough it will be seen that no matter what path is taken between two adiabatics the change of entropy will be the same.

Hence it is evident that not only is the entropy constant along any adiabatic, but also that there is a perfectly definite change in entropy between two adiabatics, no matter what the path may be.

Hence adiabatic lines are lines of uniform entropy—or "isentropics"—just as isothermal lines are lines of uniform temperature.

This property of adiabatics forms a useful aid in problems connected with heat engines generally.

Entropy Change for a given Change of State in a Gas.

For any change we have (Equation 18)

$$dH = C_v dT + P dV$$

$$= C_v dT + \frac{RT}{V} dV, \text{ since } PV = RT \quad (\text{Eq. } 3)$$

$$\text{that is } \frac{dH}{T} = C_v \frac{dT}{T} + R.\frac{dV}{V}$$

$$\text{But } \frac{dH}{T} = d\varphi = \text{entropy change}$$

$$\text{Integrating } \therefore \int_{T_1}^{T_2} \frac{dH}{T} = \int_{\varphi_1}^{\varphi_2} d\varphi = C_v \int_{T_1}^{T_2} \frac{dT}{T} + R. \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\text{That is } \varphi_2 - \varphi_1 = C_v \log_e\left(\frac{T_2}{T_1}\right) + R \log_e\left(\frac{V_2}{V_1}\right) \quad . \quad . \quad . \quad (32)$$

This expression gives the entropy change in terms of the temperature and volume for any given change of state.

The above equation can also be written

$$dH = C_p dT - \frac{RT}{P}.dP$$

$$\text{so that } \varphi_2 - \varphi_1 = C_p \log_e\left(\frac{T_2}{T_1}\right) - R \log_e\left(\frac{P_2}{P_1}\right) \quad . \quad . \quad . \quad (33)$$

In the case of a gas with a variable specific heat as in the case already discussed, in which $C_v = C_o + bT$, we should have

$$\varphi_2 - \varphi_1 = C_o \log_e\left(\frac{T_2}{T_1}\right) + b(T_2 - T_1) + R \log_e\left(\frac{P_2}{P_1}\right)$$

showing that the entropy change is greater in this case.

Example.—As an example of the use of these equations for the general change in entropy for a change of state, consider one pound of dry air initially at 32° F. and at atmospheric pressure to be compressed from its initial volume (12·4 cub. feet) to 3 cub. feet, its temperature being thereby raised to 439° F., given that $R = 53{\cdot}15$ (mech. units).

Initially $P = 14{\cdot}7$ lbs. per sq. inch $= 2{,}116$ lbs. per sq. foot

$$\text{Finally } \frac{P.3}{900} = \frac{2116 \times 12{\cdot}4}{493} = R$$

from which $p = 15{,}950$ lbs. per sq. foot

$$\text{Now } R = \frac{53{\cdot}15}{778} = {\cdot}063$$

$$\text{Hence } \varphi_2 - \varphi_1 = C_v \log_e\left(\frac{T_2}{T_1}\right) + R \log_e\left(\frac{V_2}{V_1}\right)$$

$$= {\cdot}1748 \log_e\frac{900}{493} + {\cdot}063 \log_e \frac{3}{12{\cdot}4}$$

$$= {\cdot}0043$$

The values for C_p and C_v are obtained by using

$$\left.\begin{aligned} \frac{C_p}{C_v} &= \gamma = 1{\cdot}404 \\ \text{and } C_p - C_v &= {\cdot}063 \end{aligned}\right\}$$

$$\text{so that } C_p = {\cdot}2375$$

$$\text{and } C_v = {\cdot}1748$$

This result can also be obtained by using the alternate form

$$\varphi_2 - \varphi_1 = C_p \log_e\left(\frac{T_2}{T_1}\right) - R \log_e\left(\frac{P_1}{P_2}\right) \qquad \text{(Equation 33)}$$

$$= {\cdot}2375 \log_e\frac{900}{493} - {\cdot}063 \log_e\frac{15950}{2116}$$

$$= {\cdot}0043$$

The Entropy Temperature Diagram.

It is now proposed to show how the change from the P.V to the equivalent T.φ diagram for any gas can be made.

Consider a given mass of gas, to change in any manner from A to B on the temperature entropy diagram.

Since for a small change $d\varphi = \frac{dH}{T}$

$$\text{Then } \int dH = H = \int^{b} T.d\varphi$$

Hence the heat taken in or required for a given change AB is represented by the area of the Tφ diagram under AB (Fig. 20).

Consider an isothermal line AB on the P.V diagram, this will be represented on the Tφ diagram by a line A^1B^1 at temperature T_1 parallel to the φ axis (Figs. 21 and 22).

Again, an adiabatic curve CD will be represented on the Tφ diagram by a straight line C^1D^1 parallel to the T axis and at a distance from it by an amount equal to the constant entropy of the adiabatic CD; similarly, a constant volume line MN on the PV diagram will now become a curved line M^1N^1 on the Tφ diagram (Figs. 23 and 24), and similarly for the constant pressure lines PQ and P^1Q^1.

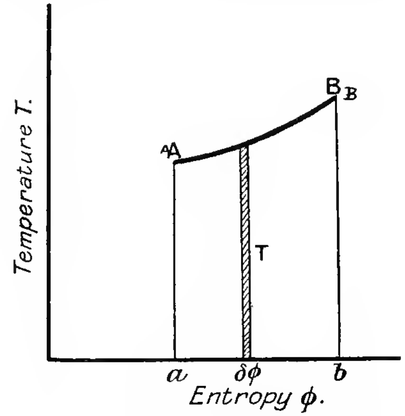

Fig. 20.—The Entropy Temperature Diagram.

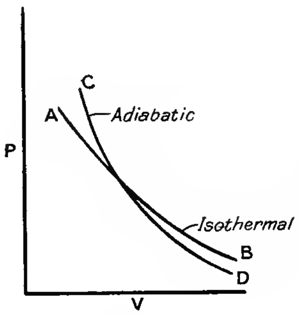

Fig. 21.

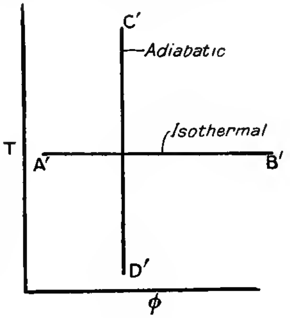

Fig. 22.

From an examination of these diagrams it will be evident why the Tφ diagram is better adapted to the representation of isothermal and adiabatic changes, and the PV diagram to pressure and volume changes.

It will be also noticed, in the same way, that an adiabatic has a greater slope than an isothermal curve in a PV diagram, so a constant pressure line generally has a greater slope than a constant volume line in the Tφ diagram.

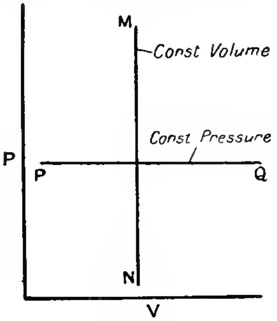

FIG. 23.

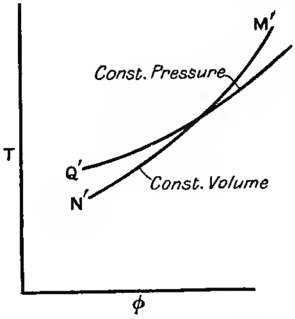

FIG. 24.

Representation of Internal Energy on the Tφ Diagram.

When a gas is heated at constant volume from a temperature T_1 to T_2 (absolute) no external work is done, so that the heat taken in must therefore be equal to the change of the internal energy

$$\text{for } dH = dW + dE, \text{ and by hypothesis } dW = 0$$

$$\text{So that } dH = dE$$

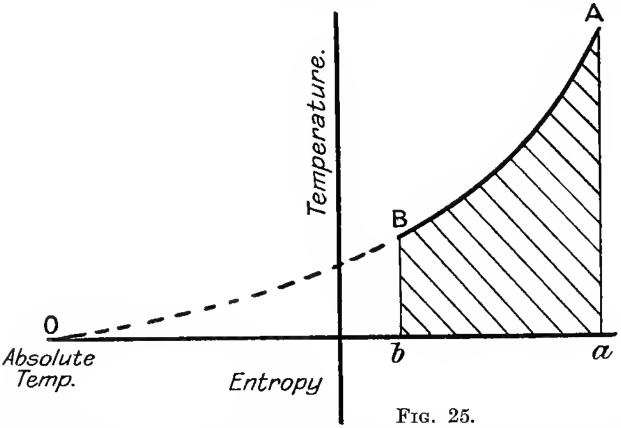

FIG. 25.

If now on the Tφ diagram (Fig. 25), in which OBA is the constant volume line, the gas changes from condition A to B.

$$\text{Then}\ \Big[\text{change of internal energy}\Big]_B^A = \text{heat supplied} = H$$

$$= \int_b^a T.d\varphi$$

$$= \text{Area } ABba$$

$$= \text{Area } AOa - \text{area } BOb$$

The internal energy at A, reckoned from absolute zero temperature, is represented by the area A0*a*, and the internal energy at B, reckoned from absolute zero temperature, is represented by the area BO*b*.

Thus the area subtended by the constant volume curve through a given point on the $T\varphi$ curve represents the internal energy of a gas defined by the co-ordinates of that point.

Change of Entropy for Expansion Law PV^n = constant.

Let $P.V^n$ = constant.

It has already been shown that from the above equation the new relation is obtained :—

$$\left(\frac{V}{V_o}\right)^{n-1} = \frac{T}{T_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

We can substitute the value for $\frac{V}{V_o}$ in terms of $\frac{T}{T_o}$ in the Entropy

$$\text{Equation } \varphi - \varphi_o = C_v \log_e\left(\frac{T}{T_o}\right) + R \log_e\left(\frac{V}{V_o}\right) \quad . \quad . \quad . \quad (32)$$

$$\text{or } \varphi - \varphi_o = C_v \log_e \frac{T}{T_o} + \frac{R}{1-n} \log_e \frac{T}{T_o}$$

$$= \left(C_v - \frac{R}{n-1}\right) \log_e \frac{T}{T_o}$$

And since $C_v.(\gamma - 1) = R$ this becomes

$$\varphi - \varphi_o = C_v \log\left(\frac{T}{T_o}\right)\left(\frac{n-\gamma}{n-1}\right) \quad . \quad . \quad . \quad (34)$$

Entropy Temperature Diagrams for Standard Cycles.

(1) *The Carnot Cycle.*

This cycle, as previously pointed out, consists of two adiabatic and two isothermal changes, as represented on the PV diagram by the area ABCD.

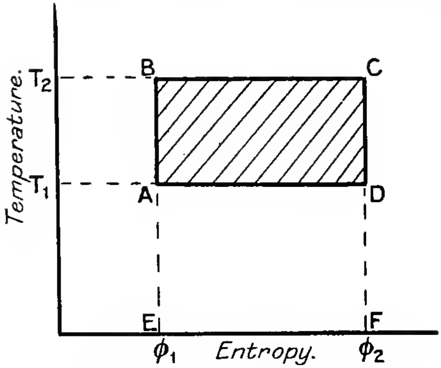

FIG. 26.—ENTROPY TEMPERATURE DIAGRAM FOR CARNOT CYCLE.

Now if these changes are considered upon the entropy - temperature diagram (Fig. 26), the adiabatic lines AB and DC will be straight lines of constant entropy parallel to the temperature axis; the isothermal lines BC and AD will be straight lines of constant temperature parallel to the entropy axis.

Also the area of the $T\varphi$ diagram, namely, ABCD, represents the work done during the cycle.

The heat taken in is represented by the area EBCF.

$$\text{The efficiency of the cycle} = \frac{\text{Work done}}{\text{Heat taken in}}$$

$$= \frac{\text{Area ABCD}}{\text{Area EBCD}}$$

$$= \frac{(T_2 - T_1)(\varphi_2 - \varphi_1)}{T_2(\varphi_2 - \varphi_1)}$$

$$= \frac{T_2 - T_1}{T_2} = 1 - \left(\frac{T_1}{T_2}\right)$$

This is the same result as obtained previously, and is obviously arrived at in an easier manner by the use of $T - \varphi$ diagram.

(2) *The " Otto " or Constant Volume Cycle.*

This cycle on the P.V diagram consists of two constant volume changes AD and BC, and two adiabatic changes AB and CD. Referring the same changes to the $T\text{-}\varphi$ diagram (Fig. 27), the two constant volume lines AD and BC are now represented by the curved lines

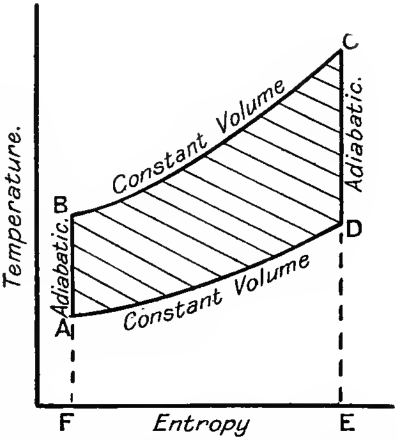

FIG. 27.—THE ENTROPY TEMPERATURE DIAGRAM FOR THE CONSTANT VOLUME CYCLE.

in similar notation, and, as before, the adiabatics by straight lines parallel to the axis.

(3) *The Constant Pressure Cycle.*

Here we have two constant pressure changes AD and BC represented upon the Tφ diagram by the curved lines of constant pressure AD and BC (Fig. 28), the adiabatics CD and BA being represented by the straight lines CD and BA.

The proof of the method for obtaining the efficiency of the cycle is exactly the same as in the case treated,* for the same cycle referred to the PV diagram, and it is useful to note that the area on the Tφ diagram, ABCD, represents, in suitable units, the useful work done during the cycle, and that the ratio of the areas ABCD and BCEF represents the efficiency of the cycle:

For the heat absorbed
is represented by
area BCEF
and the heat rejected
is represented by
area DEFA.

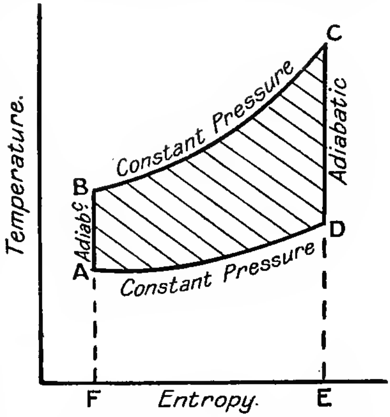

FIG. 28.—THE ENTROPY TEMPERATURE DIAGRAM FOR THE CONSTANT PRESSURE CYCLE.

In drawing the entropy temperature diagram for any given cycle of operations, it should be remembered that temperatures are reckoned on the absolute scale, and that entropies are usually calculated from some datum such as the absolute zero of temperature. That is to say, at −273° C. or −461° F. the entropy of 1 lb. of the gas under consideration is taken as zero.

In the entropy equation

$$\varphi_2 - \varphi_1 = C_v \log_e\left(\frac{T_2}{T_1}\right) + R \log_e\left(\frac{V_2}{V_1}\right) \quad . \quad . \quad . \quad (32)$$

If $\varphi_1 = -273°$ C. or $-461°$ F., then $V_1 = 0$.

And by substitution $\varphi_2 = C_v.\log_e.T_2 + R \log_e V$ . . . (35)
which is the entropy at any temperature T_2 and volume V_2.

It will be noticed that the hyperbolic logarithm is used in these expressions; for ordinary purposes of calculation the relation hyperbolic logarithm = 2·3026 logarithm to base 10 will enable the ordinary tables of logarithms to be used.

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* Vide p. 36.

Practical Methods of converting Indicator Diagrams to Tφ Scale.

It very frequently happens in internal combustion engine calculations that the changes which occur within the cylinder can be much better analysed and studied from the temperature entropy diagram, so that as the ordinary "indicator" diagram representing the cycle of changes is drawn in the form of a PV diagram, it is necessary to have some means for converting this latter diagram into a Tφ one.

There are two useful methods of performing this conversion, one being mathematical and the other graphical.

We have already obtained expressions for the entropy in terms of pressures and temperatures, and volumes and temperatures,

$$\text{thus } \varphi_2 - \varphi_1 = C_p \log_e\left(\frac{T_2}{T_1}\right) - R \log_e\left(\frac{P_2}{P_1}\right) \quad . \quad . \quad . \quad . \quad (33)$$

$$\text{and } \varphi_2 - \varphi_1 = C_v \log_e\left(\frac{T_2}{T_1}\right) + R \log_e\left(\frac{V_2}{V_1}\right) \quad . \quad . \quad . \quad . \quad (32)$$

If now we reckon entropy from the absolute zero of temperature the entropy at temperature T, volume V, and pressure P is given by

$$\varphi_T = C_v \log_e T + R \log_e V$$
$$\text{and } \varphi_T = C_v \log_e T - R \log_e P.$$

Since we can express the temperature as a function of the volume or pressure for a given change, it is an easy matter to plot φ_T and log V or log P upon squared paper, and for any future calculation to find instantly the entropy for any given pressure or volume satisfying the above conditions; the use of semi-logarithmic squared paper is convenient for this purpose.

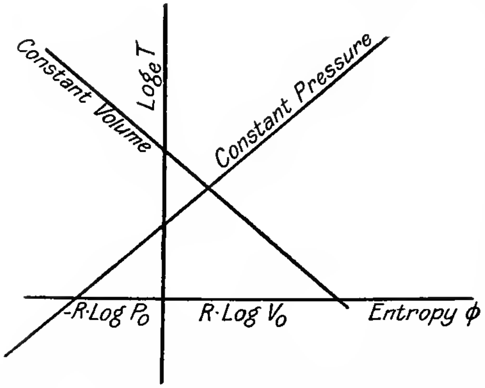

Fig. 29.—Entropy Temperature Conversion Diagram.

Again, for constant volume lines we have

$$\varphi_T = C_v \log T + \text{constant}$$

and similarly for constant pressure lines.

So that by plotting φ_T with log T, the constant volume lines and constant pressure lines are straight lines of linear equation, as illustrated in Fig. 29.

The intercepts upon the co-ordinate axes are easily determined by putting $\varphi = 0$ and $\log T = 0$ respectively in the above equations.

Alternatively we can employ the second or graphical method for obtaining the entropy diagram from the indicator diagram, in terms of the pressure and volume, both of which can be read off direct.

Suppose the actual cycle of operations is that shown in Fig. 30, which represents a theoretical indicator diagram for an engine working upon the Otto cycle.

Take any point B on the diagram and measure off in suitable units the pressure P_B and volume V_B respectively. At some other convenient place A, the quantities P_A and V_A respectively can also be obtained, and the entropy of the gases employed can be expressed at any other point in terms of the entropy at the point A.

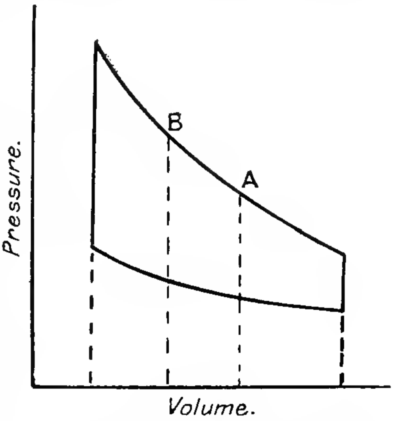

Fig. 30.—Conversion of Indicator Diagram into T—ϕ Diagram.

Thus we have $\varphi_B - \varphi_A = C_v \log_e \frac{T_B}{T_A} + R \log_e \frac{V_B}{V_A}$

and $\varphi_B - \varphi_A = C_p \log_e \frac{T_B}{T_A} - R \log_e \frac{P_B}{P_A}$

By dividing all through by C_v and C_p respectively, and putting $R = C_p - C_v$ so as to eliminate $\log_e \frac{T_B}{T_A}$

$$\begin{aligned} \text{we get } \varphi_B - \varphi_A &= C_p \log_e \frac{V_B}{V_A} + C_v \log_e \frac{P_B}{P_A} \\ &= C_v \left\{ \gamma \log_e \frac{V_B}{V_A} + \log_e \frac{P_B}{P_A} \right\} \\ &= 2{\cdot}3026\ C_v \left\{ \gamma \log_{10} \frac{V_B}{V_A} + \log_{10} \frac{P_B}{P_A} \right\} \end{aligned}$$

So that from measurements of pressure and volume from the indicator diagram we can find φ_B in terms of φ_A; further, by methods previously given, we can calculate the temperatures corresponding to these pressures and volumes, so that we are able to plot the $T.\varphi$ diagram.

These conversion methods assume C_p and C_v, the specific heats, to be constant, so far as temperature variation is concerned.

T.φ Diagram for variable Specific Heats.

If the law of variation of the specific heats with the temperature be known, the Tφ diagram can also be drawn for this case.

It will be necessary to take a number of points all around the indicator diagram, and to find by known methods the temperatures at these points.

The diagram must then be treated in small steps, assuming the specific heats constant over each successive step and in value equal to the specific heat at the middle of each step respectively.

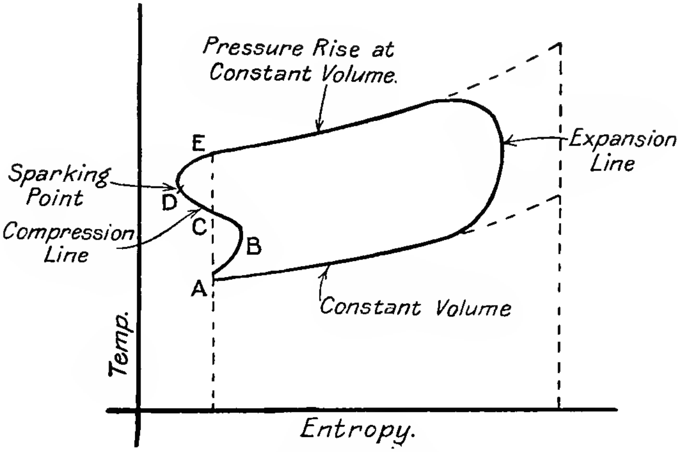

FIG. 31.

The value of φ can then be calculated in terms of T for each small step.

Thus $\delta\varphi = C_v\dfrac{\delta T}{T} = C_p\dfrac{\delta T}{T}$ over each element.

An example of an actual conversion from an ordinary petrol engine indicator diagram, which is not to scale, however, is shown in Fig. 31. After a thorough study of the principles set forth in Chapter V, dealing with indicator diagrams, some of the features which are only brought out by the Tφ diagram will be appreciated. Among such points are—(1) the deviation of the expansion and compression lines from true straight line adiabatics ; (2) the heat losses and absorptions from the cylinder walls during the final and the initial stages of the compression stroke ABCDE ; and (3) the jacket loss during the ignition and expansion stages.

CHAPTER II

EXPLOSION AND COMBUSTION IN THE ENGINE

THE changes which occur within the cylinder of an internal combustion engine are complex, and although considerable attention has been given to the phenomena observed, by eminent scientists and engineers, with a partial elucidation of the difficulties, yet there is still much to be done in this connexion.

Unfortunately, so far as petrol engines are concerned, very little scientific work has been performed in the past with the direct object of solving the special problems peculiar to this type of engine, as compared with gas and oil engines. It is only due to the efforts of one or two noted investigators that our present knowledge of the phenomena occurring exists.

The petrol engine, running at very high speeds compared with large gas and oil engines, presents great difficulties to the experimenter, so that many of our modern conclusions regarding this type of engine are drawn by inference from the experimental results and observations made upon slower running combustion engines, which offer much greater facilities for these purposes.

The comparatively recent development of the petrol engine, as compared with the older type, the gas engine, has not a little to do with our greater accumulation of knowledge and data concerning the latter.

In this chapter, therefore, it will be necessary to refer to the results of gas engine research from time to time, in instances where the parallel work is unobtainable in the case of high-speed internal combustion engines.

Explosive Mixtures.

When an inflammable vapour or gas is mixed with air (or oxygen) in certain proportions, the resulting mixture is found to be explosive, and the presence of a small flame or spark will cause the elements of the mixture to chemically combine with more or less violence.

If the mixture ignite slowly, the resulting flame taking a comparatively long time to spread through the mixture from particle to particle until the whole volume is combusted, this process is called *Inflammation.*

In this mode of combustion, the burning, or chemical combination, commences from the point of ignition of the mixture, and each portion of the mixture as it burns heats the neighbouring portion until its temperature is so high that it burns, and so on through the whole mass of the mixture.

When the mixture is completely inflamed, explosion follows, and the maximum pressure is attained; the moment of complete inflammation is also that of maximum pressure.

Combustion is complete when the chemical combination of the elements present is effected, and this point of complete combustion does not necessarily coincide with that of explosion.

In gas engines the point of maximum pressure practically always precedes the point of complete combustion.

There is another mode of combustion of an explosive mixture which can occur, quite distinct from that of inflammation, and this method is much more violent, and is known as "*Detonation.*"

Detonation appears to be caused when a wave of compression is set up in the explosive mixture, and it is the heat of this compression which fires the mixture as it traverses its whole mass.

It is evident that combustion of the mixture will spread just as rapidly as the compression wave travels through it; this speed of combustion is very considerable as compared with the process of inflammation.

Thus in the case of an explosive mixture composed of equal volumes of carbon monoxide and oxygen, at atmospheric pressure and temperature, the rate of flame travel, or inflammation, is about 3·3 feet per second, whereas if the same mixture be ignited, not at a point but in a considerable volume at once, then detonation occurs, the velocity of flame propagation being about 5,000 feet per second.

Detonation can also be made to occur in other gaseous mixtures such as hydrogen and oxygen; when ignited in the proportions of 2 volumes of H to 1 volume of O, a detonating flame occurs which travels with a velocity of about 10,000 feet per second.

The difference between inflammation and detonation of an explosive charge may be illustrated, first by igniting with a match a quantity of guncotton lying in the open air—it will burn harmlessly by the former mode of combustion; if, on the other hand, the same guncotton, under exactly the same conditions, be fired by means of a percussion cap, a very violent explosion or detonation will occur, due to the starting of an initial compression wave by the percussion cap.

These two modes of combustion are very important in petrol and other engines, and can be approached under certain conditions which are considered in the following paragraphs. It is as well to note that the normal combustion in a petrol (or like) engine is neither true detonation nor inflammation; thus a very weak mixture of petrol vapour and air will burn comparatively slowly, whereas if a mixture of either petrol vapour and oxygen, or acetylene gas (C_2H_2) and air, be introduced into the engine and ignited in the ordinary way, detonation would occur, and exceedingly higher pressures and temperatures would be developed.

The dilution of the proper quantity of oxygen required for the combustion of a given quantity of petrol vapour, by means of the nitrogen of the atmosphere (in the proportions of 79 per cent. of nitrogen to 21 per cent. of oxygen), prevents such detonating explosions from taking place in the case of petrol engines; and this from the engineer's point of view is a great advantage, for it would otherwise be obviously impossible to run the engine quietly and without much shock and hammering action; engines would necessarily have to be made considerably stronger and heavier, and engine speeds would also increase very rapidly, thus involving numerous radical changes from present design.

Experiments * in connexion with the rates of flame propagation in mixtures of different gases, in diluted mixtures, and in mixtures under different degrees of compression, have established the following facts, namely that the rate of flame propagation, or travel, in an explosive mixture:—

(1) is diminished as the proportion of an inert gas present is increased;
(2) is dependent upon the proportions of the active constituents;
(3) is diminished as the rarefaction of the mixture increases;
(4) is increased as the temperature of the mixture is raised;
(5) is much greater when the mixture is ignited at constant volume than when ignited at constant pressure.

Dealing with these factors set out, reference might be made to experiments in which it was originally shown by Sir Humphry Davy that true explosive mixtures may lose their explosive properties either by extreme excess of one of the constituents, or by excess of an inert constituent such as nitrogen, or by reducing the pressure of the gases below a certain point.

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* Bunsen, Le Chatelier and Mallard, *Recherches*, Paris, 1883. Berthelot and Vieille, Witz, *Trăité Theorique et Practique des Moteurs à Gaz et à Pètrole*, Paris, 1892, 1895 and 1899.

In the case of mixtures of petrol vapour and air, if the proportion of air to petrol by weight exceeds about 22, or if the mixture is so "rich" in petrol vapour that this ratio is below about 8, it will be impossible to obtain an explosion in the ordinary petrol engine.

Assuming there is some value for the proportions of air to petrol which gives the normal combustion in an efficient petrol engine, then the effect of changing the proportions slightly so as to give a richer or weaker mixture results in combustion tending to the inflammation type.

As one proceeds to the extremes of the proportions given, namely, 22 : 1 and 8 : 1 the weakened effects are due to different causes. For a rich mixture the poor explosion follows from dilution consequent on the presence of inert gas in the air, whilst for a weak mixture it is the disproportion of the active constituents that is the deciding factor.

Again, if two volumes of hydrogen be mixed with one volume of oxygen at ordinary atmospheric pressure, and if to this mixture about twenty-five volumes of oxygen be added, the mixture will no longer explode. Now if the density of the resulting mixture be raised by means of external pressure, or by increasing its temperature, the explosive property is restored.

On the other hand, if the original atmospheric pressure be reduced until it is about one-eighteenth of its initial value, a mixture of two volumes of hydrogen with one volume of oxygen cannot be exploded, but by heating this rarefied mixture to a fairly high temperature it can again be rendered explosive.

A further investigation of the experimental facts shows that the effect of increased temperature is to widen the range of dilution, whether the dilution is produced by the presence of an inert gas or by an excess of one of the constituent gases.

The effect of increasing the pressure of the explosive mixture before ignition is also shown by the examples given—and it is important to notice that increasing the compression pressure of the gaseous charge in a petrol engine has the effect of increasing the rate of flame propagation throughout the mixture; this fact is taken advantage of in high speed and racing engines.

The Effect of Mixture Proportions upon Flame Velocity (continued).

Dugald Clerk, from experiments upon mixtures of coal gas and air,* found that the point where inflammability ceases, when the gaseous mixture is diluted with air, is fairly sharply defined ; he found that whereas a mixture of 14 parts of air to 1 of gas would explode with a spark, a mixture of 15 parts of air to 1 of gas would not do so; and that if this "critical proportion" of gases be subjected to a slight increase of temperature or of pressure, or if a small addition of coal gas be made, the mixture then becomes inflammable.

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* *The Gas, Petrol and Oil Engine,* Dugald Clerk, p. 112.

At the same time it has been proved that in passing from the explosive mixture (in which the exact proportions of the gases for chemical combination are present) to the mixture of the "critical proportion" by successive dilutions, the inflammability slowly becomes less and less until at the critical proportion the progress of the flame travel is very slow and the noise of the explosion practically unheard.

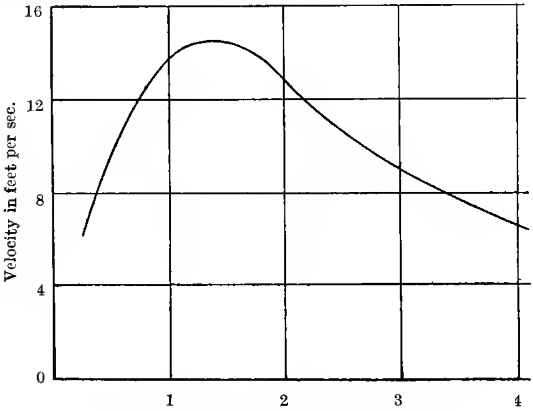

FIG. 32.—SPEEDS OF FLAME TRAVEL.

This is easily demonstrated by exploding in a glass tube mixtures of carbon monoxide (CO) and oxygen (O) by means of a spark, so as to obtain "inflammation." If the proportion of oxygen be increased, the report will become less marked, and the rate at which the flame travels along the tube will be noticeably slower, until with the "critical proportions" the results are as stated.

The effect of the proportion of the gases or vapours present, has been determined by Bunsen, and Mallard and Le Chatelier and other experimenters.

The results of Mallard and Le Chatelier's experiments are shown graphically in Fig. 32. The results of these experiments showed that if a mixture of one volume of hydrogen and four volumes of air was first taken and exploded, the velocity of the flame travel was 6·5 feet per second, whereas with the proportions of equal volumes of air and hydrogen this velocity was increased to 12·4 feet per second; and finally, as the proportion of hydrogen was still further increased until the mixture was composed of 3 parts of hydrogen to 1 part of air, the velocity of the flame was decreased to 7·5 feet per second.

It will be noticed that the velocity of flame propagation becomes a maximum for a mixture of 1 part hydrogen and $1\frac{1}{2}$ of air, and is then 14·4 feet per second.

As the air consists of 4 parts nitrogen to 1 part of oxygen by volume, it follows that the above mixture which gives the maximum flame travel consists of about 10 parts of hydrogen to 3 parts of oxygen.

Other experiments which have been made upon mixtures of hydrogen and oxygen alone, show that the mixture of these two gases in the proportions for complete combustion, namely, 2 parts of hydrogen to 1 part oxygen by volume, does not give the greatest rate of flame travel, but that the velocity is a maximum when there is an excess of hydrogen present.

This is borne out by the data given graphically in Fig. 32. The mixture of hydrogen and air (namely, $1 : 2\frac{1}{2}$) in which the hydrogen and oxygen are present in their correct proportions for complete chemical combination, does not give the greatest flame speed. But when the hydrogen is increased beyond this proportion to 1 volume of H to $1\frac{1}{2}$ volumes air, the velocity is a maximum.

Other experiments with mixtures of coal gas and air show that the maximum velocity of flame travel occurs for mixtures in which the gas is present somewhat in excess of the actual proportion required for complete combustion.

In the case of mixtures of petrol vapour and air, the speeds of flame travel have been measured for different proportions of these two substances by Dr. Neumann of Dresden, and he has shown that the velocity of flame travel is a maximum for a mixture of petrol vapour and air in the proportions by weight of 1 to 12. The proportions for complete combustion are approximately as 1 to 15, dependent upon the quality of the spirit, so that in this case also the maximum flame velocity occurs when the petrol vapour is slightly in excess.

As a general deduction, it would appear that for maximum flame propagation the proportion of the combustible constituent to the oxygen carrying one must be somewhat greater than that necessary for complete chemical combination.

The actual results of Dr. Neumann's experiments are given in Fig. 33, showing the variation in the rate of flame travel as the mixture is gradually enriched with petrol vapour until the maximum is reached, after which any further enrichment causes a decrease in the flame speed. This factor of flame travel with mixture proportion is of considerable importance in petrol and other explosion engines.

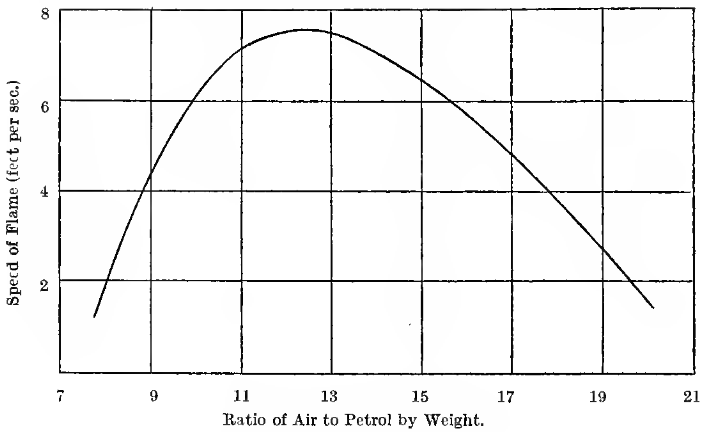

FIG. 33.

Explosion at Constant Volume.

It has been stated that the flame velocity is much greater when the mixture is exploded under conditions of constant volume than when under constant pressure. This fact is established by Mallard and Le Chatelier's experiments upon mixtures of hydrogen and oxygen (or air) in open and closed tubes.

In the case of a mixture of 2 volumes of hydrogen to 1 volume of oxygen, exploded in a tube, one end of which was closed and the other open, as shown in Fig. 34, so that the ignited gases issued from the open end and performed external work upon the atmosphere under constant (atmospheric) pressure, the measured flame speed was 65·6 feet per second.

When a similarly proportioned mixture was exploded in a closed tube (Fig. 34) so as to obtain constant volume conditions, the velocity of the flame measured was 3,280 feet per second.

This exceedingly great increase in the flame velocity in the second case is probably due to the fact that the particles of gas as they ignite increase both in temperature and pressure rapidly and thus greatly assist the flame propagation which takes place from particle to particle.

When the tube is open, expansion into the external air can occur; but when closed the particles cannot expand into the external air, so that the tendency to expansion can only have effect inwardly, causing the ignited portions of the gas to be projected into the portions of the gas as yet unignited, and thus greatly assisting the rate of flame travel; further, the absence in the case of constant volume explosion of external work performance no doubt affects the rate of flame travel.

In the case of a petrol engine, ignition at constant volume can only occur at the dead centre position, when the piston is momentarily stationary, so that for maximum flame velocity it is important to ignite as near the dead centre as possible, at the same time bearing in mind the necessary time interval elapsing between the points of ignition and the attainment of maximum pressure. When much hydrogen is present in the explosive mixture the flame velocity is greatly increased thereby.

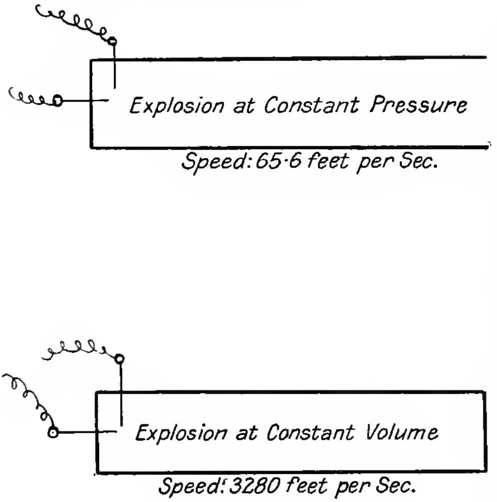
Explosion at Constant Pressure
Speed: 65·6 feet per Sec.
Explosion at Constant Volume
Speed: 3280 feet per Sec.

FIG. 34.

In connexion with Berthelot and Vielle's experiments upon the rates of flame propagation of gases at constant volume in a closed vessel, and also by using explosion vessels of different capacities, it was found that the larger the capacity of the vessel the longer the period of time elapsing between the ignition of the gas and the attainment of maximum pressure. This point has an important bearing upon the design of large internal combustion engines.

Combustion Data.

Before proceeding to further considerations of the real changes occurring within the cylinder and the important results of closed vessel experiments, it will be necessary to devote some space to the subject of combustion, from the chemical and thermal points of view.

The basis of the working of practically every type of internal combustion engine is the chemical combination of the elements hydrogen (H) and carbon (C) with the oxygen (O) of the air.

From experiments made by authorities such as Favre and Silverman, Andrews, and others, the actual amount of heat evolved in the combustion of the above elements with oxygen has been determined in each case.

These "Heats of Combustion" are given in the Tables IX and X below, and are expressed in B.T.U.s per pound of the element combusted, and also in Centigrade Heat Units.

TABLE IX

Calorific Values

Element or Compound.	Symbol.	B.T.U.s per lb.	Cent. T.U.s per lb.	B.T.U.s per cub. foot at 32° F. and 760 mm. pressure.
Hydrogen	H	62,030*	34,460*	293
Carbon	C	14,540	8,080	976
Carbon monoxide	CO	4,320	2,400	342
Sulphur	S	4,000	2,220	715
Ethylene	C_2H_4	21,350	11,860	1,676
Benzene	C_6H_6	17,840	9,910	4,020
Methane	CH_4	23,510	13,060	1,066

* The value given for hydrogen includes the latent heat of the steam. If this latent heat remains in the gas, the heat of combustion in B.T.U.s per lb. becomes 52,500.

TABLE X

Calorific Values of Internal Combustion Engine Fuels

Name of Fuel.	Composition per cent.			Calorific Value in B.T.U.s per lb.	Remarks.
	Carbon.	Hydrogen.	Oxygen.		
Alcohol	52·2	13·0	34·8	12,600	C_2H_6O
Methyl alcohol	37·5	12·5	50·0	9,500	CH_4O, known as wood spirit or wood naphtha
Methylated spirits	—	—	—	11,000	90% ethyl alcohol 10% methyl ,, Small quantity of paraffin
Benzol	92·3	7·7	0	18,100	C_6H_6
(1) Petrol, ·680 density	—	—	—	19,200*	Also known as Gasolene. Light petrol
(2) Petrol, ·720 density	85	15	0	18,700*	Medium petrol
(3) Petrol, ·760 density	—	—	—	18,250*	Heavy petrol
Paraffin	85	15	0	18,900	Known as kerosene
Hexane	83·7	16·3	0	20,000	C_6H_{14}
Heptane	84	16	0	20,760	C_7H_{16}
Toluol	91·3	8·7	0	18,300	C_7H_8
Xylol	90·5	9·5	0	18,460	C_8H_{10}
Acetylene	92·3	7·7	0	21,600	C_2H_2

* Lower calorific values.

It will be noticed that the evolution of heat in the case of hydrogen combining with oxygen is roughly four times that of carbon and oxygen, so that fuels rich in hydrogen should have high "Calorific" values.

Here it should be explained that by the Calorific value of a fuel is meant the total number of heat units evolved during the combustion of unit weight of the fuel.

In the following considerations the atomic constitution of the molecules of each element is expressed by its chemical symbol. Thus H_2, O_2, N_2, etc., are the chemical symbols for hydrogen, oxygen, and nitrogen respectively, and further express the fact that one molecule of each of these elements contains two atoms.

Avagadro's law states that equal volumes of gases contain equal numbers of molecules, the gases being under similar pressure and temperature conditions.

It follows from this, that the weights of unit volumes of any of the gases whose atomic weights are known can be found. The atomic weights of all of the elements and compounds mentioned herewith are given for convenience in the following Table XI.

TABLE XI

Table of Atomic and Molecular Weights

Element or Compound.	Series.	Symbol.	Atomic Weight.	Molecular Weight.
Hydrogen	—	H_2	1	—
Oxygen . .	—	O_2	16	—
Nitrogen . .	—	N_2	14	—
Carbon	—	C	12	—
Sulphur .	—	S	32	—
Carbon monoxide .	—	CO	—	28
Carbon dioxide .	—	CO_2	—	44
1. Methane . .	The Paraffin Series	CH_4	—	16
Ethane . .	C_nH_{2n+2}	C_2H_6	—	30
Propane .	gaseous	C_3H_8	—	44
Butane .	constituents	C_4H_{10}	—	58
Pentane	Liquid constituents	C_5H_{12}	—	72
Hexane . .		C_6H_{14}	—	86
Neptane .	Petrol	C_7H_{16}	—	100
Octane . .		C_8H_{18}	—	114
2. Ethylene. .	The Olefine Series, C_nH_{2n}	C_2H_4	—	28
3. Benzene . . .	The Benzene Series	C_6H_6	—	78
Toluene . . .	C_nH_{2n-6}	C_7H_8	—	92
Xylene . . .	Benzole	C_8H_{10}	—	106
4. Acetylene . .	The Acetylene Series C_nH_{2n-2}	C_2H_2	—	26
5. Naphthalene .	The Naphthalene Series C_nH_{2n-12}	$C_{10}H_8$	—	128
6. Alcohol . . .	—	C_2H_6O	—	46
Methylic alcohol .	—	CH_4O	—	32

Lower and Higher Heating Values.

In expressing the heat of combustion of hydrogen, methane, or in fact of any hydrocarbon, allowance is sometimes made for the fact that the water formed during combustion is either condensed to the liquid state, or is present as water vapour or steam.

If present as a liquid, the water vapour in condensing yields up an extra quantity of heat equal to its latent heat of evaporation, which is 966 B.T.U.s per lb., or 537 Centigrade heat units per lb. at 212° F. or 500° C. respectively.

Thus by the "higher heating value" of a fuel is meant the actual heat of combustion plus the latent heat of condensation of the water formed during combustion.

By the "lower heating value" of an element or compound containing hydrogen is meant the heat of combustion. This is the value one is concerned with when the water formed during combustion remains uncondensed as either saturated or superheated steam.

If the fuel and the oxygen admitted for combustion be quite dry before burning, then a small portion of the water formed will be utilized in saturating the products of combustion, and so a small correction for the quantity of water uncondensed upon this account must be made in expressing the higher calorific value of the fuel, knowing the pressure, volume, and temperature of the final products of combustion.

Further, it is usual to define the temperature of the final products at which the condensed water vapour is supposed to be. Thus if, as is frequently the case, the temperature of 60° F. be taken as the standard, then the total heat given up by 1 lb. of water vapour in condensing to 60° F. will be

$$966 + (212 - 60) = 1{,}118 \text{ B.T.U.s.}$$

In the Tables IX and X given on p. 63 the higher heating values are given for the hydrogen constituents.

Most fuels used in petrol and oil engines are nearly pure hydrocarbons and can be represented by the general chemical formula C_xH_y, where x and y represent the number of atoms in the molecule.

For example, American petroleum consists chiefly of hydrocarbons belonging to the Paraffin Series, and represented by the general formula C_nH_{2+}; whilst petrol consists almost entirely of pure hexane of the same series.

It is often necessary to be able to calculate the heat of combustion of a hydrocarbon fuel from its chemical composition, and for this purpose, besides knowing the chemical formula, it is also necessary to know the atomic weights of the elements.

Thus the atomic weight of hydrogen is taken as unity, and the atomic weight of carbon is taken as 12.

If the chemical formula of the fuel is C_xH_y, from Table IX it can be shown that 1 gramme of carbon yields 32·29 B.T.U.s in complete combustion to CO_2, and 1 gramme of hydrogen yields 115·15 B.T.U.s in complete combustion to H_2O.

The weight of carbon present in the fuel is proportional to $12 \times x$, and hence the heat of combustion per gramme is proportional to $12 \times x \times 32{\cdot}29$. Similarly the heat of combustion of the hydrogen (per gramme) is proportional to $1 \times y \times 115{\cdot}15$, so that the heat of combustion per gramme of $C_xH_y = \dfrac{12 \times 32{\cdot}29x + 1 \times 115{\cdot}15y}{12x + y}$

For example, if the fuel be composed chiefly of the Olefine Series represented by C_nH_{2n}.

$$\text{The Heat of Combustion} = \frac{12 \times 32{\cdot}29 \times n + 1 \times 115{\cdot}15 \times 2n}{12 + 2n}$$

$$= 44{\cdot}13 \text{ B.T.U.s per gramme}$$
$$= 20{,}016 \text{ B.T.U.s per lb.}$$

Similarly for a hydrocarbon of the form C_nH_n.

Heat of Combustion = 38·67 B.T.U.s per gramme.
= 17,540 B.T.U.s per lb.

In the case of a gas where the proportions of hydrocarbons cannot be estimated exactly, Dr. Slaby* gives the following general formula for the heat of combustion :—

$$H = 1000 + 10500y \text{ calories per cubic metre}$$
$$= [112 + 18880y] \text{ B.T.U.s per cubic foot.}$$

Where y = the weight of 1 cubic metre (or foot) of the hydrocarbon in kilograms (or pounds).

The value of the quantity y is obtained by an analysis of the gas.

There is one other relation for calculating the calorific value of a solid or liquid hydrocarbon, when the percentage weights of hydrogen, oxygen and carbon in the fuel are known from analysis, namely, $H = 14500\ C + 52230 \left(H - \dfrac{O}{8}\right)$

It will be seen that the percentages of hydrogen and carbon present have each been multiplied by their respective heats of combustion, and further it is assumed that the oxygen present is combined with one-eighth its weight of hydrogen in the form of water, so that the equivalent quantity of heat for this combination is deducted from that of the hydrogen.

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* *Calorimetrische Untersuchungen*, Berlin, 1893.

In the case of solid or liquid fuels which show upon analysis that other elements are present besides hydrogen, oxygen and carbon, the general formula must be modified accordingly.

Thus if the fuel is found to contain sulphur (S) and free water (H_2O) besides the above elements, then the heat of combustion or calorific value is given by

$$H = 14500\ C + 52230 \left[H - \frac{O}{8}\right] + 4000\ S - 1000\ H_2O.$$

Volumetric Proportions in Combustion.

It is often important to be able to calculate the weight or volume of air required for the complete combustion of a given weight of fuel.

If the fuel be given by the chemical formula C_xH_y the chemical changes occurring are that the carbon is burnt to carbon dioxide thus, $C + O_2 = CO_2$ and the hydrogen to water, $2H + O_2 = 2H_2O$.

So that for the fuel C_xH_y we have the volumetric equation (that is, the relation between the number of molecules of each compound present).

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 = xCO_2 + \frac{y}{2}H_2O.$$

So that 1 volume of C_xH_y combines with $\left(x + \frac{y}{4}\right)$ volumes of oxygen to form x volumes of CO_2 and $\frac{y}{2}$ volumes of H_2O.

Since the composition of the air by volume is 79·1 per cent. nitrogen and 20·9 per cent. oxygen, it follows that 4·78 cub. feet air contains 1 cub. foot of O and 3·78 cub. feet of N.

Hence 1 volume of C_xH_y will require $4{\cdot}78\left(x + \frac{y}{4}\right)$ volumes of air for complete combustion, and if the water formed be regarded as occupying no appreciable volume, the volume of the products of combustion will be $\left[x + O + 3{\cdot}78\left(x + \frac{y}{4}\right)\right]$

$$\text{or } 4{\cdot}78x + 3{\cdot}78\frac{y}{4}$$

So that the volume of the products of combustion is less than the initial volume upon the assumption that the water vapour condenses.

If, as is actually the case, no condensation within the cylinder of the engine occurs during the expansion stroke then an increase in volume due to combustion will occur on the ratio

$$\frac{\text{Initial volume}}{\text{Final volume}} = \frac{4{\cdot}78\left(x+\frac{y}{4}\right)}{4{\cdot}78x+\frac{y}{2}+3{\cdot}78\frac{y}{4}}$$

$$\text{or } \frac{4{\cdot}78x+1{\cdot}19y}{4{\cdot}78x+1{\cdot}44y}$$

The actual increase in volume rarely reaches 15 per cent.*

Proceeding next to a consideration of the weights of the substances already mentioned, the general volumetric (or molecular) equation for the chemical changes occurring must be converted into the molecular weight equation.

Thus in the usual notation we have

$$\begin{array}{lllll} & C_xH_y + & \left(x+\frac{7}{4}\right) O_2 = & xCO_2 + & \frac{y}{2}\ H_2O \\ & | & | & | & | \\ \text{In molecular weights} & (12x+y) & \left(x+\frac{y}{4}\right).32 & x\left[12+32\right] & \frac{y}{2}\left[2+16\right] \end{array}$$

Hence it follows that 1 lb. of the fuel C_xH_y will require $\frac{32x+8y}{12x+y}$ lbs. of oxygen and will yield $\frac{44x}{12x+y}$ lbs. of CO_2 and $\frac{9y}{12x+y}$ lbs. of H_2O.

Since the composition of the air by weight is 76·8 per cent. N and 23·2 per cent. O, 4·31 lbs. of air contains 1 lb. of O and 3·31 of N.

Then 1 lb. of the fuel will require $4{\cdot}31 \times \frac{32x+8y}{12x+y}$ lbs. of air for complete combustion.

The ratio $\frac{\text{Initial weight of explosive mixture}}{\text{Final weight of exhaust products}}$

$$= \frac{(12x+y)+32\left(x+\frac{y}{4}\right)}{44x} = \frac{44x+9y}{44x}$$

This is based upon the supposition that, in the case of an internal combustion engine, the water vapour condenses in the exhaust; so that the real weight of exhaust products ejected will be somewhat less than the weight of the explosive mixture, upon this assumption.

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* Vide p. 77.

Application of Results to case of a Petrol Engine.

In the case of the petrol engine it is sufficiently accurate to consider the hydrocarbon fuel to be pure hexane C_6H_{14}.

The equation of chemical changes then becomes

$$C_6H_{14} + \left(6 + \frac{14}{4}\right)O_2 = 6CO_2 + \frac{14}{2}H_2O$$

It follows from this equation that 1 cubic foot of petrol would require for complete combustion $4{\cdot}78 \times \frac{19}{2}$ (cubic feet) or 45·41 cub. feet of air, and that the volume of the exhaust products, including that of the water vapour will be 48·91 cub. feet.

$$\text{Hence } \frac{\text{Initial volume}}{\text{Final volume}} = \frac{46{\cdot}41}{48{\cdot}91}$$

So that the volume is about 5·6 per cent. greater after combustion than before.

$$\text{Again, from the equation } C_6H_{14} + \left(6 + \frac{7}{2}\right)O_2 = 6CO_2 + 7H_2O$$

$$\text{and by molecular weight} \qquad 86 \qquad 304 \qquad 264 \qquad 126$$

1 lb. of petrol requires $\frac{304}{86} = 3{\cdot}53$ lbs. of oxygen, or $3{\cdot}53 \times 4{\cdot}31 = 15{\cdot}21$ lbs. of air for complete combustion ; and the volume of the exhaust products, the condensed water being ignored, is 14·75 lbs.

The final weight of exhaust products is therefore about 91 per cent. of the original weight of explosive mixture. After an appreciable period of engine running, however, the water would be ejected and the above remarks would not hold, in so far as the alteration in weight of the products is concerned.

Composition of Exhaust Gases.

From the preceding results, in the case of nearly pure petrol or "hexane" mixed with air in the proportions for complete combustion, namely, 15·21 lbs. of air to 1 lb. of petrol by weight, it follows that theoretically the composition of the exhaust gases should be as follows :—

$$\begin{aligned} CO_2 &= 14{\cdot}3 \text{ volumes} \\ N_2 &= 85{\cdot}7 \quad ,, \\ \hline &\quad 100{\cdot}0 \quad ,, \end{aligned}$$

The water formed being considered as condensed.

Let us now examine in the case of a petrol engine the composition of the exhaust gases for mixtures with different ratios of air to fuel.

In the first place, petrol vapour and air have a fairly wide range of explosibility, from a weak mixture consisting of about 20 parts air to 1 part petrol by weight to the richest mixture upon which an engine will fire, namely, 8 parts air to 1 part of petrol.

It is evident then, that for weaker mixtures there is more than a sufficiency of oxygen necessary to combust completely the petrol present, so that after explosion one would anticipate the presence of free oxygen in the exhaust gases in addition to carbon dioxide and nitrogen.

In the case of mixtures in which the quantity of petrol vapour present is in excess of the theoretically correct amount, it follows that since there is an insufficiency of oxygen the mixture will be only partially combusted, so that instead of carbon dioxide and nitrogen alone, there will be carbon monoxide (CO) as representing the only partially complete combustion of the carbon in the petrol. It is further evident that as the richness of the mixture increases so will the incompleteness of combustion, and hence the percentage of carbon monoxide in the exhaust gases will become greater.

These theoretical predictions are fairly closely borne out in practice, for upon analysing the products of combustion of a petrol engine, it is found that the exhaust consists of carbon dioxide, oxygen, and nitrogen for weaker mixtures, and of carbon dioxide, carbon monoxide, and nitrogen for richer mixtures.

The mixture which gives the nearest resemblance to perfect combustion is found in practice to consist of about 14·75 parts of air to 1 part of petrol for petrol of density 0·720 at 15° C., and the products of combustion consist of

(Carbon dioxide)	CO_2	= 13·3	per cent.	by volume
(Carbon monoxide)	CO	= 0·5	,,	,,
(Oxygen)	O_2	= 0·5	,,	,,
(Methane)	CH_4	= 0·06	,,	,,
(Hydrogen)	H	= 0·18	,,	,,
(Nitrogen)	N_2	= 85·46	,,	,,
Aldehyde		= traces		
		100·00		

These figures will of course vary to a small extent with the quality or grade of fuel used.

It will be noticed that this analysis (which represents the average of a large number of tests) reveals the facts that combustion within the cylinder of a petrol engine is not perfect, although the incompleteness is not very marked, as evidenced by the presence of CO, O, CH_4 and H in small quantities. In addition, chemical tests of the condensed water in the exhaust reveal the presence of an aldehyde. From the results of a large number of tests and analyses of the exhaust gases in four-cylinder petrol engines, Dr. Watson has plotted the results in the form given in Fig. 35, the proportions (expressed as percentages) of the constituents of the exhaust products being plotted against mixture strengths, by weight.

The weights of air and petrol used by the engines during the tests were accurately measured, and the exhaust gases were analysed; the grade of petrol was that previously mentioned

It is interesting and of value to notice that the percentage of oxygen present becomes greater as the mixture gets weaker, and that as the mixture becomes succeedingly richer in petrol, after the point of complete combustion is passed, the percentage of carbon monoxide becomes greater; and that for weaker mixtures no carbon monoxide is present, whilst for richer mixtures no oxygen is present. These facts are made use of later, in the case of the two-stroke engine, for calculating from the exhaust products the amount of escaping unburnt charge.

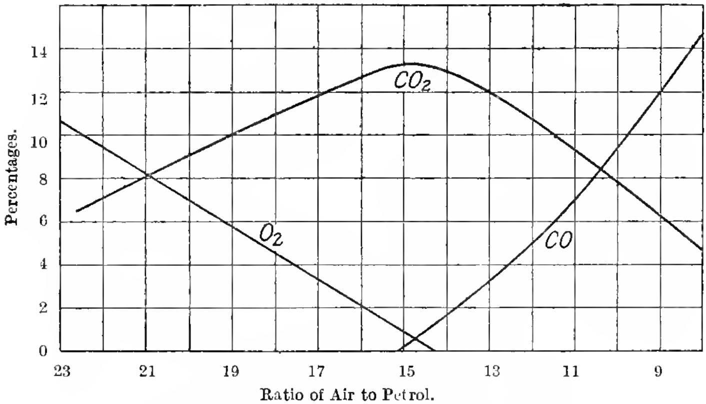

FIG. 35.—EXHAUST GAS ANALYSIS CURVES.

The curve for the oxygen proportions is a straight line, whereas that of the carbon monoxide is a curved line as shown, and the two curves overlap by a small amount near the mixtures approaching correct proportions.

The quantity of nitrogen present is easily obtained by the difference method, from the curves.

The presence of free oxygen and carbon monoxide together in the exhaust in the region of the complete combustion mixtures may possibly be due to variation in the mixture strength in the different cylinders in the multi-cylindered engines tested, which would not be evident from the average measurements made, or may be due to one cylinder failing to fire the charge and so discharging it into the exhaust pipe, where "after-burning" might occur. Air leakage to the exhaust pipe may also produce the variation.

That both CO and O_2 do not occur together in the exhaust *in any appreciable quantity* has therefore been conclusively established.*

It frequently happens that the ratio of the proportions of air and petrol in the mixture have to be determined from such an analysis, and very often this method is the only one used for finding mixture strengths. Table XII gives the results of exhaust gas analyses for petrol-engine mixtures.

TABLE XII

Analyses of Exhaust Gas

Air/Petrol Ratio.	%CO_2	%CO	%O_2	%H_2	%H_2O	%CH_4	%N	Total.
10 . . .	7·12	8·45	—	3·01	13·20	1·00	67·20	99·98
14 . . .	11·75	—	—	—	15·30	—	73·0	100·05
18 . . .	9·07	—	4·85	—	11·75	—	74·3	99·98

It is thus necessary to be able to calculate backwards from the exhaust analysis results to obtain the mixture proportions. A concrete example will make the method clear.

The composition of the exhaust gases in an actual case was :—

CO_2 = 12·5 per cent.
CO = 2·6 ,,
O_2 = 0·2 ,,
H_2 = 0·9 ,,
CH_4 = 0·3 ,,
N_2 = 83·5 ,,

100·0 per cent.

Now 12·5 volumes of CO_2 contain 6·25 volumes C
2·6 ,, ,, CO ,, 1·3 ,, C
0·3 ,, ,, CH_4 ,, 0·15 ,, C

Hence total volumes of C = 7·7.

Again, 12·5 volumes of CO_2 contain 12·5 volumes of O_2
2·6 ,, ,, CO ,, 2·5 ,, ,, O_2
0·2 ,, ,, O_2 ,, 0·2

Total 15·2 volumes of O_2

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* Some interesting curves of exhaust gas analysis results for alcohol and benzol mixtures are given in a Paper entitled "Benzol, etc., as Fuels for Internal Combustion Engines" (W. Watson), *Proc. Inst. Auto Engrs.* 1914–15.

Since the composition of air by volume is 79·1 per cent. N and 20·9 per cent. O, it follows that 1 volume of N is mixed with $\frac{20{\cdot}9}{79{\cdot}1}$ or ·266 volumes of O in atmospheric air.

Hence the 83·5 volumes of N in the exhaust are mixed with 83·5 × ·266 or 22·2 volumes of oxygen.

As we have found 15·2 volumes of O_2 in the exhaust gases, we can argue that the remainder, or 22·2—15·2, has gone to form water, i.e., 7·0 volumes of O_2, which is equivalent to 14·0 volumes of H in water.

Reckoning up the hydrogen we have

$$\begin{array}{lll} H_2 \text{ contained in water} & = 14{\cdot}0 & \text{volumes} \\ \quad ,, \qquad ,, \quad CH_4 & = 0{\cdot}6 & ,, \\ \quad ,, \qquad ,, \quad H_2 & = 0{\cdot}9 & ,, \\ \hline \qquad\qquad \text{Total} & = 15{\cdot}5 & \text{volumes.} \end{array}$$

Hence in the exhaust we have

$$\begin{array}{ll} H & = 15{\cdot}5 \text{ volumes} \\ C & = 7{\cdot}7 \quad ,, \\ N_2 & = 83{\cdot}5 \quad ,, \end{array}$$

Expressed as weights these are

$$\begin{array}{ll} H & = 15{\cdot}1 \times 1 = 15{\cdot}1 \\ C & = 7{\cdot}7 \times 12 = 92{\cdot}4 \\ N_2 & = 83{\cdot}5 \times 14 = 189{\cdot}0 \end{array}$$

Now 83·5 volumes of N correspond with 83·5 × 1·266 volumes of air which weigh 83·5 × 1·266 × 14·4, which gives us

$$\frac{\text{Weight of air}}{\text{Weight of petrol}} = \frac{152{\cdot}8}{15{\cdot}1 + 92{\cdot}4} = 14{\cdot}2$$

$$\text{Also } \frac{\text{Volume of H}}{\text{Volume of C}} = \frac{15{\cdot}1}{7{\cdot}7} = 1{\cdot}965$$

The above method assumes the whole of the carbon and hydrogen of the petrol as estimated by direct analysis to be present in the exhaust gases.

Dr. Watson * has pointed out that (1) the ratio of the volumes of hydrogen to carbon should be a constant, as obtained by a chemical analysis of the petrol, and (2) that the ratio of air to petrol as calculated from the exhaust gases ought to be the same as that obtained from actual direct measurement.

That this is not the case is illustrated by the curves given in Fig. 36, where it will be seen that the two ratios obtained from direct measurement and exhaust analysis do not agree, for they

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* *Proc. Inst. of Auto Engs.*, 1909.

should both give the straight line AB, whereas the curve CD obtained from exhaust analyses lies above AB.

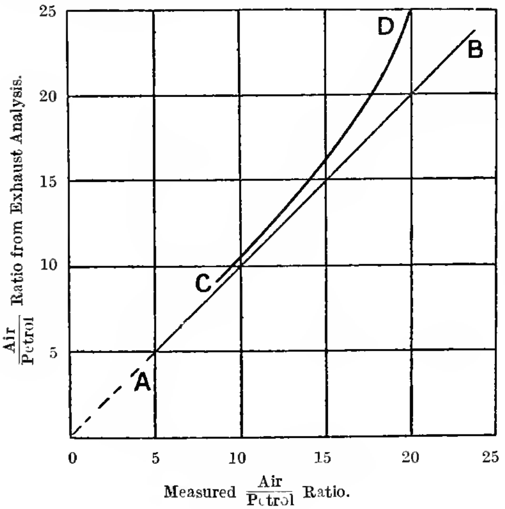

FIG. 36.

The exhaust gas ratios depart more and more from the direct measurement ratios as the mixture becomes weaker. This is again borne out by the curves in Fig. 37, in which it will be noticed that the ratio of H/C for weaker mixtures is greater than for richer, and that in the latter case the ratio is nearly the same as that obtained from an analysis of the petrol. It thus indicates that for weaker mixtures the exhaust gas analysis reveals a deficiency of carbon.

The results obtained from the air/petrol and H/C ratios all serve to show that the chemical changes occurring in the cylinder are more or less complex, and that other chemical compounds may be formed which are not found by the ordinary analysis methods

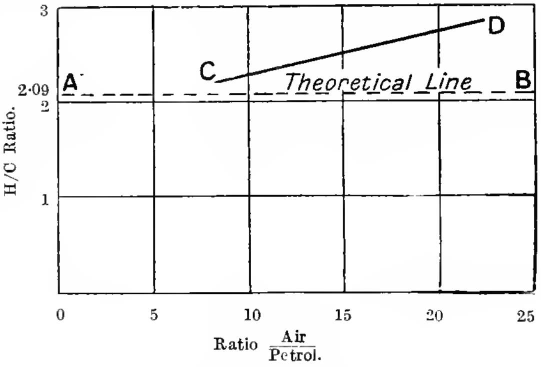

FIG. 37.—HYDROGEN-CARBON RATIOS.

It is well known that the water which condenses in the exhaust pipes contains in solution appreciable quantities of aldehyde $C_nH_{2n}O$, and it is possible that this may account for the observed deviations mentioned above.

It is thus evident that in the case of the petrol engine the fuel is never completely combusted. Experiments made by Prof. Hopkinson * have led to the same conclusions.

Prof. Bone, in the Gaseous Explosions Committee's Report, states that he is not surprised at the disappearance of carbon (as a soluble product) in the above experiments, as from his own work in connexion with the combustion of dense hydrocarbons in a Diesel engine he found considerable "disappearance" of both hydrogen and carbon, which was at once explained when the water condensed from the exhaust gases was analysed, as it was found to give a very strong aldehydic reaction.

Dugald Clerk's experiments upon mixtures of coal gas and air in various proportions, which were exploded in a cylinder in which the piston overran an exhaust port through which samples of the exhaust gases could be taken, showed that in all the cases taken there was a certain amount of unburnt fuel in the exhaust. The quantity of carbon ranged from 2 to 4·3 per cent. by volume, and of hydrogen by 0·8 to 3·7 per cent. by volume. These results indicate that combustion continues even in the explosion of a strong mixture for some little time after attainment of maximum pressure.

Since the combustion within the cylinder is incomplete and combustible gases are found in the exhaust, it is obvious that the whole of the calorific heat of the fuel is not liberated during the explosion process, so that the efficiency of the engine as calculated upon the assumption that the whole of the calorific equivalent of the fuel is available will give much lower results than if calculated upon the heat evolved corresponding to the chemical changes actually occurring,

Using Thomsen's values for the heats of combustion of CO, H, and CH_4, at constant volume, namely :—

1 gramme of CO burning to CO_2 evolves 2,417 calories
1 ,, H ,, H_2O ,, 28,685 ,,
1 ,, CH_4 ,, CO_2 and H_2O evolves 11,870 calories,

TABLE XIII

Ratio of Air to Petrol by Weight.	Composition of Exhaust Gases.						Percentage of Heat of Fuel used.
	CO_2	O	CO	H	CH_4	N	
9	6·7	0	12·0	4·3	1·4	75·6	58
10	8·2	0	9·6	3·5	1·2	77·5	64
11	9·6	0	7·3	2·6	0·9	79·6	71
12	11·1	0	5·0	1·8	0·6	81·5	79
13	12·5	0	2·6	0·9	0·3	82·7	89
14	13·5	0·4	0·4	0·1	0·0	85·6	99

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* *Engineering*, August 9, 1907.

and calculating the heats of combustion from the exhaust gas analyses results, Dr. Watson gives the above table (XIII). The results are also shown plotted in Fig. 38.

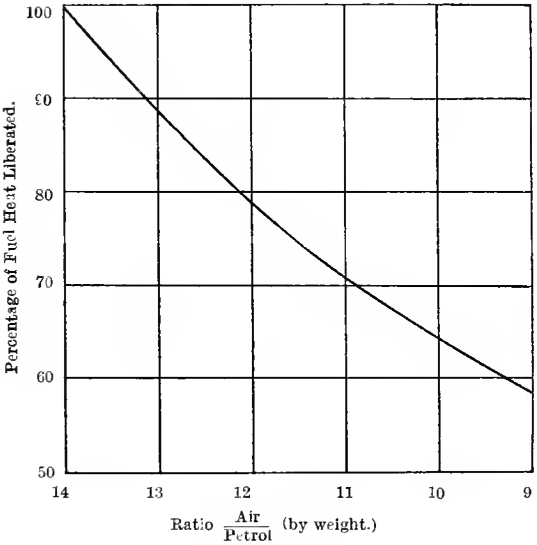

FIG. 38.

From both the diagram and the table it is evident that as the mixture becomes richer the percentage of the heat of the fuel liberated becomes smaller.

Further, if the "thermal efficiency" (by which is meant the ratio of the measured work done on the piston, or indicated work, to the equivalent energy of combustion of the fuel used) be calculated from the actual heats evolved in the chemical changes occurring, it will be found to be much greater for richer mixtures, but naturally uninfluenced by weaker mixtures, for no combustible matter is found in the exhaust.

This question of thermal efficiency is dealt with at greater length in Chapter III.

Increase of Volume after Combustion.

It has already been mentioned that the volume of the products of combustion is not the same as the volume of the original explosive mixture, measured at the same pressure and temperature, but that the final volume of the combustion products will be greater than the initial volume of air and petrol for the case of complete combustion into CO_2 and water, the nitrogen remaining unaltered.

In the case of mixtures both richer and weaker than this perfect mixture, it is easy to calculate from the exhaust gas analysis results the changes of volume which occur after combustion. The results of such calculations and analyses are shown graphically in Fig. 39.

From this curve it will be seen that the increase in volume for weaker mixtures is not very marked, and on the average is about 5 per cent., whereas for the "perfect" mixture the volume after combustion is between 5 per cent. and 6 per cent. of the initial volume.

For richer mixtures, the increase in volume is much more marked, and increases as the mixture becomes richer, until for the richest mixtures used it is about 15 per cent. of the initial volume.

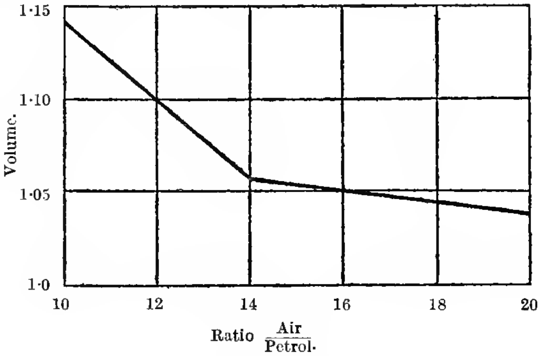

FIG. 39.

Closed Explosion Vessel Experiments.

The results of combustion experiments, made in closed explosion vessels, have thrown a good deal of light upon the subject of combustion in its relation to internal combustion engines.

It is not intended here to dwell upon the methods adopted, or the apparatus employed in these experiments, but to briefly summarize the results of these tests, in so far as they relate to internal combustion engines.

Clerk in 1884 employed a closed explosion vessel which was provided with an indicator and recorder for measuring the pressures attained, primarily with the object of discovering the pressures and pressure durations of coal gas and air mixtures. The explosive mixture of gas and air which gives the greatest explosion pressure for a given volume, and initial pressure, and which maintains the maximum pressure for the longest time period when subjected to the cooling influence of the colder walls of the vessel, is the one most suited to combustion engines.

The rapidity of the rise in pressure from the moment of ignition, is a measure of the explosibility of the mixture, and the period elapsing between the times of ignition and attainment of maximum pressure is termed the "time of explosion" in closed vessel experiments.

The combustion process is not necessarily complete when the maximum pressure is reached.

Using mixtures of coal gas and air initially at 17° C. and atmospheric pressure, the following results were obtained by Clerk :—

TABLE XIV

Mixture Proportions $\frac{\text{Air}}{\text{Gas}}$	Maximum Pressure in lbs. per sq. inch above atmospheric.	Time of Explosion in seconds.	Temperature of Explosion.*
4	80	0·16	1,595° C.
5	91	0·055	1,812° C.
6	90	0·04	1,792° C.
7	87	0·06	1,733° C.
9	78	0·08	1,557° C.
11	61	0·17	1,220° C.
12	60	0·24	1,202° C.
13	51·5	0·31	1,033° C.
14	40	0·45	806° C.

* Temperature calculated from observed pressure.

The results show that there is a certain mixture strength of about 5 which yields the maximum pressure and temperature, and a very similar mixture proportion of 6 which gives the most rapid time of explosion.

Evidently in closed-vessel experiments the explosions will take place under constant volume conditions. In this type of explosion the portion of the mixture first ignited expands rapidly, and in so doing projects the flame into the bulk of the mixture and compresses the remaining unignited portions.

The rate of inflammation at constant volume is therefore greater than that at constant pressure by the rate of flame projection (by its expansion) and also by the compression of the unignited portions by the portion first inflamed.

The explosion indicator records bring out these points, and one is therefore reproduced here (Fig. 40).

In this curve the pressure rises comparatively slowly at first, and then, as a result of the cumulative inflammation effects, the explosion rate rapidly increases.

An interesting point brought out by explosion-vessel results is that the time of explosion for the same mixture proportions can be considerably increased by igniting an appreciable mass of the gaseous mixture at once, either by several sparks at different parts of the explosion vessel, a long powerful spark, or by a flame of large dimensions.

Mixtures which are weak in gas or explosive vapour inflame at comparatively slow rates, but by igniting the mixtures in the above

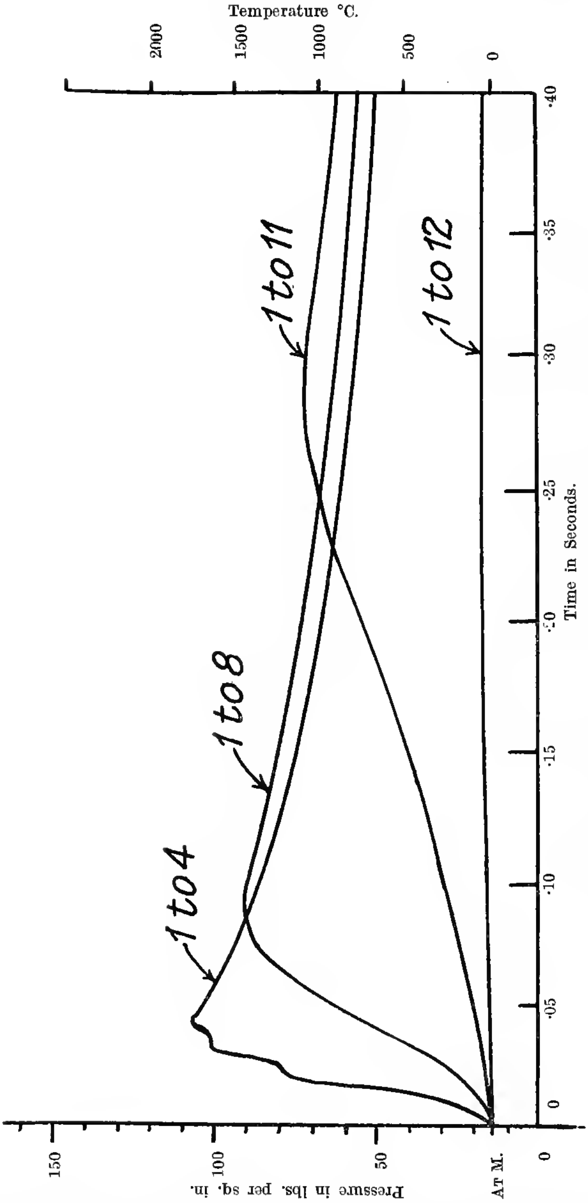

Fig. 40.—Results of Closed Explosion Vessel Experiments (Clerk).

manner very much higher explosion periods can be attained; it is also shown that the shape of the explosion vessel has a direct influence upon the explosion period—a long cylindrical vessel with the igniter at one end giving a very much longer explosion period than one of short bore to length ratio.

It is found that the maximum pressure is not increased by rapid ignition.

The larger the volume of the explosion vessel the longer is the explosion period, due no doubt to the increased path of flame travel before complete inflammation occurs.

With coal gas and air mixtures, initially at atmospheric pressure and temperature, the greatest explosion pressure is found to be about 114 lbs. per sq. inch absolute, and the highest value of the mean temperature attained is about 2,000° C.; the actual temperature at any one spot near the centre of vessel, or igniter, may be higher than this, and near the walls of the vessel will be much lower than the value given.

The effect of increasing the density of the gaseous mixture is found to result in an almost proportional increase in the maximum explosion pressure; thus if the initial pressure be trebled for the same mixture proportions, the explosion pressure will also be trebled.

Professor Petavel's * closed explosion vessel experiments which were made upon coal gas and air mixtures at initial pressures of over 1,000 lbs. per sq. inch, and which resulted in maximum explosion pressures of about 9,000 lbs. per sq. inch and temperatures reaching 2,300° C. with a coal gas to air proportion of $\frac{1}{6}$, show that the above statement holds even at these high initial pressures.

The time of explosion for these high initial pressures was about ·058 seconds, and therefore is not very different from that obtained with mixtures initially at atmospheric pressure. This fact is not, however, directly applicable to the case of high speed internal combustion engines, for reasons which are considered later.

The effect of increase in density results also in a small increase in the explosion temperature.

The explosion temperatures attained also decrease as the proportion of inflammable gas or vapour present decreases, and with the weakest coal-gas mixtures that can be electrically ignited the highest temperature attained is about 900° C.

The above facts have an important bearing, as will be shown later, upon the heat efficiency of actual engines.

The rate of rise of pressure due to explosion is found to be

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* *Phil. Mag. Roy. Soc.*, 1902.

practically the same for all pressures yet attained in closed vessel experiments; and further, this rate is scarcely affected by the value of the initial pressure.

The rate of pressure rise may be taken to be a measure of the rate of flame travel in the explosion vessel, and if the maximum pressure is assumed to be attained when the flame fills the vessel, then the actual flame velocity can be calculated from indicator records. Clerk gives the rate of flame travel estimated in this manner as varying from 2 feet to 13 feet per second in small explosion vessels, and it may be added that Hopkinson measured a flame velocity of 5 feet per second in a cylindrical vessel 23·4 inch diameter by 28·5 inches deep for an air to gas ratio of 9.

The diminution of the rate of flame travel after the attainment of the maximum rate, which is brought out by the explosion records, is not entirely due to the cooling action of the walls of the vessel, and it is now known that chemical combination still proceeds, but at a reduced rate, after the explosion vessel is filled with flame.

An interesting experiment in this connexion by Hopkinson consisted in measuring the temperatures at different parts of the explosion vessel by means of platinum resistance thermometers. Thermometers *A*, *B*, and *C* were situated in the cylindrical vessel as follows: *A* at the centre, just near the point of occurrence of the spark, *B* at a distance from the centre of roughly one-half the radius, and *C* very close to the walls. The explosion pressures and temperatures were optically recorded, and it was found that at first the pressure rose slowly, and the temperature at *A* rose rapidly. The pressure rose for 0·22 seconds at a rapid rate, and then at a decreasing rate to the maximum pressure after 0·26 seconds.

The temperature at *C* was found to suddenly rise after 0·23 seconds; this corresponded with the moment when the flame reached the walls of the vessel, and indicated that the vessel was full of flame. Further, the pressure continued to rise after the maximum temperature at the walls was attained.

The mean temperature in the vessel was 1,600° C., and at the centre *A* the platinum always melted, showing that the temperature there always reached or exceeded 1,750° C., the melting point of platinum. The temperature at *A* is given as 1,900° C. At *B* the maximum temperature was 1,700° C., whilst at *C* it was about 1,200° C.; a thermometer inserted inside, near the wall, recorded 850° C.

These experiments show that there is a large temperature variation within the explosion vessel at different points, and the same statement no doubt holds in the case of internal combustion engines.

Rates of Cooling in Explosion Vessels.

The rates of cooling in closed explosion vessel experiments, made by different experimenters, show that the larger the explosion vessel is the slower is the rate of cooling from a given temperature. The curves given in Fig. 41 illustrate this point. Curve *A* shows the temperature fall with time in the case of Hopkinson's explosion vessel of 6·2 cub. feet capacity, whilst curve *B* refers to the cooling rate for Clerk's small explosion vessel of 0·15 cub. feet.

The mean temperature of the large vessel fell from 1,600° C. to 1,075° C. in 0·5 second, while in the small vessel the temperature fell from 1,600° C. to 520° C. in the same time.

It is further found that if the density of the explosive mixture be increased in the same vessel the rate of cooling is diminished. It should be here mentioned that the actual heat flow increases with an increase in density, although the rate of cooling is diminished.

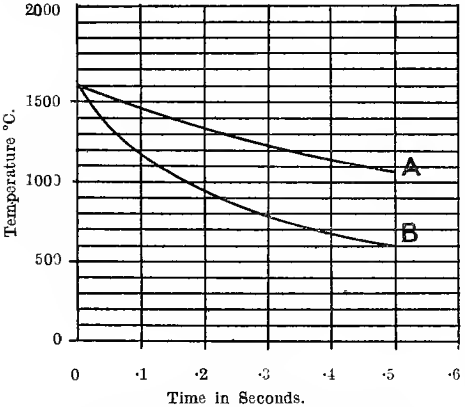

Fig. 41.—Effect of Dimensions upon Cooling Rates.

Effect of Combustion Products upon Combustion.

Some interesting experiments have been made by Grover * upon the pressures obtained in closed explosion vessels when the explosive mixtures of coal gas and air were diluted with the burnt gases of previous explosion.

The apparatus employed consisted of a cast-iron cylinder closed at both ends, and of about 1 cub. foot capacity.

The charge was ignited electrically, the resulting pressures being recorded by a Crosby indicator in conjunction with a rotating drum, upon which time intervals were also automatically recorded.

Temperatures were taken by means of a mercury thermometer.

The explosive mixture was mixed with a certain proportion of the residual products of previous explosion, and the pressures of explosion of the mixture were measured.

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* "The Effects of the Products of Combustion upon Explosive Mixtures," *The Practical Engineer,* 1895.

The results of the tests showed that the highest pressures are obtained when the volume of fresh air admitted is only a little more than that required for complete combustion, and that if these mixture proportions be employed the charge may be diluted up to nearly 60 per cent. with the products of combustion; that the latter should always take the place of an excess of air, and that when these products are employed in place of an excess of air the time of explosion is appreciably reduced.

Mr. Grover mentions that products of combustion when mixed with fresh explosive mixture have been generally supposed to reduce the efficiency of the charge, but that his experiments showed that the presence of the products of combustion in certain explosive mixtures actually has the effect of increasing the maximum pressure obtained.

The maximum increase of pressure due to the presence of the exhaust products was found to occur with the weakest mixture used, and the increase of pressure diminished as the mixture became richer in coal gas.

In order that an idea may be formed of the influence of exhaust dilution Table XV is given.

TABLE XV

RESULTS OF EXPERIMENTS UPON DILUTION OF EXPLOSIVE MIXTURES OF COAL GAS AND AIR OF RESIDUAL PRODUCTS

INITIAL PRESSURE ATMOSPHERIC

Mixture Proportions.			Maximum Pressure in lbs. per sq. inch above Atmospheric.	Maximum Rise of Pressure over that of undiluted charge, per cent.
Coal Gas.	Air.	Products of Combustion.		
6·2	93·8	0	16	
6·2	88·8	5	22	118
6·2	68·8	25	34	
6·2	63·8	30	35	
7·1	92·9	0	31	
7·1	87·9	5	27	19·4
7·1	67·9	25	30	
7·1	57·9	35	37	
7·7	92·3	0	36	
7·7	77·3	15	42	19·4
7·7	67·3	25	43	

Dugald Clerk, in criticizing the results of these experiments, points out that the presence of combustible products in the residual gases may account for the increased pressures observed.

In high speed internal combustion engines the residual gas plays an important part, and the problem is not so much one of beneficial dilution of the fresh charge, but rather one of being able, at the high speeds, to scavenge the exhaust products sufficiently to prevent low charge or volumetric efficiency and preignition.

The temperature of the residual gases has an important influence upon the initial charge temperature, and therefore upon the thermal efficiency.

Application of Closed Explosion Vessel Results to Actual Engines.

The results of closed vessel experiments cannot in most cases be quantitatively applied to internal combustion engines, on account of the different conditions existing in the two cases.

Nevertheless, valuable information and principles have been derived from these experiments which have a direct bearing upon practice; thus the different rates of cooling in large and small vessels, and in different density mixtures, the explosion pressure relations, temperature distribution, and numerous other instances may be cited.

Moreover, many of the conditions known to exist in actual engines may be artificially imitated in explosion vessel experiments; thus the effect of turbulence was very completely studied by observing the temperature and pressure rates in closed vessels by means of a fan driven at a high rate of revolution. Again, the nature of heat losses to cylinder walls of different kinds of surface has been studied by coating the explosion vessel walls with silver, highly polished, and with lamp-black.

The rates of flame travel in closed explosion vessel experiments vary from 2 to 13 feet per second, and if these results were directly applicable to internal combustion engines, it would limit very seriously their speed and practicability.

In gas and petrol engines, measurements from indicator diagrams show that the rate of flame propagation varies from 35 to 100 feet per second.

The explosion occurs only approximately under constant volume conditions, the piston travelling outwards and causing an increasing capacity during the latter stages of flame travel and combustion.

The effect of turbulence is the chief explanation of the greatly increased rates of flame propagation in the real engine.

It will be seen that the results of closed vessel experiments have an important bearing upon the practical side of the subject of internal combustion engines.

Theoretical Explosion Temperatures.

From the values given in Table IX, the amount of heat evolved by the complete combustion of 1 lb. of any of these elements or compounds is known ; thus it will be seen that when 1 lb. of hydrogen is completely burned to water 34,400 heat units ° C. are evolved, or the heat evolved would raise 34,400 lbs. of water through 1° C.

If it is required to know the theoretical temperature of combustion it will be necessary to know :—

(1) The constituents in the products of combustion.
(2) The weights of each constituent.
(3) The specific heat of each constituent, at different temperatures if variable.

Item (1) can be obtained from a knowledge of the chemical reactions or changes occurring.

Item (2) is readily calculable from the atomic and molecular weights of the elements and compounds which take part in the chemical changes (conveniently expressed as a chemical equation).

Item (3). The specific heats of the different constituents, either at constant pressure or constant volume, are not known very accurately at the present time; but in so far as the results of the researches of the authorities mentioned in Chapter I are concerned, the values for CO_2, steam and nitrogen, are approximately known for different ranges of temperature, and at different temperatures. For the purpose of being able to calculate the theoretical temperatures of combustion it would be necessary to know the values of the specific heats of the different combustion products at all temperatures.

Assuming for the present that the specific heats of the products are known to be constant at all temperatures and to be denoted by s_1, s_2, s_3 etc., and the masses of the different products formed by m_1, m_2, m_3, etc.

Then if H be the calorific value of the original fuel

$$\left.\begin{array}{l}\text{the calorific intensity or}\\ \text{theoretical combustion}\\ \text{temperature would be}\end{array}\right\} \frac{H}{m_1s_1 + m_2s_2 + m_3s_3 + \text{etc.}}$$

$$\text{or } \frac{H}{\Sigma ms}$$

For example, if we consider the case of hydrogen burning to water, we know that 1 lb. of hydrogen evolves 34,460° C. heat units (Table IX).

$$\text{Also since } \underset{4}{2H_2} + \underset{32}{O_2} = \underset{2(2+16)}{2H_2O}$$

it follows that 1 lb. of hydrogen in complete combustion forms 9 lbs. of water. Assuming that the specific heat of the water, in the state of steam, be 0·490 (from Holborn and Austin's experiments for temperatures of from 110° C. to 1,000° C.), we have

$$\left.\begin{array}{c}\text{Theoretical temperature}\\ \text{of combustion}\end{array}\right\} = \frac{34{,}460}{9 \times 0{\cdot}490} = 7{,}840^\circ\ \text{C.}$$

It will be evident that the mean specific heat for this very large temperature range will be much higher than the value 0·490 taken for the range 110°—1,000° C., so that the calculated temperature of combustion will be somewhat *lower* on this account.

This estimated temperature of combustion is very much higher than any temperature that has ever been attained in practice or even known to exist under the conditions. If the same method of calculation be applied to other cases of combustion the temperatures calculated upon the assumptions previously made, will be found to be exceedingly higher than any actually known, so that other changes must occur to account for the discrepancy between the theoretical and observed facts; it is therefore proposed to inquire more fully into the possible explanations of the discrepancies.

If a given quantity of an explosive mixture be admitted into a closed explosion vessel of any form and exploded, and further, if the rate of rise of pressure and the maximum pressure attained be measured by some means, it will be found that the pressure measured is only about 50 per cent. or 60 per cent. of the pressure that ought to be theoretically attained.

Again, if the temperature of explosion be calculated from the observed pressure rise (it cannot be measured by any known means at present) and be compared with the temperature which should be theoretically attained, as calculated from a knowledge of the products of combustion and their specific heats as previously mentioned, these observed temperatures are always found to about half of the theoretical temperatures calculated upon the assumption that all the heat of combustion is evolved or liberated. Dugald Clerk, Mallard and Le Chatelier and other authorities have found experimentally that the actual temperatures and pressures obtained in explosion vessels are very much lower than the calculated quantities.

In the experiments of Clerk various mixtures of coal gas and air were exploded in a closed explosion vessel, and the pressures and durations of pressure of explosion were measured. Further, from the pressures observed the maximum temperatures attained during explosion were calculated. Figures illustrating these results are given in Table XVI, and from these it will be noticed that neither the theoretical temperatures nor pressures are attained.

TABLE XVI

EXPLOSION OF COAL GAS AND AIR

Ratio $\frac{\text{air}}{\text{gas}}$ by Volume.	Observed Pressure lbs. per sq. inch above Atmosphere.	Calculated Pressure.	Temperature of Explosion calculated from observed Pressure.	Theoretical Temperature of Explosion if all Heat were evolved.
14	40	89·5	806° C.	1,786° C.
13	51·5	96	1,033° C.	1,912° C.
12	60	103	1,202° C.	2,058° C.
11	61	112	1,220° C.	2,228° C.
9	78	134	1,557° C.	2,670° C.
7	87	168	1,733° C.	3,334° C.
6	90	192	1,792° C.	3,808° C.
5	91	—	1,812° C.	—
4	80	—	1,595° C.	—

The experiments of other investigators, such as Bunsen, Hirn, Mallard and Le Chatelier, etc., confirm these results, which go to show that only about one-half of the heat energy of the fuel is apparently developed in the explosion.

Several interesting explanations for this apparent loss of heat energy have been advanced by different authorities, and will be here considered.

The four chief theories put forward in explanation of the above facts are—

(1) The Dissociation Theory.
(2) The Wall Action or Cooling Theory.
(3) The Delayed or After Burning Theory.
(4) The Increasing Specific Heat Theory.

(1) The Dissociation Theory.

In order to account for the observed suppression of temperature in explosions of gaseous mixtures Dugald Clerk has suggested that the very high temperatures of explosion are sufficient to cause the gases present, such as steam and CO_2, to split up or to "dissociate" into their respective elements, and in so doing to absorb heat, thus reducing the temperature and pressure to values not very different from the observed ones.

It is well known that when gases dissociate, heat is absorbed in the operation in the same way as heat is evolved when these gases combine together.

Experiments relating to dissociation temperatures of steam and carbon dioxide have been made by Grove and Deville,* which show definitely that at temperatures of from 900° to 1,000° C. in the case of the steam, and at 1,300° C. for carbon dioxide, dissociation commences to occur.

Deville has also showed that carbon monoxide was partly dissociated into carbon and carbon dioxide (for the oxygen evolved or set free combines with the undissociated carbon monoxide) at temperatures of about 1,300° C., and from the results of later experiments upon carbon dioxide he concludes that CO_2 is "strongly dissociated at 1,200° C.," to use his own words. As the actual temperatures attained during combustion in explosion vessels and engine cylinders is somewhere between 1,000° C. and 2,000° C., it is quite reasonable to conclude that some of the products of combustion, at least, cannot exist in a state of perfect chemical combination, but that the undissociated and dissociated products will exist together until the temperature falls sufficiently for the free elements of dissociation to recombine. The evolution of heat during the recombination will evidently occur after maximum temperature has been reached, and will cause the expansion curve to then lie above the theoretical one.

Dissociation would then appear to consist of a suppression of heat at the maximum temperature of explosion, and an evolution of heat during the expansion stroke.

The exact amount of dissociation occurring forms a point of controversy amongst different authorities at the present moment, although there appears to be no doubt that dissociation does prevent the attainment of very high temperatures during combustion. It has previously been shown that with no dissociation, hydrogen burning in oxygen would give a theoretical temperature of at least 7,000° C., yet the experiments of Bunsen, Deville and others show that even the temperature of 3,000° C. is not attained.

Again, in the case of hydrogen and air exploded at constant volume, it has been shown that whereas without "dissociation" the temperature attained should be 4,000° C. actually it only reaches about 2,000° C.

It is, therefore, evident that dissociation plays an important part in combustion, and that the temperatures of compound gases are thereby limited. The general opinion of scientific authorities appears to be that the whole cause of the heat suppression is not explained on the dissociation theory, but that dissociation is only one of the contributing causes.

It is of interest to notice in comparing the results of tests upon wide ranges of explosive mixtures, at different pressures and temperatures, that the amount of apparent heat suppression is about the same in all cases; thus for the richest mixture given in Table XVI this amount as indicated by the ratio of the observed to the calculated pressures and temperatures is nearly the same as for the weakest mixture. This and the fact first mentioned by Schöttler,* that the expansion curve does not suddenly vary its course and shape as it should do in accordance with the above theory, are evidence that dissociation by itself does not fully account for the actual changes observed.

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* *Leçons sur la Dissociation professées devant la Société Chimique de Paris*, 1864.

(2) Influence of the Cylinder Walls.

The whole subject of "wall action," as it is termed, is of great importance to students and designers of internal combustion engines, apart from affording a reasonable explanation of the observed facts of heat suppression already alluded to.

During the whole of the cycle of operations of an internal combustion engine, heat is being exchanged between the working gases within the cylinder and the walls of the combustion chamber and cylinder.

During the combustion of the charge within the cylinder the average temperature of the exploding gases is, of course, exceedingly higher than that of the walls of the combustion chamber, so that the charge loses heat to the walls both by radiation and by conduction. There is also another factor influencing the heat flow, namely, the degree of agitation or turbulence of the gases; this factor will be considered later.

From the point of view of heat suppression, it will be evident that as the temperature of the combusting gases is very high as compared with that of the cylinder walls (or in the case of explosion vessel experiments, the walls of the explosion chamber) there is a certain proportion of the heat of combustion absorbed by the cylinder walls by the process of radiation of heat from the hot gases. Conduction from particle to particle, and turbulence, also contribute towards the total loss of heat to the cylinder walls, but radiation is probably the most important process of heat loss involved.

If during combustion heat is lost from the combusting gases to the cylinder or combustion chamber walls, there will result an apparent heat suppression similar to that observed in engine and closed explosion vessel experiments.

Wall action, however, does not afford a full explanation of the experimental facts, and the general opinion here, also, is that it is only one of the important influencing factors.

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* *Die Gasmaschine*, by Prof. R. Schöttler, Brunswick, 1902.

Radiation and the other contributing influences discussed under the wall action theory cannot directly account for the nearly constant heat suppression observed over wide ranges of temperature and pressure ; this will be evident from the following considerations.

There are three important factors concerned in wall action, which are as follows :—

(1) Radiation.
(2) Conduction.
(3) Turbulence.

Dealing with these in the order named, the first factor, radiation, is very important from both the theoretical, and from the practical aspect.

Radiation is almost of the same degree of importance in petrol and other explosion engines as the conduction of heat through the cylinder walls is, in determining the nature and amount of the heat flow through the walls.

In the case of large Diesel and gas engines, the chief characteristics of the designs are the arrangements made for the cooling ; this is apparent in the weight and size of these engines in proportion to their power output.

Heat flow and the temperatures thereby involved are of great importance to engine designers, in connexion with temperature stresses and distortions, so that if the principles governing heat flow are known, it will enable the designer to allow for the effects of temperature changes, density, composition of the gases, and other important factors. The heat loss by radiation from the exploding and expanding gases to the cylinder walls depends upon several factors; to give an example, the absolute mean temperature of the gases will partly govern their heat loss by radiation, whilst the density, or compactness, of the gases, the area and shape, and the state of the cylinder walls each influence the radiation effect, quantitatively.

The Temperature Effect of Radiation.

In the case of an incandescent mass of gas at an absolute temperature T ° C. it has been shown by Stefan, Dulong and Petit and others that the total quantity of heat radiated in unit time is a function of the absolute temperature T, and can be expressed in symbols thus :—$H_T = 16 \times 10^{-10} T^4$

Where H_T is the quantity of heat radiated per square foot of area of the radiating body per hour, expressed in B.T.U.s.

If we assume the area of the cylinder walls to be A sq. inches, and T_w their mean temperature, and further if T_c is the mean temperature of the exploding gases; then, if there were no other causes contributing to heat flow but the temperature, *the theoretical loss of heat by radiation to the walls per second would be given by* $3{\cdot}8 \times 10^{-15} \times A(T_c^4 = T_w^4)$ *B.T.U.s.*

The curve shown in Fig. 42 illustrates the radiation heat loss variation with the temperature, but is only intended to represent this effect qualitatively.

It will generally be found in using the above expression that the term T_w^4 is negligible, this term being very small compared with T_c^4.

Examples * of the application of the above expression to the case of steam engine boilers and fireboxes suffice to show that it can, in these cases, yield quantitative results which at least enable one to form ideas of the magnitude of the temperature effect upon the quantity of heat lost to the walls by radiation. The reader would do well to consult the papers mentioned in the footnote.

Much interesting and useful information has been obtained from radiation experiments upon flames of coal gas burning in air, in which the thickness, volume, and constitution of the flame could be altered.

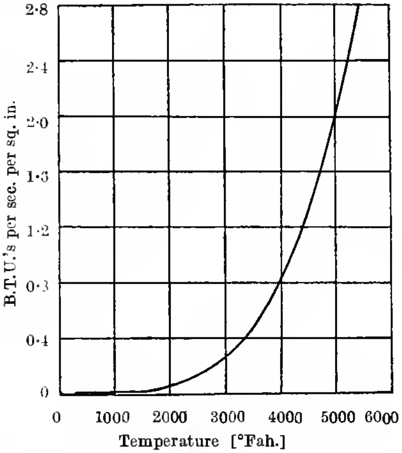

FIG. 42.—RADIATION LOSSES AT DIFFERENT TEMPERATURES.

Professor Callendar † was the first to point out that the loss of heat of combustion in closed vessel explosions, including internal combustion engines, is of the same order as the heat loss measured from non-luminous Bunsen flames which radiate from 15 to 20 per cent. of their heat of combustion. Further, ‡ he mentions that although it is impossible to estimate, separately, the exact amount of radiation heat to the cylinder walls, it is probable that the loss by radiation is proportional to the area of wall surface exposed, and is practically independent of the time, since the period of duration of the flame is exceedingly short in the most efficient mixtures.

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* *Proc. I.M.E.*, 1909, "Heat Transmission" (Dalby).

† *Proc. R.S.A.*, lxxvii, p. 400, April, 1906.

‡ *Proc. I.A.E.*, April, 1907.

From this it follows that the heat lost by radiation to the cylinder walls during combustion is independent of the engine speed.

It is also probable that part of the radiation loss which occurs during combustion (or propagation of the flame throughout its mass) is proportional to the volume and not to the surface, and so this part of the radiation loss is constant.

From these experiments upon the heat radiated from flames of various sizes and burning under different conditions Prof. Callendar showed that when the air and gas were in the proportions for complete combustion the average amount of heat radiated was 12·5 per cent. of the heat of combustion.

As the air supply to the burners was reduced for the same rate of gas consumption, the size of the flame increased, as also did the quantity of heat radiated.

A maximum of about 20 per cent. was reached under these conditions, corresponding with enrichment of the mixture; as the gas supply was reduced, relatively to the air supply, the radiation fell off greatly as a result of the falling off of flame temperature and size.

With the steady luminous flames of the bat's wing type of burner a considerable increase of radiation was obtained by excluding air from the flame.

With low temperature flames the proportion of heat radiated was very low, often only 2 per cent.; this also follows from Stefan's law, previously given.

The effect of composition of mixture upon the radiation from the open flame is shown in Fig. 43, which illustrates some of the results of Callendar's experiments.

The total radiation of a Mêker burner, expressed as a percentage of the heat of combustion, is shown plotted against the ratio of air to gas by volume in the mixture. This air-gas proportion will of course affect both the temperature and construction of the flame.

It will be evident from Fig. 43 that the net effect of enrichment of the mixture strength is to increase the amount of heat radiated.

In connexion with these results it should be mentioned that mixtures in the given proportions, if burnt without further addition of air, would not radiate the same percentage of heat.

It is interesting to note that for a ratio of air to gas of 5 the duration of the luminous flame was about $\frac{1}{50}$ of a second.

It must be borne in mind that the flames in the above experiments were burning in air, and therefore these experiments were not quite analogous to closed explosion vessel ones.

As regards the effect of gas and air proportions upon the amount of the radiation loss, some experiments by Hopkinson * show some interesting results.

Experimenting upon a 40 h.p. gas engine the following results were obtained :—

	Case (1).	Case (2).
Percentage of gas in cylinder contents .	8·5 per cent.	11·0 per cent.
Total heat loss per minute	1,510	2,300 B.T.U.s.
$\frac{\text{Total heat loss}}{\text{Total heat supply}}$	29 per cent.	34 per cent.
Piston temperature	300° C.	430° C.

The effect of increasing the quantity of gas in the mixture relatively to the air, case (2), will be seen in the increased heat loss to the walls, for whereas the quantity of gas is increased by 30 per cent., the piston temperature is 43 per cent. higher, and the total heat loss is over 50 per cent. greater than in case (1).

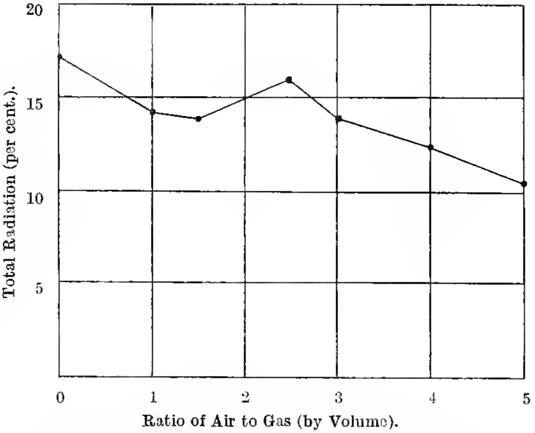

Fig. 43.—Radiation Loss in Mêker Burner Flame for different Air-Gas Ratios.

If the loss of heat were due to conduction, it would diminish with an increase of gas in the mixture, because the temperature with the stronger mixture would be relatively less on account of the increased volumetric heat.

The existence of radiation, however, which increases more rapidly in proportion with the temperature, would account for this increased heat flow to the walls.

Effect of Volume upon Radiation.

Reverting once again to the open flame experiments of Callendar, it was shown by measurements of the radiation from different burners grouped together, that the amount of radiated heat was proportional to the volume for small flames.

This was illustrated by first measuring the radiation from a

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* Hopkinson, *Proc. Roy. Soc. A.*, vol. lxxvii, p. 400.

single Mêker burner, whose cone of radiation was measured, and then by placing another similar burner in a line with the first one and the measuring instrument.

The quantity of radiated heat measured was greatly increased, but was not, however, doubled.

A third flame placed behind the other two caused a further, but relatively smaller, increase in the radiated heat measured. As the row of flames was increased the radiation tended towards a finite limit.

The general result of these experiments shows that for small flames of three centimetres or less the radiation is proportional to the volume.

If, however, the diameter of the flame be increased beyond three centimetres the radiation will also increase, although not in proportion with the volume of the flame.

In the case of very large flames the radiation tends to become proportional to the surface.

The radiation from hot gases is rendered more complex owing to the fact that the gas is to an appreciable extent transparent to its own radiation; that is to say, the quantity of heat radiated depends upon the thickness of the mass of the gas instead of being purely a surface phenomenon, as is the case with solid bodies.

In the Report for 1910 of the Gaseous Explosions Committee of the British Association it is stated that:—

" The flames met with in internal combustion engine cylinders or in explosion vessels differ from the open flames producible in the laboratory, both in respect of the lateral extension and also in respect of density.

" In both of these particulars the difference is rather great, the least dimension of the mass of flame in a gas engine cylinder being only in the smallest sizes comparable with the diameter of the Mêker burner flame, while the density of the gas just after firing in the gas engine is from twenty to thirty times that of the burner flame gases."

It does not appear possible to determine the effect of these two factors, from theoretical considerations, with enough accuracy to enable any quantitative inference as to radiation in the case of gas engines to be drawn from flame experiments, although their qualitative results are important.

Effect of the Time Factor upon Heat Loss.

Although, as has been already mentioned, the period of explosion in petrol and like engines is very small, yet it is of some interest to consider the effect of the time factor upon the heat loss.

From explosion vessel and other experiments, Hopkinson has shown * that the heat loss measured from the moment of ignition varies as the square root of the time elapsing,

or $H = k\sqrt{t}$, where k is a constant, and t = the time of the explosion from the moment of ignition.

For the cooling of a flame the heat loss is expressed in the following manner :—$H = 2T\sqrt{\frac{k.c.t}{\pi}}$

where T = the flame temperature
k = thermal conductivity
and c = the thermal capacity per unit volume.

It has already been pointed out (page 91) that the heat loss during explosion is of small proportion, and in most cases of internal combustion engines, is negligible.

Effect of Pressure upon Radiation.

The real effect of pressure upon radiation cannot be stated exactly, in view of the difficulties encountered in the experimental determinations, but it is possible to predict the effects with some degree of accuracy.

Callendar states that within moderate limits of pressure the radiation and absorption powers of a flame of unit thickness at a given temperature and of a given composition should vary directly as the pressure or density.

The value of the radiation from a layer of thickness 1 cm. at a pressure of 10 atmospheres should be the same as that from a layer of 10 cms. at 1 atmosphere, assuming that the quality of the radiation or the nature of the combustion is not affected by pressure.

To a first approximation, therefore, it can be assumed that the radiation will be proportional to the density for a given mass of gas at a constant temperature ; that is to say, two similar masses of flame in which the temperatures at corresponding points are the same, and in which the densities are inversely proportional to the volumes (i.e., constant masses), will radiate heat similarly and equally.

From the point of view of the molecular theory of gases, an increase of density implies a proportionate increase in the frequency of molecular collisions, and this would result in greater facility for interchange in the translational and atomic energies, besides which an increase of density would lessen the distance between consecutive molecules, thus causing an interaction and consequent variation in their separate vibrations.

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* *Proc. Roy. Soc. A.*, vol. lxxix.

For all practical purposes it has been shown that it is sufficiently accurate to assume that the radiation varies as the density to the power 0·9, or expressed in symbols, $R = k.\rho^{0·9}$, where k is a constant, and ρ = the density.

It follows that in the case of an engine working upon the Otto cycle, as the density of the mixture is a maximum towards the end of the firing stroke (at the moment of ignition), the radiation will here be greatest in amount.

Effect of Nature of Wall Surfaces upon Radiation.

It has been found that the quantity of heat lost to the walls of the explosion vessel or combustion chamber, other conditions remaining the same, depends upon whether the surfaces of the walls are polished or are coated with a good heat absorbent, such as carbon.

Hopkinson * has made experiments upon this subject and has determined the relation between the heat loss and surface condition in explosion vessels.

In these experiments a mixture of gas and air was exploded in an explosion vessel the walls of which were coated with tinfoil, and the pressure developed, together with the rate of heat loss to the walls, were compared with the values obtained in the case in which the walls were coated with lamp-black.

It was found that the difference in maximum pressure was very small, but that the rate of fall of pressure during cooling was considerably less with the bright lining than with the lamp-black surface.

Experiments which have been made upon gas engines with polished combustion chambers also show that a perceptible increase in mean effective pressure results.

Additional evidence in support of these results has also been obtained from experiments made with explosion vessels coated internally with silver.

In these experiments exactly similar proportions of mixtures were exploded, firstly with the interior surface coated with lamp-black, and secondly with the silver coating.

It was found that in the second case the maximum pressure attained was increased by 3 per cent., whilst the subsequent rate of cooling of the exploded products was reduced to about one-third.

These results would tend to show that the heat efficiency and mean effective pressure of an internal combustion engine will be appreciably greater with a polished combustion chamber than with a carbon-coated surface.

Clerk estimates the heat loss by radiation on the explosion line

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* *Gaseous Explosions Committee's Report,* 1909.

in a gas engine using the most economical mixture and a maximum temperature of 1,800° C. to be about 3·0 per cent., when the explosion period is $\frac{1}{30}$ second under ordinary compression conditions.

Heat Flow by Conduction.

As distinct from the radiation of heat from the mass of the burning and burnt gases to the cylinder walls, heat is also lost, or rather interchanged, between the cylinder contents and the walls by the process of conduction through the gases themselves and through the metal walls of the cylinder to the water jacket or the air beyond.

It will be evident that the conduction processes within the cylinder are not very simple in their action, when it is remembered that there is always a film of gas clinging to the exposed combustion chamber walls, and in addition often a layer of carbon, or an oil film, interposed between the gases and the metal of the cylinder walls.

It is a generally accepted fact that there is a thin film of gas clinging to the walls of the combustion chamber, which is uninfluenced and unaffected by convection currents or turbulence of the gases, and which, further, offers an appreciable resistance to the passage of heat by conduction.

Similarly upon the other outer wall of the cylinder there is a water film which is more or less unaffected by the motion of the main body of circulating water in the water jackets, and which also offers resistance (but which in this case is much smaller than in the case of the gaseous film) to the transfer of heat from the walls to the water itself.

If the cooling medium be air, there is reason to suppose that there is an air film clinging to the outer wall surfaces which offers a similar resistance to the internal gas film.

It will thus be seen that the heat flow has to travel through the following surfaces :—

(*a*) Between the main body of the gas and the gas film.
(*b*) The surface between the gas film and the metal wall of the cylinder.
(*c*) The outer surface between the metal wall and the water (or air) film.
(*d*) The junction of the water (or air) film and the main body of the water (or air).

If in addition to this the cylinder walls be coated with carbon deposit or oil, and the outer surface be covered with a water sedimentary deposit or dirt, then additional resistance paths to the heat flow will be established—such resistances are commonly known to occur in internal combustion engine cylinders or walls.

In considering the conduction of heat through the main body of the hot gases, the temperature gradient (which largely governs the heat flow) is comparatively small through the main body of the gases to the gas film, so that to all intents and purposes the amount of heat conducted through the hot gases themselves is negligible.

If we next consider the temperature of the hot gases at some instant during combustion or expansion, it is possible to obtain an idea as to the relative magnitudes of the resistances encountered, which are due to the interposed media.

The "thermal resistances" of different thicknesses of various substances vary inversely as their conductivities and inversely as their thicknesses, or expressed in symbols $R_T = \frac{c}{k.t}$

where k = conductivity in suitable units
and t = thickness of substance
c = a constant
R_T = the resistance to heat flow.

The difference in temperature between two sides of a plate of given thickness of any substance is a measure of the heat flowing through this substance, in unit time.

Further, for a given heat flow through different thicknesses of various substances, the temperature drop necessary for this given flow will be proportional to the "thermal resistance" of the substance, as defined above.

In a diagram in which the existing temperatures at different depths of the substance are shown by vertical ordinates, these ordinates will also show the relative values of the resistances offered; the greater the temperature drop for a given thickness (or depth) the greater is the resistance encountered by the heat in its passage.

In the case of a petrol engine, the walls are comparatively thin, and the distribution of temperature across them is practically uniform, so that the general expression given later can be greatly simplified.

Thus for approximate purposes of calculation, if T_0 and T_1 are the temperatures of the inner and outer surfaces of the cylinder wall, of thickness d, and k the conductivity, then the heat flow per unit time per unit area of walls $= \frac{(T_0 - T_1).k.}{d}$

Most of the published values of the constant k, which is termed the coefficient of conductivity, are given in C.G.S. units, the areas being in sq. centimetres, the thicknesses in centimetres, and temperatures Centigrade. If, however, the B.T.U. of heat and the Fahrenheit scale of temperature be employed, keeping the linear dimensions in centimetres we have the heat flowing in time t seconds, expressed in B.T.U.s $= \dfrac{k^1(T_o - T_1)}{d}.A.t$

Where t = the time in seconds
A = area in sq. centimetres
d = thickness in centimetres.

Values for this new constant k^1 are given below, for different metals.

Metal	k^1
Iron	$1{\cdot}8 \times 10^{-4}$
Steel	$4{\cdot}0 \times 10^{-4}$
Copper	$20{\cdot}0 \times 10^{-4}$
Brass	$5{\cdot}5 \times 10^{-4}$
Carbon	$\cdot 036 \times 10^{-4}$

As the temperature of the inner wall of an explosion vessel, or cylinder, is constantly changing, and at a very rapid rate, the actual heat flow which occurs is more complicated.

Resuming, for the moment, the consideration of the loss of heat to the outer cooling medium, it will be evident that, the films and thicknesses being small, the temperature gradients can be represented by straight lines, and with a knowledge of the conductivities of the different substances concerned, we can, for steady conditions, find the corresponding heat flows.

For the purposes of illustrating the nature of the relative resistances offered to the transfer of heat in the case of an internal combustion engine cylinder, Fig. 44 is reproduced.

The relative temperature gradients are shown quantitatively in this figure; the resistances offered to the heat flow are also indicated by these temperature gradients.

The diagram shows that the resistance of the gas film is very high, compared with that of the liquid films, and at *ef* there is probably a temperature drop due to change of contact surface, and which is analogous to the potential difference set up electrically between the surfaces of contact of two different metals.

The fall of temperature in the metal itself is very small, comparatively speaking, and there is another contact surface fall of temperature at *fg*, followed by the water film drop, and finally by the water body gradient, which is also very small.

The total resistance offered to the flow of heat is the integral of the respective resistances shown.

It has previously been stated that the gas film offers an exceedingly high resistance to the heat flow; and in further evidence of this fact, the resistance of the gas film in the analogous case of a steam boiler is stated * as being 98 per cent. of the total resistance encountered, the other 2 per cent. representing the resistance of the remaining influences.

It was also mentioned, in the Paper referred to, that in the case of hot furnace gases impinging upon one side of the firebox, the other being in contact with the water film and water at 212° F., the hot side of the metal plate was only about 40° above the temperature of the water side.

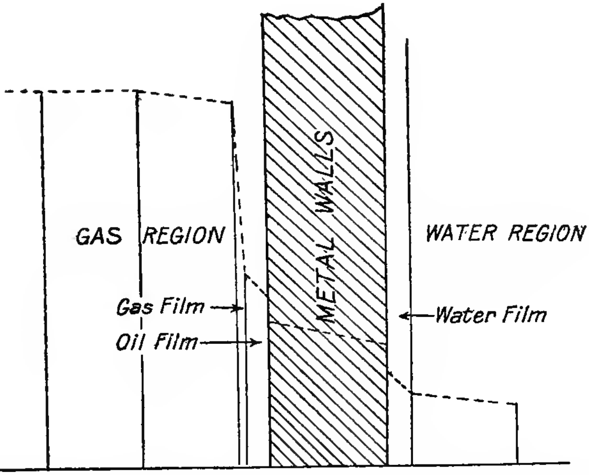

FIG. 44.—TEMPERATURE GRADIENTS IN PETROL ENGINE CYLINDER.

Another interesting result was obtained by coating the water side of the heating surface with grease; when a greasy deposit $\frac{1}{16}$ inch thick was present the temperature difference rose to 550° above the temperature of the water. This was equivalent to an increase in the thermal resistance nearly fourteen times that in the case of clean surfaces.

Very much the same effect probably occurs in the cylinder and jackets of an internal combustion engine, the layer of carbon deposit and oil upon the walls offering a high resistance to the heat flow. Any sedimentary deposit or dirty surfaces within the water jacket would have a similar effect upon the heat flow.

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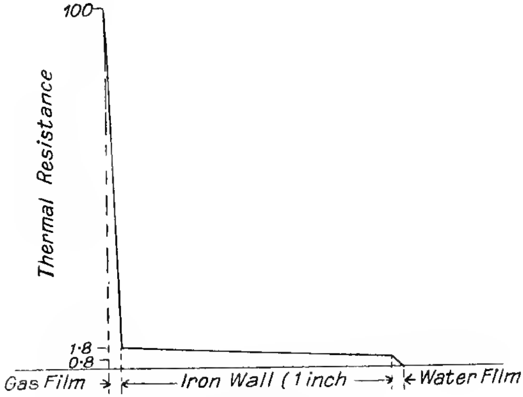
* *The Engineer*, vol. lxx, p. 523 (Hudson).

Example of Approximate Temperature Gradients.

Consider as a further quantitative illustration of the relative temperature gradients existing, the case of a cast-iron plate 1 inch thick, one side being exposed to water and the other to air. The air and water films are probably of the order of $\frac{1}{100}$ inch thick.

The thermal resistance of the iron plate is taken as unity, so that the resistance of the air film is $\frac{3,500}{100} = 35$, and that of the water film $\frac{80}{100} = 0{\cdot}8$.

The numbers for the relative conductivities of air, water, and iron are taken from Kelvin's figures,* which give the conductivity of iron as 80 times that of water, and that of copper 500 times that of water.



Note: The film thicknesses are shown exaggerated.

FIG. 45.—HEAT FLOW RESISTANCES.

Also the conductivity of iron is given as 3,500 times that of air, whilst that of copper is 20,000 times that of air.

Hence the relative resistances of the air film, the iron and the water film will be as 35 : 1 : 0·8.

Fig. 45 shows the corresponding temperature gradients for this type of heat flow. It follows from the above figures that roughly about 95 per cent. of the total resistance to the heat flow is due to the gas film, 3 per cent. the iron plate, and 2 per cent. due to the water film.

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* "Heat," *Ency. Britt.*, 9th Edition, p. 577.

In the case of the cast-iron walls of a motor engine cylinder the resistance of the metal to the heat flow would be about one-quarter of above figure, or less than 1 per cent. of the total resistance encountered.

Heat Flow in Petrol and Gas Engines.

Some interesting data has been obtained in connexion with a series of tests made upon engines by Hopkinson, the results of which are given in full in a Paper * read before the Institution of Automobile Engineers in 1909.

From the results of these tests, it appears that about 30 to 50 per cent. of the heat of explosion ultimately passes into the walls of the cylinder; this is the heat which has to be removed by the water jacket.

The temperature gradients, which must exist in order that the heat may be conducted from the explosion side of the cylinder to the water side, give rise to rather high temperature differences between different parts of the metal, and to high local temperatures at such places as the centre of the piston and the valves, which are remote from the water jackets.

The effect of these temperature differences is to cause unequal expansion and corresponding stresses, besides which is the tendency towards preignition of local places at high temperatures.

The factors which determine the temperature of the metal at any point on the inner surface of the walls are the mean rate of heat flow per unit area into the wall at that point and the thickness of the walls at the same point.

The real heat flow will, as we have pointed out in connexion with the subject of radiation, depend upon the density, temperature and turbulence of the gas; the latter effect is dealt with at length later.

The thicknesses of the air and other existing surface films, the thickness of the metal of the walls, and the conductivity of the different substances concerned, will determine the temperatures of the metal walls. The size and design of the engine also influence to a great extent the temperature distribution.

Hopkinson arrives at a working idea of the magnitude of the heat flow in the cases of a Crossley gas engine of 11·5 inches diameter and 21-inch stroke, and a four-cylinder Siddeley petrol engine tested by him with a bore and stroke, respectively, of 4·6 and 5·1 inches. He assumes the rate of heat flow per unit area to

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* "The Effect of Size and Speed upon the Performance of an Internal Combustion Engine," Inst. Aut. Engrs., February, 1909.

be obtainable by dividing the mean area exposed to the hot gases into the total heat carried away by jacket water and radiation.

This estimate of heat flow gives a general idea of the average heat flow, but does not take into account the want of uniformity of the cylinder, walls, piston, etc.

The figures given in Table XVII illustrate the results obtained in the two cases. It will be seen that the heat flow per unit area into the petrol engine cylinder walls is nearly twice as great as in the case of the gas engine, and in numerical value is about five thermal units per sq. inch per minute, upon the average.

TABLE XVII

	Crossley.	Siddeley.	Ratio.
In centre surface area in sq. inches . .	400	82	4·9
Out ,, ,, ,, ,, ,, ...	1,160	156	7·4
Mean surface ,, ,, ,, .	780	119	5·5
$\frac{\text{Mean surface}}{\text{Total cylinder volume}}$	0·30	1·06	0·28
Percentage of heat lost to jackets, etc.	40·5	34	1·2
Total heat loss per minute	2,250	2,300	—
$\frac{\text{Mean heat loss}}{\text{Mean surface}}$	2·9	4·85	1·7

Hopkinson gives as the temperature gradient necessary for this heat flow through cast iron 100° C. per inch, or with the usual thicknesses of cylinder of petrol engine practice about 20° C. difference of temperature.

If we consider the existence of gas, liquid or solid films, this temperature gradient through the metal itself will be considerably less than the figure above given.

Under all circumstances, therefore, it would appear that the temperature of the jacketed portions of the cylinder metal can be taken as being that of the jacket water with sufficient accuracy.

The same general results apply to the case of the gas engine in which the cylinder walls are nearly four times as thick as those of the petrol engine.

In some tests made in connexion with temperatures of gas engine cylinders, the inside of the combustion chamber was partly lined with tinfoil, the melting point of which was about 230° C. It was found that the tinfoil was quite unaffected by the explosions, thus showing that the inner walls could not have been at a very high temperature.

With reference to the effect of a layer of carbon deposit, the conductivity of which is probably about $\frac{1}{50}$ that of iron, it appears from the above figures that for the necessary heat flow per sq. inch given a layer of carbon $\frac{1}{20}$ inch thick would have a mean temperature of about 350° C., and a layer of twice this thickness would have a temperature of about 700° C. This temperature would be quite sufficient to set up preignition, or premature ignition, of the compression charge.

Other interesting points brought out by these experiments in connexion with the temperatures of the piston and valves and the stresses occurring in the metal of the piston due to the high temperature effects are dealt with in the original Paper, to which the reader is referred.

Mathematical Investigation of Heat Flow.

In order to obtain a true theoretical conception of the heat flow in thick masses of material, under given conditions of temperature and time, it is necessary to consider the flow of heat through an infinite block of material one face of which is plane surface. The general results obtained can be then modified to suit the special cases occurring in practice.

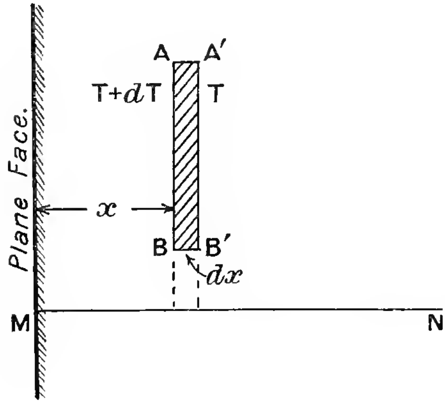

FIG. 46.

Referring to Fig. 46, imagine one face of this block of material of conductivity k, specific heat s, and density ρ, to be at a temperature T.

Let distances in the direction of the heat flow lines MN be represented by x.

Consider an elementary section of thickness dx and area A.

Then the flow of heat across AB in unit time $= k\mathrm{A} \cdot \dfrac{d\mathrm{T}}{dx}$

The flow of heat across $\mathrm{A}^1\mathrm{B}^1$ at a distance $x + dx$ will be $-\,k\mathrm{A}\,\dfrac{d}{dx}\left(\mathrm{T} + \dfrac{d\mathrm{T}}{dx} \cdot dx\right)$

Hence the gain of heat by the element $AA^1B^1B = kA.\frac{d^2T}{dx^2}.dx$ (*a*) in the limit.

This is a general expression for the heat absorbed by any elementary section.

The effect of this heat will be to raise the temperature of the element. If we consider the rate of rise of the temperature of the element, then the gain of heat can be expressed as mass × specific heat × rate of heat gain,

$$\text{that is by } (\varrho.Adx).s.\frac{dT}{dt} \quad . \quad . \quad . \quad . \quad . \quad (b)$$

This gain of heat must evidently be the same as that previously obtained in (*a*).

Hence equating these expressions we have

$$kA\frac{d^2T}{dx^2}dx = -\varrho As\frac{dT}{dt}dx$$

$$\text{or } \frac{d^2T}{dx^2} = -\frac{\varrho s}{k}.\frac{dT}{dt} \quad . \quad . \quad . \quad . \quad . \quad (c)$$

From this differential equation, the relation between temperature, time, and distance can be obtained, knowing the conditions existing in any special case, assuming that the specific heat is constant.

Next, consider the heat of conduction by the element under discussion to be radiated or emitted from the surface of the element; then a further relation can be obtained.

If we denote the quantity of heat emitted per unit area of the surface, per unit difference of temperature, per unit time, by E, then heat lost by emission by element AA^1B^1B per unit of time $= E.p.T.dx$ where p = its perimeter.

Hence we have from (*a*)

$$kA.\frac{d^2T}{dx^2}dx = EpTdx$$

$$\text{or } \frac{d^2T}{dx^2} = \frac{Ep}{kA}.T \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

The solution of this equation is of the form

$$T = T_0 e^{-\mu x} \text{ where } \mu = \sqrt{\frac{Ep}{kA}}$$

This result indicates that the temperature falls from initial value T_0 according to an exponential law, with the distance from the plane face of the solid.

And by combining (*d*) with (*c*) we have

$$\mathrm{E}p\mathrm{T} = -\mathrm{A}s\varrho.\frac{d\mathrm{T}}{dt}$$

from which $\mathrm{T} = \mathrm{T}_0 e^{-mt}$ where $m = \frac{\mathrm{E}p}{\mathrm{A}s\varrho}$

From this result it will be seen that the temperature varies with the time, at any given point in the solid, according to an exponential law.

Heat Flow for Cyclical Variations of Temperature.

In cases of steam and internal combustion engines, the temperature of the side of the metal walls exposed to the steam or hot gases is continually changing with time, and periodically undergoes a cycle of changes.

The effect of this cyclical variation of temperature is to cause waves of heat to travel into the metal walls.

The simplest form of temperature oscillation applicable to such cases can be represented by the sine law of variation.

Assume that $\mathrm{T}_0 = a \sin 2\pi n.t$ is such a law of variation where n = frequency of temperature cycles and a is a constant. This expression symbolically shows that T_0 is the temperature of the skin of the metal, where $x = 0$ at any time t.

The actual value of the constant a can be obtained from the differential equation (*c*) by differentiating and equating differential coefficients in the usual way.

This result can, however, be obtained by working conversely from the relation (*c*), that is,

$$\frac{d^2\mathrm{T}}{dx^2} = -\frac{\varrho s}{k}\cdot\frac{d\mathrm{T}}{dt}$$

The solution of this well-known type of equation is, in its simplest form,

$$\mathrm{T} = \mathrm{A}\varepsilon^{-ax} \sin (qt - \gamma x) \quad . \quad . \quad . \quad . \quad (e)$$

$$\text{where } a = \pm\sqrt{\frac{\pi n\varrho s}{k}}$$

If now $\mathrm{T} = 0$ when $x = \infty$, that is to say the temperature at infinity is zero, and if $\mathrm{T} = a \sin qt$ when $x = 0$ (the cyclical temperature variation at the plane face).

We can write (*e*) as

$$\underline{\mathrm{T} = \mathrm{A}\varepsilon^{-ax} \sin (2\pi nt - ax)} \quad . \quad . \quad . \quad . \quad (f)$$

This result gives the temperature at any depth x of the metal of a solid infinite mass of material with one surface plane exposed to a sine variation of temperature of frequency n.

This result, for approximate purposes, can be applied to the walls of thick cylinders.

If the outer wall is at a temperature T_e and d is the thickness of the walls, the inner surface being at an average temperature T_i

Then the expression

$$T = A\varepsilon^{-ax} \sin (2\pi nt - ax) + T_i + \frac{T_i - T_e}{d}.x \quad . \quad . \quad (g)$$

gives the temperature at any depth x in the walls under these conditions.

Application of Mathematical Results.

It follows from the general solution of the differential equation (c) that if a temperature variation approximating to a simple harmonic law occurs at the inner surface of the cylinder walls, the amplitude of this temperature vibration (ae^{-ax}) decreases as one proceeds outwards through the walls, and that the temperature, as one goes from the inside to the outside, lags behind the temperature occurring at the inner wall, the amount of this lag increasing from inside to outside.

It can be easily shown by substituting reasonable values for the constants involved (such as for the density, specific heat, conductivity and cyclical frequency) that in the case of a cylinder wall of 0·25 inches thickness the temperature oscillation which originates within the cylinder dies out before it reaches the outside of the metal.

Experiments have been made upon the temperatures of steam engine cylinder walls by Prof. Callendar.* He found that in the case of a steam engine running at 100 revolutions per minute, the range of steam temperatures being 46°, the temperature variation at a depth of ·01 inch in the cylinder walls was only 4°, whereas the calculated fluctuation should theoretically have been 5°.

Actual measurements of the temperatures of gas engine cylinder walls by Prof. Coker † show that the above results are substantially correct, and the maximum temperature of the cylinder walls was found to be only 7° F. in excess of the mean temperature, and at a depth of 0·4 inches from the inside of the cylinder the amplitude of temperature fluctuation was only about $\frac{1}{500}$ of the amplitude at the inner surface exposed to the gases.

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* *Min. Proc. Inst. Civil Engineers*, 1897–8 (Callendar and Nicolson).

† "The Temperatures of the Walls of a Gas Engine Cylinder," *Eng.*, 1908, p. 497.

An example of the amplitudes at different depths in a cast-iron wall has been worked out by Wimperis,* taking a cyclical frequency equivalent to a speed of 300 revolutions per minute.

The figure given (Fig. 47) illustrates the results of this calculation graphically, and further indicates how the amplitude of the temperature waves decreases rapidly as we go outwards, until at a depth of $\frac{3}{16}$ inch the temperature oscillations practically die out.

Prof. Coker in a more recent paper † deals with the flow of heat and cylinder wall temperatures in the case of a gas engine.

About 1,400 B.T.U.s were supplied to the engine per minute, of which about 500 B.T.U.s were lost, or rather transferred to the cooling water (although he cites one exceptional case in which as many as 750 B.T.U.s were given to the jacket water).

The total surface available for the passage of this heat to the jackets, etc., including the area of the piston top, was about $3\frac{1}{2}$ sq. feet, and of the compression space was about 1 sq. foot, thus giving an average flow of heat of about 300 B.T.U.s per sq. foot, or roughly two B.T.U.s per sq. inch.

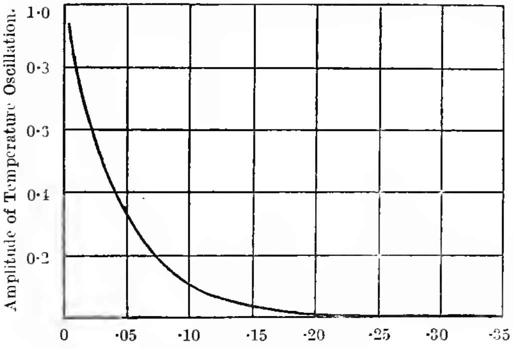

FIG. 47.

The maximum heat flow shown to exist amounts to about five B.T.U.s per sq. inch, and from these figures (which compare very favourably with Hopkinson's results for a gas engine) it can be readily shown that the mean difference of temperature between the inner and outer walls of the cylinder is less than 72° F. (or 40° C.), and so the mean temperature at the centre of the walls must be fairly low, assuming the cyclical variation to be not unduly great.

It has also been calculated, upon the assumption that the variation of temperature in the region of the walls follows a simple harmonic or sine law, that the temperature fluctuation at a depth of about $\frac{3}{8}$ inch is less than 0·2° C.

As the assumed surface variation of temperature, namely, 100° C., is higher than experimental determinations have established, it is evident that the fluctuations of temperature within the cylinder walls will be much less, and that the mean temperature of the cylinder walls will be fairly low.

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* *The Internal Combustion Engine* (Wimperis).

† "The Temperature in a Gas Engine Cylinder," *Proc. I.C.E.*, 1913.

It is indeed possible to use ordinary mercury thermometers to measure the mean wall temperatures in such cases as the above, with a sufficient degree of accuracy.

It therefore follows from the above considerations that the type of heat flow which occurs in the case of the petrol engine should closely resemble that in the nearly parallel case of the gas engine, and that to allow the usual quantity of heat (occurring under practical conditions) to pass, a comparatively small temperature gradient is sufficient, being usually of the order of about 80° C. per inch of the metal, and further, that this average temperature gradient is not appreciably affected by the temperature variations at the walls.

The Effect of Turbulence upon Heat Flow.

We have hitherto considered some of the factors concerned, directly and indirectly, with the heat flow losses in the engine cylinder, primarily with an idea of affording a partial explanation of the well-known heat suppression effects occurring in internal combustion engines. This subject of heat flow, apart from its above relation, is also important in other ways, in connexion with the subject of internal combustion engines.

It is not possible, here, to dwell at any great length upon the subject of heat flow, but the reader is referred for fuller details and explanations to the Papers, and the references given in the footnotes.*

Before concluding this important branch of the subject, it may be of interest to consider the influence of another factor, namely, the effect of the agitation, motion, or turbulence of the explosive mixture before ignition upon the rate of ignition and of heat flow.

Consider first the case of a gas engine running at 120 revolutions per minute, and working upon the four-cycle principle. The period of the suction stroke is about 0·25 second, and the average velocity of the explosive gases during this suction period is about 100 feet per second through the valves upon the average in the case of larger types.

In examples of high speed petrol engines, the period of the suction stroke may be only $\frac{1}{100}$ second, that is, only about one-twenty-fifth of the corresponding period in the case of the gas engine.

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* Vide *The Gas, Petrol and Oil Engine* (Dugald Clerk); *The Gaseous Explosions Committee's Reports*, 1912–14.

Also it may be remarked that whereas the time of inflammation necessary in the case of a gas engine is about $\frac{1}{30}$ second, as the experiments of Hopkinson and Clerk have shown, in the case of the petrol engine of motor car practice running at 1,200 revolutions per minute the time of explosion is about $\frac{1}{120}$ second.

This period may even be $\frac{1}{300}$ second for small petrol engines, and if the flame travels for a cylinder length of four inches, then its rate must be 100 feet per second, or about 68 miles an hour.

In most gas and petrol engines the gas flows through the valves at a mean velocity, which is on the average about 100 feet per second.

These very high velocities of the gases produce considerable eddying effects within the cylinder, and experiments have shown that this eddying motion, or turbulence, persists throughout the compression stroke, and is still in existence when ignition occurs.

Hopkinson * has made experiments upon the degree and the amount of turbulence occurring in the case of gas engines, which throw considerable light upon a hitherto abstruse subject.

In the above case of a gas engine running at 120 revolutions per minute, the amount of heat flowing through the cylinder walls is roughly about 30 per cent. of the total heat of the fuel. If the speed of the engine be doubled, the exploding gases will remain in contact for only half the period of time occupied in the former case, so that one would be led to think from this that the heat loss at this higher speed would only be about half that occurring at the lower speed, or 15 per cent.

This is not the case, however, for the heat loss to the walls is about the same in each case, thus showing that the heat loss is twice as great at the higher speed.

This observed result is explainable upon the idea of turbulence of motion existing, for it is well known in connexion with boiler-flue gases, often termed the "scrubbing effect," that when a gas is in motion it exchanges heat more readily with a still object in its vicinity than if at rest, and further, that the higher its speed of motion or degree of agitation the more readily does it part with or absorb heat, as the case may be. Thus in the above case, the engine speed being doubled implies a corresponding increase in the degree of turbulence, and therefore the heat loss to the walls is enhanced upon this account; or in other words, the decrease in the heat loss due to the smaller time intervals is to all intents counterbalanced by the greater turbulence due to the higher speed.

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* *Phil. Trans. Royal Society*, 1911–12, and *Report No.* 21 *Gaseous Explosions Committee.*

In order to investigate this phenomenon of turbulence Hopkinson carried out some experiments with a gas engine, but instead of allowing it to fire in the usual way, it was electrically motored around without allowing a charge to enter, or the explosion to occur, thus causing the air inside to be alternately compressed and expanded. An electrical resistance, insulated from the cylinder walls, was inserted within the combustion chamber, and its temperature, which was raised by means of an electric current, was measured.

By measuring the temperature when the engine was at rest, at the end of the compression stroke, and when it was motored around at different speeds, it was found that the temperature was much lower when the engine was running than when at rest, and that the temperature fell more and more as the speed was increased.

These effects are explainable upon the notion of turbulence existing, for evidently the higher the speed the greater the turbulence, and the more readily the gases would receive heat from the electrical resistance.

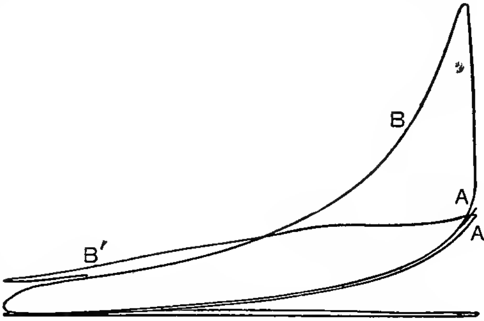

Fig. 48.—Showing the Effect of Turbulence on the Indicator Diagram.

Dugald Clerk * has made some experiments in connexion with the subject of turbulence, the results of which may be of interest here.

A gas engine was employed for these tests, and indicator diagrams were taken, firstly, when the engine was working normally, and secondly after two idle compression strokes with the engine not firing. In this latter case the turbulence of the gases was damped down considerably, as the valves were shut down during the idle strokes and simple compression and expansion alone occurred.

Examples of the indicator records are shown in Fig. 48 for the two cases; the full line diagram AB corresponds with the first case of normal firing, whilst the expansion line A^1B^1 corresponds with the second case in which the turbulence effect had been allowed to subside. The ignition periods in the two cases illustrated in the diagram were ·037 and ·092 second respectively.

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* *Reports* 23 *and* 24 *Gaseous Explosions Committee,* 1912.

The very marked effect of this damping upon the rate of inflammation, or explosion, is at once evident from the diagram, and it is concluded that the suction turbulence is of vital importance in producing the high rates of flame travel necessary in internal combustion engines.

Further, it has been shown that by artificially agitating an explosive mixture in a suitable explosion vessel or chamber, the speed of flame propagation was greater than when the mixture was ignited from rest, and also that the maximum pressure attained in the former case was greater.

The experiments made in connexion with the subject of turbulence indicate that in the gas and petrol engine one effect of turbulence is to increase the rate of inflammation considerably. Clerk showed by means of indicator diagrams that for a gas engine the time of inflammation without turbulence was about two and a half times that which occurred with some turbulence.

It follows from the above results that experiments made upon rates of flame travel in closed explosion vessels do not give results comparable with those of real explosion engines, unless the explosion mixtures are agitated to about the same degree of turbulence actually occurring in practice; this, of course, is difficult to ensure.

From these, and other equally conclusive experiments, it has been established that not only is the maximum pressure attained increased by turbulence, but that the period of time of attainment of this pressure is reduced—in fact the actual ignition velocities and pressures attained in gas, oil and petrol engines are only possible as a result of turbulence.

From the experiments mentioned, the real significance of turbulence in affecting heat flow will be understood, and it will be evident that the counteracting effects of speed and turbulence upon heat flow are extremely satisfactory in permitting wide ranges of speeds to be attained in petrol and other internal combustion engines.

The design of the combustion chamber, valve ports and pipes, and their disposition, etc., have also an important bearing upon the maximum and mean rates of ignition, corresponding with different modes and degrees of turbulence effect.

Dr. Watson's experiments * upon the sleeve valve type of engine, which was free from valve pockets and had a central igniter, showed that rapid rates of ignition occurred, in other words, that the maximum pressures were attained sooner than in the case of the **T**-headed engine with valve pockets on either side of the cylinder head.

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* "Test of a Daimler Sleeve Valve Engine," *Proc. Inst. Aut. Engrs.*, 1912.

Wall Action in Petrol Engines.

The effect upon heat loss of the combustion chamber and cylinder walls during the explosion period has already been mentioned, but it will also be evident that throughout the whole cycle of operations in an internal combustion engine heat will be exchanged between the cylinder contents and the exposed surfaces. Thus if the cylinder walls be assumed to be at some mean temperature, say in the neighbourhood of 100° C., then during the explosion or expansion stroke the gases will be at a considerably higher temperature (usually about 2,000° C.) than the walls, and so heat will be lost to the walls as a consequence.

During the period of the ensuing exhaust stroke the gases will still be at a much higher temperature (usually about 700° C.) than the walls, thus facilitating heat loss to the latter.

During the following suction stroke the gases, initially at atmospheric temperature (usually in the region of 20° C.), will now receive heat from the hot walls, increasing their temperature in consequence.

Heat will be received along the compression stroke up to a certain point, when the temperature attained by compression is the same as that of the walls, after which a reverse interchange of heat ensues.

In practice, however, the wall temperature varies, and owing to the existence of oil, carbon films, and to other factors, the interchange of heat is not quite the same as that just stated, during the compression stroke, although sensibly correct in the other cases.

It is fairly well established that the fresh charge assumes the temperature of the cylinder walls at least early in the compression stroke, on account of its turbulence, etc., so that during the latter stages of this stroke, or indeed over the greater part of its range, heat is given to the walls on this account.

The temperature-entropy diagram, deduced from considerations of the indicator diagram, affords a valuable practical means of studying the interchange of heat between the gases and the cylinder walls. The reader should refer to the diagram of temperature entropy given on page 54, in this connexion.

The considerations of the second explanation of the observed facts relating to the apparent suppression of maximum explosion temperatures and pressures in closed vessel and internal combustion engine experiments have led to a short study of some of the other factors which are more or less allied with the subject of heat loss to the walls, but it is now intended to resume the consideration of the two remaining factors, enumerated on page 87.

(3) The After Burning Theory.

Coming next to the third explanation put forward to account for the difference between the theoretical and observed results, it has been pointed out that possibly combustion is not complete at the moment of attainment of maximum pressure, but that combustion occurs, although at a gradually diminishing rate, all along the expansion stroke, at least, and in some cases during the exhaust. The part of combustion which occurs after the point of maximum pressure is termed the "after burning" portion.

It is well known that the expansion curve of pressure-volume, obtained from actual engines, by means of indicator diagrams, differs from the theoretical expansion curve in that the fall of pressure along the stroke is not so rapid as in the latter case, as is shown in Fig. 49 (which is not, however, shown to scale).

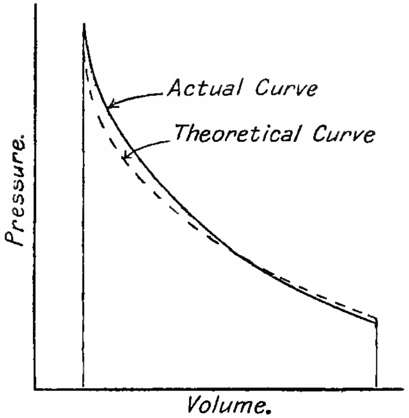

FIG. 49.—REAL AND ACTUAL EXPANSION CURVES.

In practice the expansion curve often lies considerably above the theoretical expansion curve, drawn according to the law $PV^n =$ constant, where n is the constant chosen to suit the particular gases present.

This fact would appear to show that if the pressure and temperature do not decrease as rapidly as they theoretically should do in an engine, then heat must in some way be added during the expansion stroke.

As the cylinder walls have been shown to be at a very much lower temperature than the gases during the expansion stroke, it is evident that heat cannot be given by them to the gases, in order to account for this observed heat addition.

Upon the assumption that combustion is slow, and occurs throughout the expansion stroke, the evolution of heat by the gases after maximum pressure can be accounted for.

It is also evident from many experiments that the moment of maximum pressure is not always that of complete combustion.

Combustion in Petrol Engines.

The author has recently made experiments, in co-operation with Dr. Watson, in connexion with the subject of combustion in petrol engines. By means of a spectroscope and a transparent cylinder window, the progress of combustion was studied throughout the explosion stroke.

In order to examine the nature of the combustion at any given point in the explosion, or expansion stroke, a rotating disk with a perforated sector slit was made (by means of suitable mechanism) to uncover the cylinder window, and expose it to the collimator of the spectroscope at any part of the stroke.

Although a consideration of the actual spectroscopic work is somewhat outside the scope of the present work, yet it may be of interest to refer to some of the results relating to combustion in the petrol engine.

The principle upon which most of the experiments were based was that a line spectrum of the gases within the cylinder was only obtainable during the actual process of chemical combustion; and that the general intensity of the lines in the spectrum obtained from the exploding gases was a measure of the degree or intensity of the combustion occurring. The petrol engine employed for these experiments was one of fairly low compression, namely, 60 lbs. per sq. inch (ex. gauge), and ran at speeds varying from 700 to 900 revolutions per minute.

By arranging the apparatus so as to obtain a spectrum for any given position of the piston, both before and after the dead centre position at the end of the compression stroke, it was an easy matter to trace the process of combustion from the moment of firing the charge to the point of complete combustion.

One of the first facts which was noticed was that in order for the point of maximum pressure (as shown by the indicator diagram) to occur at, or very shortly after the dead centre position of the piston, the spark must occur at a crank angle of about 40° before dead centre for normal mixtures, corresponding, for the engine in question, with a period of ·01 second.

Another point shown was that the point of maximum intensity of combustion was nearly always either just before that of maximum pressure or coincident with it, the difference being very small and amounting only to about 10 degrees of crank angle at the most, and corresponding with a period of about ·002 second.

The changes in the intensity of the combustion as the piston moves down the explosion stroke are illustrated in Fig. 50, in which the vertical ordinates indicate the intensity of the combustion and abscissae the piston positions expressed in terms of the crank angles.

Two curves are shown, one for a weak mixture (Curve A) and the other (Curve B) for a correctly proportioned mixture. The prolonged process of combustion in the former case is well illustrated.

The general colour of the explosion, as viewed visually through the quartz glass window in the combustion head, was observed to change as the process of combustion occurred, for any given mixture strength.

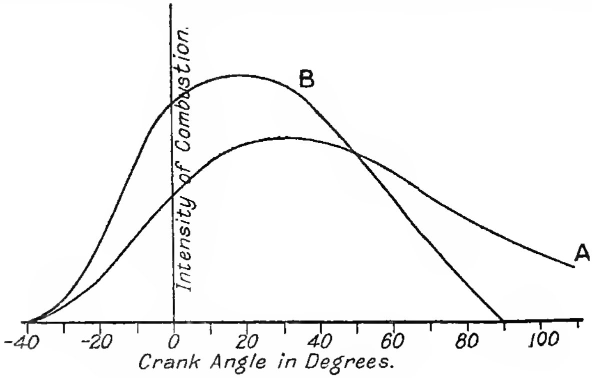

FIG. 50.

The following Table XVIII will serve to indicate these colour changes for different piston positions after the firing point, in the case of a mixture composed of fifteen parts air to one of petrol, by weight.

TABLE XVIII

COMBUSTION COLOURS DURING THE PROCESS OF COMBUSTION IN A PETROL ENGINE

Air petrol ratio = 15·0

Crank Angle in degrees.	Flame Colour.
—40°	Spark occurs. No colour visible.
—20	Very deep blue. Cylinder walls just visible.
0	Deep blue.
10	Blue (cobalt).
20	Brighter blue (cobalt). Maximum intensity.
30	Bright blue with slight yellow tinge.
45	Yellowish white, less intense but with blue tinge.
60	Greenish yellow, blue tinged.
80	Dark green, mottled with blue.
90	Non-luminous.

In this case the spark occurred at an angle of 40° before the dead centre, and the point of maximum explosion pressure (200 lbs. per sq. inch) at 27° down the firing stroke, whilst the point of attainment of maximum combustion was 20° down the firing stroke. Combustion was fully complete at a crank angle of 90°, or when the piston was just over half-way down its firing stroke.

The actual period of combustion was ·0180 second from point of firing.

This period of combustion was found to vary from 0·01 second to ·025 second, according as the mixture was either richly or weakly proportioned.

It was also found that the average colour of the explosion, viewed over the entire range of combustion, varied with the mixture strength; this is shown in Table XIX. This table also gives the periods of complete combustion, but here it should be pointed out that these periods depend upon the compression pressure, which in the above case was low. The fact that the colour of the explosion varies with the mixture strength is often made use of in carburettor setting for obtaining the best mixture proportions, more particularly when tuning up aeroplane engines, with open exhausts.

TABLE XIX

COMBUSTION COLOURS IN THE PETROL ENGINE

$\frac{\text{Air}}{\text{Petrol}}$ Ratio	Period of Combustion in seconds.	Flame Colour.
8·5	·010	Bright orange yellow.
9·0	·011	Bright yellow.
9·7	·016	Bluish white with faint yellow tinge.
10·0	·017	Light blue with trace of yellow.
11·3	·017	Light blue.
13·6	·018	Intense light blue.
14·6	·018	Maximum intensity light blue.
17·3	·025	Whitish blue, less intense.

There is no doubt that some relations exist between the colour of the explosion flame and the proportion of different gases undergoing chemical changes, as is evident from the facts that the character of the spectra of the hydrocarbon, carbon monoxide and carbon dioxide compounds change in a definite manner as the mixture strength is varied.

It may, in the future, be shown by means of the spectroscope whether dissociation of the gases taking part in combustion occurs, but at the present stage the subject is rendered more or less complex by the unsatisfactory explanations put forward for the spectra which occur.

One other interesting point brought out by the preceding research was that the nature of the combustion process in the petrol engine

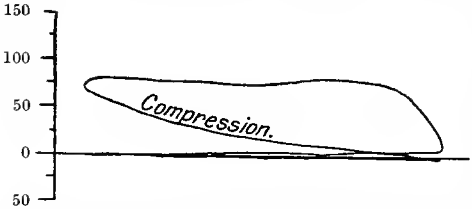

FIG. 51.—LIGHT LOAD DIAGRAM FROM GAS ENGINE RUNNING UPON A WEAK MIXTURE.

was the same as that which occurs in the ordinary Bunsen or Mêker burner gas flame; the spectra yielded in the two cases, as the proportions of the combustibles were varied, being practically identical.

The spectrum of the luminous gas flame, such as that given by a Batswing burner was found to be the same as that taken from a petrol engine cylinder when a very rich petrol mixture was employed.

In both cases the spectrum of luminous carbon was pronounced.

In the light of the above experiments, and in their relation to the after-burning theory, it would appear that combustion goes on after the point of attainment of maximum pressure; that the period of combustion varies with the mixture strength being shorter for mixtures rich in petrol and relatively longer for weaker mixtures (incidentally, this latter fact affords an explanation of the phenomenon well known to motorists of "popping-back," for the weaker mixtures, burning comparatively slowly, are still burning at the end of the exhaust stroke, and when the inlet valve opens, ignite the fresh charge in the induction pipe); that the period of combustion is probably dependent upon the compression pressure, being smaller, the greater the pressure of compression.

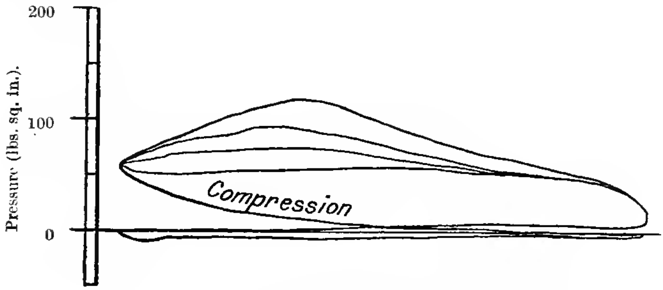

FIG. 52.—LIGHT LOAD DIAGRAM FROM PETROL ENGINE RUNNING UPON A WEAK MIXTURE.

The effect of this definite period of combustion during the early part of the explosion, or firing stroke, will evidently cause the expansion curve of pressure-volume to depart from the hypothetical one.

It will be seen from diagrams of pressure taken from engines working with weak mixtures, given in Figs. 51 and 52 for the cases of gas engines and petrol engines respectively, that the shape of the expansion curve differs very considerably from that which one would theoretically expect to occur if all the heat were developed by the time maximum explosion pressure was attained.

In both cases the expansion curve is practically parallel with the datum line of volumes, and quite distinct from any theoretical expansion curve obtained upon the usual assumptions of complete combustion before expansion.

Dugald Clerk has shown by means of closed explosion vessel experiments, in which the time-rate of pressure variation could be automatically recorded, that after-combustion may exist.

The reports of the Gaseous Explosions Committee mention this phenomenon of *after-combustion*, or "postponement of chemical equilibrium until after maximum pressure," and point out that a considerable amount of the energy at the moment of maximum pressure is in the form of chemical energy; and further, that this delayed combustion may be regarded as a surface effect, for considerable differences of temperature may occur within the mass of the gases. The gases in the neighbourhood of the cylinder walls being at comparatively low temperatures are conducive to after-burning results, whereas in the main body of the gases chemical equilibrium will probably occur practically at the moment of maximum pressure.

Hopkinson has indicated, by means of temperature measurements at different points of the interior of an explosion vessel, that combustion is complete in the main body of the gases very much earlier than the moment of maximum pressure, thus showing that the incomplete combustion in an explosion is mainly a surface phenomenon.

It is also evident that combustion must be retarded in the neighbourhood of the cold walls, and there is no doubt that this at least affords a partial explanation of the observed facts.

There is another possible explanation of this delayed or after-burning phenomenon, apart from the dissociation and slow-combustion theories, namely, upon the assumption of variable specific heats of the expanding gases.

(4) The Variable Specific Heat Hypothesis.

Mallard and Le Chatelier have suggested that the increase in the specific heats of the gases at very high temperatures may partially account for the observed phenomenon, and also for the "heat suppression" effects, dwelt upon earlier in this chapter.

The previous explanation of after-burning might be considered as a chemical one, whereas the present one under consideration would be regarded as a thermal one.

Whilst there appears to be no doubt that delayed burning does exist, and further affords a partial explanation of the observed "heat suppression" and other well-known phenomena, yet it is also the opinion of several of the leading authorities upon the subject, that the increase in the specific heats of the gases concerned with temperature must also be considered in the complete explanation to account quantitatively for these effects.

Wimperis * has suggested that although the increase of specific heat leaves a large proportion of loss to be accounted for at low temperatures, as apart from high temperatures, this defect in the variable specific heat explanation might be accounted for by the longer ignition period at low temperatures, allowing wall action to exert a greater influence.

Wall action depends also upon the temperature and other factors, as well as upon time, and the theory of wall action alone does not offer a full explanation of the 50 per cent. heat suppression observed.

Using Mallard and Le Chatelier's results for the specific heats of gases at different temperatures, the theoretical explosion pressures can be calculated for different mixtures in Dugald Clerk's experiments, and if this be done the calculated pressures upon this basis will give results in much better agreement with the actually observed ones.

It is found, however, that if the explosion pressures are so calculated, then for all mixtures the pressures thus obtained appear to be still higher than those actually measured, but the discrepancy is reduced upon the average to about 27 per cent., from 50 per cent.

If now allowance be made for the fact that the pressure rise is retarded by the cooling action of the walls, it is possible to still further reduce, or rather to bring the calculated pressures into accordance with the observed ones.

The rate of cooling is calculable from the indicator diagram and can be approximately obtained at the actual maximum temperature of explosion.

Wimperis assumes the mean rate of cooling to be half the maximum during the attainment of maximum pressure, and in an example taken from a cooling curve finds that for coal gas the relation between the pressure (p) and time (t) could be expressed as $p = \frac{31}{\sqrt{t}}$, and upon this basis has calculated the losses due to cooling.

If this cooling factor be considered in calculating the explosion

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* *Engineer*, October 10, 1902.

pressures by the variable specific heat method, a much better concordance with the observed pressures will result.

This better agreement can be at once seen from the curves given in Fig. 53,* which represent the relation between the pressures and mixture proportions for explosive mixtures of coal gas and air.

Curve A represents the pressures calculated upon the ordinary constant specific heat hypothesis, and represents Dugald Clerk's calculated results already given.

Curve B is drawn upon the variable specific heat theory, making no allowance for cooling ; and the Curve C represents the pressures obtained, after allowing for the cooling effect, which brings the calculated pressures into very close agreement with those actually measured by Dugald Clerk.

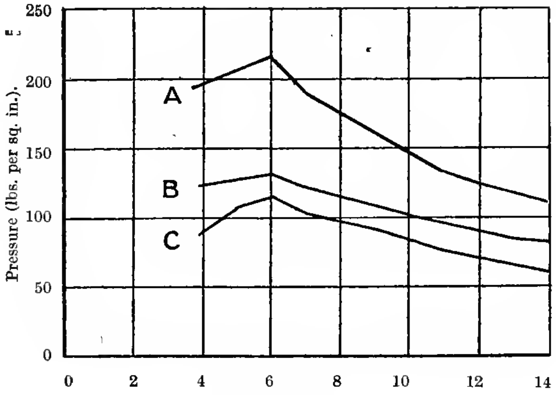

FIG. 53.

It should be here mentioned that the cylinder walls are by no means at a uniform temperature, and that the actual wall action occurring is more or less complicated, so that it is difficult in most cases to know how to make an exact allowance for this, but the above example from Wimperis will indicate the significance of the factors of variable specific heat and wall action in suppressing the temperature and pressure of explosion.

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* *The Internal Combustion Engine,* Wimperis.

CHAPTER III

THE CONDITIONS OCCURRING IN ACTUAL ENGINES

The Working Cycles.

IN the first chapter, upon Thermodynamics, the three ideal cycles applicable to internal combustion engines were discussed.

The Carnot or constant temperature cycle was ruled out, in so far as its practical application was concerned, on account of its excessive dimensions and weight for a given power.

The constant pressure cycle was also considered, and it was shown that if the compression pressures were relatively great, the possible efficiency was also similarly high.

From the point of view of motor car engine requirements, the constant pressure type of engine, for example the Diesel engine, so far has made very little headway on account of its comparative heaviness for a given power and for other practical reasons, although it has been employed, recently, in Germany for heavy locomotion and transport, so that in the following considerations the constant volume cycle of operations alone will be dealt with; this being the basis of the present working cycles adopted in motor-car and aeroplane engines.

The Constant Volume Cycle in Practice.

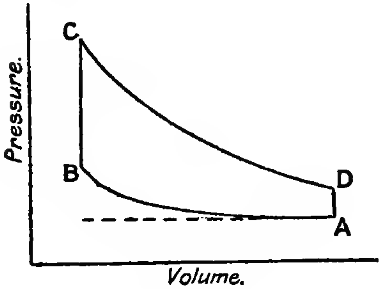

FIG. 54.

In the ideal constant volume cycle, illustrated in Fig. 54, it is assumed that adiabatic compression occurs along the line AB, followed by addition of heat from the working substance at constant volume along BC. The working substance then expands adiabatically along the curve CD, and gives up its surplus heat at constant volume along DA, thus leaving the working substance in its initial condition at A again.

The cycle of operations occurring in practice in petrol engines differs in many respects from this ideal cycle. It is proposed to consider, briefly, the chief points of difference between the ideal and real cycles of operation in the following pages.

In the real or Otto cycle of operations, the working substance can be either a mixture of gases or of gases and vapours, and the heat addition, which is more or less at constant volume and which occurs during the process indicated by the line BC in the ideal cycle, is derived from the heat of combustion, or the chemical energy, of the fuel or explosive mixture.

Consider next the manner in which the Otto cycle (with which it is here assumed the reader is acquainted) differs from the ideal one.

Commencing with the compression process AB, Fig. 54 : this is not quite an adiabatic one in practice, for instead of the explosive mixture being compressed in such a way that its store of heat energy does not alter, the cylinder walls, which are water-jacketed in practice, absorb a certain amount of the heat energy of the charge, so that the pressure at the end of the compression stroke is somewhat lower than it would be in a truly adiabatic process.

The quantity of heat absorbed by the cylinder walls is, however, insufficient to render the process isothermal, that is, to keep the temperature of the charge constant throughout the process.

It will, therefore, be seen that the equation of the compression line AB for the real cycle lies between that of the adiabatic $PV^{\gamma} =$ constant, and the isothermal $PV =$ constant.

In the case of air as the working fluid, $\gamma = 1{\cdot}408$, very nearly, and measurements from diagrams of pressure-volume obtained from petrol engines indicate that the law of compression is approximately adiabatic, and can be represented by the equation ($PV^{1\cdot25 \text{ to } 1\cdot35} =$ constant).

It will be noticed that the value of γ in this case lies between that of $\gamma = 1$ (the isothermal case) and $\gamma = 1{\cdot}408$ (the adiabatic case for air). There is no doubt that the fact of the working substance being a mixture of air with some 6 per cent., upon the average, of petrol vapour, and a certain amount of burnt gases, partially accounts for the value of the index being lower than the adiabatic one for dry air.

It also follows from this that the temperature at the end of the compression stroke will be somewhat lower than that calculated upon an assumption of adiabatic compression with air as the working substance.

The rise in pressure along BC (in the ideal diagram) has already been shown to be considerably less than is predicted upon the assumption that all of the heat of combustion of the fuel is utilized in raising the pressure and temperature.

The several factors contributing to this apparent "pressure" suppression have already been dealt with in connexion with dissociation, wall action, increasing specific heat of the explosive mixture and products, and after combustion.

In practice the explosion process is not instantaneous, but occupies a definite period of time depending upon the mixture proportions, time of occurrence of the spark, speed of engine, cylinder dimensions, compression pressure, and other factors, so that in addition to the observed after-combustion effects, strictly speaking, heat is not given to the working substance at constant volume; here again the real working cycle differs from the ideal.

Further, owing to the combustion process continuing after the point of attainment of maximum pressure, which results in an addition of heat to the expanding gases along the expansion line CD (Fig. 54), in the ideal case, and also to the opposing effect of absorption of heat by the relatively cool walls of the cylinder during expansion, the true expansion process is therefore different from the theoretical adiabatic one.

Measurements of indicator diagrams from gas and petrol engines show that the expansion curve in most normal cases follows the law $PV^{1\cdot15 \text{ to } 1\cdot35} =$ constant, the value of the index depending upon several factors, which will be considered later.

In the case of weak explosion mixtures the expansion line does not appear to follow any definite law, and in extreme cases is often a line nearly parallel, for most of its length, with volume axis.

In practically all motor-car engines the exhaust valve opens before the end of the expansion stroke, and remains open for a comparatively long period, so that the heat rejection process represented upon the ideal diagram by the line AD is not the same as the one which actually occurs.

The general differences between the real or Otto cycle and the ideal cycle can best be studied by a reference to Fig. 55, in which the cycle of operations for the case of a petrol engine is shown by the shaded area, which has been drawn upon the assumption of a constant specific heat during the process of heat addition.

The dotted diagram, B, represents the theoretical cycle of operations for the same working substance, and has been drawn upon the assumption that the specific heat of the working substance is variable, and that the cycle conforms in other respects to the ideal constant volume cycle. The other diagram, C, refers to the constant specific heat case.

It will be evident that the compression pressure (and also the temperature) is higher in this latter case, as also is the pressure and temperature at the end of the heat addition (or explosion) process.

A comparison of these two diagrams will show in what way the real cycle of operations departs from the ideal.

The lower explosion pressure and the expansion line lying below that of the ideal diagram indicate the effects, previously mentioned, of apparent heat suppression.

The effect of the early opening of the exhaust valve is apparent in the shape of the "toe" of the shaded diagram.

Further, the compression curve will be seen to be lower in the real diagram; this is in accordance with the predicted effect.

It should be pointed out, in passing, that the shaded diagram is not intended to represent an actual indicator diagram taken from an engine, but has been drawn for comparison purposes.

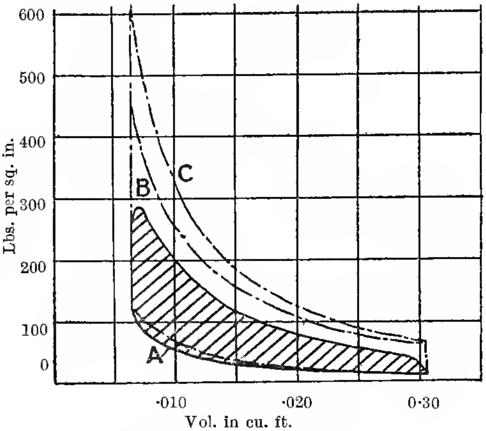

FIG. 55.

A.—Indicator Diagram from Petrol Engine.
B.—Ideal Diagram for Variable Specific Heats.
C.—Ideal Diagram for Constant Specific Heats.

The real indicator diagram will be found to reveal other interesting points of difference, and a discussion of these will form the subject of a subsequent chapter.

Ideal and Real Efficiencies.

In the case of the ideal engine working upon the Otto cycle, it is assumed that the working substance is air, obeying the conditions relating to ideal gases, and that none of the heat added during the constant volume process is lost by conduction or radiation during the expansion process.

It is further assumed that the specific heat is constant throughout the whole sequence of operations.

It has been already shown that the theoretical efficiency for a substance working upon the Otto or constant volume cycle is given by

$$E = 1 - \left(\frac{1}{r}\right)^{0{\cdot}408} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

This shows that the efficiency depends solely upon the compression ratio. The differences existing between the real working cycles and the ideal one have previously been shown to considerably modify the results obtained in practice, so that the efficiency of an actual engine will be much lower than that theoretically obtainable.

The values of the Air Standard Efficiency, as it is termed, for different compression ratios r as referred to the ideal cycle with air as the working substance and calculated from the relation (a), are given in the following Table XX, and have already been shown graphically on page 35, Fig. 12.

TABLE XX

AIR STANDARD EFFICIENCY

Compression Ratio.	Efficiency.
2	·25
3	·36
4	·43
5	·48
10	·61
100	·85

The values of the compression ratios employed in motor-car engine practice lie between 3 and 5, being upon the average about 4, so that theoretically we ought to expect efficiencies of about 0·43.

Owing, however, to the several contributing factors already mentioned, the best efficiencies yet obtained have not exceeded 0·30 (as actually measured).

It is usual, for comparative purposes, to refer the performances of real engines to that of the ideal cycle under like conditions, and for this purpose the ratio of the actual efficiency to the ideal efficiency, called the Relative Efficiency, is used.

It should here be pointed out that the term "efficiency" here employed represents the ratio $\frac{\text{work obtained}}{\text{heat expended}}$, both being in the same units.

In the case of a petrol engine this efficiency is termed the Thermal Efficiency, which is the ratio of the real work done, in suitable units (as represented by the area of the indicator diagram), to the calorific value of the quantity of petrol used in performing this work through the agency of the explosive mixture.

If the indicated horse power be represented by the letters I.H.P., and if w lbs. of petrol are used per hour, the calorific value of the fuel per pound being represented by C, then the

$$\text{Thermal Efficiency} = \frac{\text{I.H.P.} \times 33{,}000 \times 60}{778 \times w \times \text{C}}$$
$$= 254\frac{\text{I.H.P.}}{w.\text{C}}$$

The calorific values of different fuels, as employed upon petrol-type engines, are given in Table X, page 63.

The relations between the thermal efficiencies obtained in practice and the theoretical efficiencies obtainable are best understood by a reference to the experimental results * obtained with a four-cylinder car engine 85 mm. bore and 120 mm. stroke, running at a speed of about 1,100 revolutions per minute.

In these tests the compression of the engine was varied, so that the compression ratios varied between 3·92 and 4·71.

It will be shown later that the strength of mixture and the speed both affect the efficiency, so that the results here given refer to constant conditions of speed and mixture strength.

The results of these experiments are given in Table XXI for a mixture strength giving nearly the maximum possible thermal efficiency, in order to make as favourable a comparison as possible.

TABLE XXI

Compression Ratio.	Measured Efficiency.	Ideal Air Standard Efficiency.	Relative Efficiency.
3·92	·26	·42	·63
4·35	·27	·44	·61
4·71	·28	·46	·63

The air standard efficiencies have been calculated from the relation $E = 1 - \left(\frac{1}{r}\right)^{0\cdot408}$, using the compression ratios given in the first column.

The measured efficiencies obtained are therefore of the order of 60 per cent. of the theoretical efficiencies calculated upon the air standard.

If, however, the measured efficiencies be compared with those for an engine working, not upon air, but with the actual petrol air

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* "The Thermal Efficiency of a Four-Cylinder Petrol Engine," *Proc. I.A.E.*, 1908.

mixture employed, and if allowance be made for the fact that the specific heat of the gases increases with temperature, then a fairer basis of comparison will be obtained. For comparison purposes this modified ideal efficiency will be termed the *Corrected Ideal Efficiency.*

If therefore in the above results due allowance be made for these properties of the working substance, and the measured thermal efficiencies be compared with the corrected ideal efficiencies, it can be shown from the results that the relative efficiency of the petrol engine to the ideal is comparatively high.

The ideal corrected efficiencies have been calculated for petrol air mixtures and are given in the table below.

TABLE XXII

Compression Ratio.	Measured Efficiency.	Corrected Ideal Efficiency.	Relative Efficiency.
3·92	·26	·34	·76
4·35	·27	·35	·77
4·71	·28	·37	·77

The relative efficiencies are now about 75 per cent. of the maximum possible efficiencies; it is therefore evident that it is only possible to increase the actual efficiencies realized in practice by about 25 per cent. in order to attain ideal efficiency, or in other words, if all heat losses to the cooling water, by radiation and by the escaping exhaust gases, were prevented, an increase of efficiency of less than 25 per cent. would be possible. Much time and ingenuity have been wasted in connexion with attempts at drastic improvements to the efficiencies of internal combustion engines.

Variation of Efficiency with Mixture Constitution.

Since the quantity of fuel supplied per working stroke of the engine represents a certain amount of available heat energy, the thermal efficiency will depend upon the amount of fuel supplied, and upon the proportion of this fuel which is actually usefully combusted, so that the proportions of petrol and air in a given volume of explosive mixture will determine to a great extent the value of the thermal efficiency obtained.

For example, if the explosive mixture be richer in petrol than the mixture for complete combustion, or the " optimum mixture," as it is now termed, then part of the fuel will be incompletely combusted, and will therefore pass out into the exhaust without having yielded its maximum heat energy of combustion, so that it is to be expected that ultra-rich mixtures, representing fuel wastage, will show comparatively low thermal efficiency values.

In the case of ultra-weak mixtures which yield low efficiencies, slowness of combustion, and heat absorption by the surplus air present, etc., are the governing influences ; but in the exceptionally weak mixtures upon which the petrol engine can be made to run, there is no doubt that the prolonged period of combustion, and the relatively small quantity of fuel present in the cylinder contents, will explain the lower efficiencies obtained.

These contentions are supported by the results of experiments * made upon a four-cylinder Talbot engine, in which conditions such as the speed, jacket water temperature, etc., were kept constant, and measurements of the indicated horse power and mixture strength were made simultaneously.

The results of these tests are shown plotted in Fig. 56, from which it will be seen that, commencing with very weak mixtures, the efficiency rapidly rises as the mixture becomes richer in petrol, and ultimately reaches a maximum for a mixture strength in the proportions of about seventeen parts of air, by weight, to one of petrol.

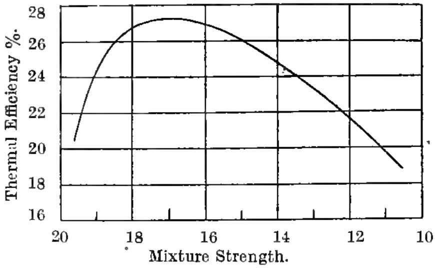

FIG. 56.

It should here be mentioned that the "optimum" mixture necessary for complete combustion is composed of about 14·75 parts of air to one of petrol, for the ·720 density petrol as used in these tests. (This ratio varies somewhat with the grade of petrol used.)

Proceeding with the above considerations, it will be seen that after reaching a maximum for the 17·0 mixture, the efficiency begins to decrease, and continues to decrease at a more or less uniform rate as the mixture becomes richer and richer in petrol, right down to the strongest mixture upon which the engine will work.

With the weakest mixtures the engine tends to run irregularly, and unless the petrol is properly atomized it is difficult to obtain consistent running ; this is further enhanced by the slowness of the propagation of the flame, which in some cases is so marked that when the inlet valve opens to admit a fresh charge this is ignited by the previous charge being still burning throughout the

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* *Proc. Inst. Aut. Engineers,* 1908.

exhaust stroke—a phenomenon already alluded to as "popping back" in the carburettor.

From the above results, it will be observed that the mixture strength giving maximum thermal efficiency is decidedly weaker than that which yields complete combustion; from this it necessarily follows that in order to obtain the highest fuel economy the engine should be run upon mixtures rather upon the weak side of the optimum mixture.

Consider next the reasons why this weaker mixture should give the highest efficiency of any mixture.

Commencing with the optimum mixture, and gradually reducing the strength of the mixture, it will be seen that the total heat of combustion becomes reduced; that is, the temperature reached after combustion becomes lower. The effect of lowering this maximum temperature is twofold. In the first place, since the heat losses from the combusting charge to the walls is dependent upon the temperature, this loss will be lower at lower temperatures, so that a smaller proportion of the heat developed will be lost to the cylinder walls. In the second case, since the specific heat of the gases increases with the temperature, the mean specific heat for the range of temperature obtained with the weaker mixtures will be lower.

From what has already been mentioned in Chapter III, dealing with Combustion and Explosion, it will be evident that the lower the mean specific heat, the greater will be the rise of pressure produced by a given quantity of heat, and hence the net effect of lowering the mean specific heat will be to increase the indicated horse power, and thus give for the same heat supply more effective work.

The combined influence of reduced wall losses and lower mean specific heats will thus cause the efficiency to increase up to a certain point of mixture weakness, beyond which the greater opposing influence of slow combustion will result in a decrease in this efficiency, this falling off becoming more and more enhanced as the mixture is progressively weakened beyond the point of maximum efficiency.

The results of experiments made at different compression pressures show that this variation of efficiency with mixture strength obeys the same type of general curve as that shown in Fig. 56, although the actual values of the maximum efficiencies reached were different, but occurred very nearly at the same value of the weaker mixture strength shown in the figure.

There is a slight tendency, however, for the maximum efficiencies to occur with the rather weaker mixtures than the one shown in Fig. 56, in the case of higher compression pressures.

Thus the curves marked A, B, and C in Fig. 57 were obtained with three different compression ratios of 4·71, 4·35, and 3·92 respectively, in the same engine and running at the same speed.

It will be seen from this figure that the most efficient mixture is at about 17·4 in the case A, whereas for B it is 17·0, and for C, the lowest compression pressure, it occurs at about 16.

The probable explanation of this effect is that with the higher compressions, the maximum temperatures attained are higher, so that the reduction in the heat loss and mean specific heat will be more effective as the mixture is weakened than in the case of lower compressions, and therefore in the maximum temperatures.

This effect of the compression upon the mixture strength at which maximum efficiency occurs must not, however, be confused with the direct influence of degree of compression upon the efficiency to be considered later.

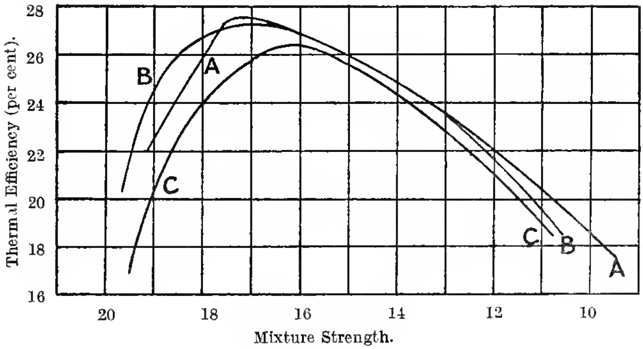

FIG. 57.

Variation of Thermal Efficiency with Nature of Fuel.

It is interesting to note the manner in which the values, and the mode of variation, of the thermal efficiency are affected by the nature of the fuel employed.

Three liquid fuels are here considered, namely, petrol, benzole, and alcohol, all of which are capable of being utilized in petrol-type engines. The results of some tests made by Dr. Watson * upon a sleeve valve engine with these respective fuels are illustrated in Fig. 58.

The same compression pressure and engine speed conditions were employed in each case, and the same carburettor, which was of the two-jet type, was used for all the fuels, with the addition

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* *Proc. Inst. Aut. Engineers,* 1914.

of a heating or vaporizing arrangement, and larger jet for the alcohol mixtures (methylated spirits being used in these cases).

Under these conditions, the curves show that the thermal efficiency with alcohol is greater than either of the other fuels, benzole being next, and petrol the lowest.

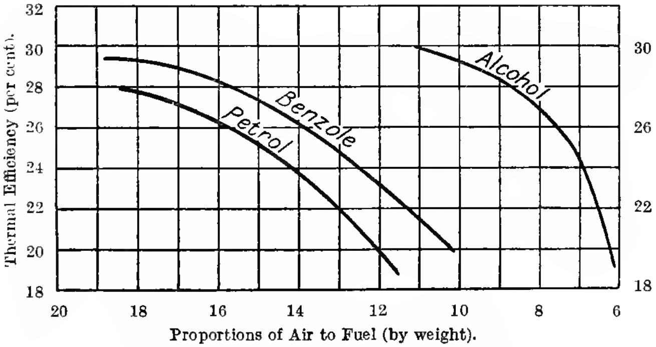


FIG. 58.—VARIATION OF THERMAL EFFICIENCY WITH FUEL MIXTURE.

The maximum values and mixture proportions for the three fuels are given below in Table XXIII.

TABLE XXIII

Fuel.	Parts $\frac{\text{Fuel}}{\text{Air}}$ by Weight.	Maximum Thermal Efficiency per cent.
Alcohol .	11·5	30·0
Benzole .	18·5	29·4
Petrol . .	19	28·0

The ranges of mixtures employable with the three fuels are about the same in each case.

It should be remembered, in making comparisons of thermal efficiencies, that different fuels, for the maximum efficiencies under working conditions, require different compression pressures.

Alcohol, when employed in an internal combustion engine, requires, for the best efficiencies, a compression pressure varying from 150 to 200 lbs. per sq. inch, and yields thermal efficiencies of from 35 to 38 per cent., which is about 25 per cent. greater than for petrol.

Again, benzole, as used in petrol engines, although giving better thermal efficiencies at the same compression pressures, will yield the best results when the compression pressure is from 10 to 15 per cent. higher than with petrol.

In connexion with the use of alcohol in petrol engines, the better thermal efficiencies obtained have been explained by Continental authorities as being due to the slower rate of flame propagation, and consequently more uniform and lower temperatures evolved. The losses of heat to the cylinder walls, which increase as the temperature of explosion increases, will be correspondingly less, although the duration of the period of high temperatures or combustion will be longer.

Dr. Ormandy,* who has experimented with different fuel mixtures in petrol-type engines, gives the interesting results illustrated in Fig. 59, showing the variation of the thermal efficiency with engine speed for different fuel mixtures.

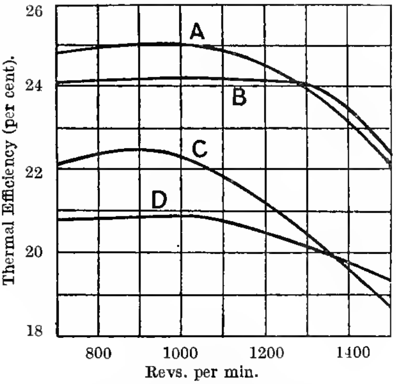
It will be noticed that the alcohol mixtures with benzole give the highest thermal efficiencies.

In connexion with the practical value of different fuels possessing high thermal efficiencies, it must be understood that a high thermal efficiency does not necessarily imply a low fuel consumption for a given horse power output.

Since the thermal efficiency varies inversely as the product of the quantity of fuel and its thermal capacity or heating value, it follows that if the latter factor be relatively small, the former must be higher for the same thermal efficiency and I.H.P.

In the case of petrol having a heating value of 19,600 B.T.U.s per lb., the consumption per B.H.P. hour on the average was 0·68 pint, whilst with alcohol, whose heating value is about 9,800, the fuel consumption worked out at ·95 pint per B.H.P. hour when used in the same engine under the same working conditions of speed and compression, etc.

Fig. 60 illustrates graphically the fuel consumptions of some of the fuel mixtures tested in a petrol engine by Dr. Ormandy. These curves correspond with those given in Fig. 59.



Fuel Mixtures.

A.—1 Benzol to 1 Meth. Spirits.
B.—2 Meth. Spirits to 1 Benzol.
C.—90's Benzol.
D.—Petrol.

FIG. 59.

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* "Some Experiments on Mixed Fuels," *Proc. Inst. Aut. Engineers*, 1913–14.

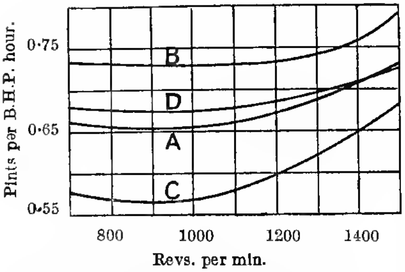


FIG. 60.—FUEL CONSUMPTIONS FOR DIFFERENT FUELS.

The relatively greater fuel consumption of the alcohol mixture is marked, and if the results be interpreted upon a basis of cost per gallon, say, of the fuel, then alcohol would appear to be a dear fuel to use, in place of petrol, in ordinary petrol engines.

Influence of Compression upon Efficiency.

From both theoretical and practical considerations it can be shown that the thermal efficiency increases with the compression pressure for the same speed and mixture strength.

Dealing with the theoretical aspect, in the first instance it has been shown, that in the case of an engine working upon the Otto cycle the efficiency can be expressed as $E = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$, where γ is the ratio of the specific heats at constant pressure and volume respectively, and r is the compression ratio (which is of course directly proportional to the compression pressure).

As this compression ratio is increased the second term upon the right-hand side diminishes, and therefore the efficiency increases, and further, when $r = \infty$ the efficiency is unity ; that is, if the gas could be compressed indefinitely, the useful work given out during the cycle of operations would be equal to the heat supplied to the gases.

From the practical point of view, the effect of increased compression ratio is to affect the thermal efficiency in several ways, which will be studied individually herewith.

The losses of heat from the explosive mixture which occur in an internal combustion engine depend, amongst other things, upon :—

(1) The total internal area of the exposed walls.
(2) The time of duration of the heat loss.
(3) The density of the charge.

The total internal area, in an engine, exposed at the end of the compression stroke will be relatively smaller as the compression ratio is increased.

This can be shown in an elementary manner by considering the combustion chamber to be cylindrical and with a flat head, the piston also being flat-headed.

The ratio of the surface exposed at the end of the compression stroke to the total area at the beginning of the same stroke will be inversely proportional to the compression ratio, that is to the final and initial cylinder lengths.

The effect of smaller area exposure is to reduce the heat loss due to area exposed, although this is to some extent, as we shall see later, counteracted by the effect of increased density.

The effect of increased compression pressure is also to cause the duration of the time of explosion, or period of attainment of maximum pressure and combustion to be shorter, so that the time during which the greater part of the heat loss occurs is reduced.

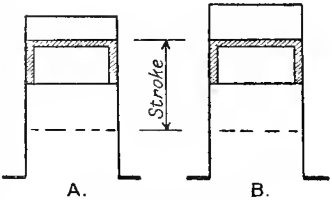
Incidentally, it might be mentioned here that as engines with higher compressions generally run at higher speeds, the effect of duration of the period of heat loss is still further reduced.

Considering, lastly, the influence of density of the charge upon heat loss, although this has already been dealt with in Chapter III, the results might be briefly summed up here by saying that as the compression ratio is increased, the density of the charge goes up and the temperature of compression increases ; this increase in temperature is not much, however, and only amounts to about 100° C. for an increase in the compression ratio of from 4 to 6.

The heat flow will slightly increase for this reason, but not in proportion to the density, and at the same time the area of the walls is smaller.

The net effect, then, of factors (1), (2) and (3) is to cause a decrease in the heat loss from the charge to the cylinder walls, as the compression pressure is increased up to a certain limit.

Further, as the compression ratio is increased, the ratio of cylinder volume to clearance volume increases, as will be seen from Fig. 61, which represents two engines of equal bore and stroke, in one of which, A, the clearance volume is, owing to the higher compression ratio, smaller than in the second, case B, the working volumes being the same.



A.—Working Volume = 4.
B.—Working Volume = 3.

FIG. 61.

The ratio of the working volume to the clearance volume is greater in case A than in case B.

From this it follows that there is a smaller volume of residual exhaust gases to mix with, or to dilute, the incoming fresh charge, and within fairly wide limits this diminution in exhaust gas dilution increases the efficiency, as a relatively larger volume of the charge enters into action as the compression ratio increases.

It has further been shown by closed vessel experiments that the higher the compression pressure the quicker the explosion period is and the higher were the pressures obtained.

Pressure Data for Petrol Engines.

In the case of petrol engines, the higher pressures of explosion for higher compression pressures are shown, both upon the indicator diagram and from power output measurements; and there exists a rough empirical relation between the compression and explosion pressures, which can be expressed as $P_E = a.P_c$, where a is a constant usually taken for design purposes as being between 3·5 and 4·0 for petrol engines working under the best conditions, and upon the four-cycle principle; P_E and P_c being respectively the explosion and compression pressures.

In the case of two-stroke engines of the three-port type, non-scavenged, and employing crankcase compression, the approximate empirical relation connecting the compression and explosion pressures is given by $P_E = 3{\cdot}3$ to $3{\cdot}8\ P_c$ for four-cycle engines, and by the relation $P_E = 2{\cdot}9$ to $3{\cdot}6$ for two-cycle engines.

These relations are deduced from the results of a large number of measurements of indicator diagrams.

In the same way, there is a certain general relation existing between the compression pressure and the mean effective pressure reckoned on the indicated horse power.

The results of the Horse Power Rating Committee's investigations,* together with those of numerous other tests which have been made upon four-cycle petrol engines, show that the following approximate relations between the compression pressure P_c and the mean effective pressure P_m, reckoned on the indicated power, exist, namely, $P_m = c{\cdot}P_o$, where c is a constant.

The value of the constant c varies, in the extreme cases, between 1·45 for highly efficient engines down to 0·40 for poor types.

Engines with compression ratios between 4·0 and 5·0 give an average value for $c = {\cdot}60$, whilst for compression ratios between 3·5 and 4·0 an average value for $c = {\cdot}65$, and finally for compression ratios between 2·7 and 3·5, on the average $c = {\cdot}75$.

These relations tend to show that engines with high compressions do not give, relatively, such high mean effective pressures as the medium compression engines.

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* *Proc. Inst. Aut. Engineers,* 1910–11.

The Effect of Compression.

From the above statements it will be seen that the chief beneficial influence of compression upon engine efficiency is to cause the maximum explosion pressure and the mean effective pressure upon the piston to be higher, due, of course, to the greater range of expansion.

With the same terminal pressure in each case, the effect of higher compression is to enlarge the area of the indicator diagram for the same quantity of gaseous mixture, that is, for the same fuel consumption. This is shown diagrammatically in Fig. 62, the dotted diagram representing the higher compression one.

The influence of compression upon thermal efficiency is well defined in the case of the gas engine, and much more information is available for this type of engine. A brief survey of gas engine results will help to show, by analogy, the effects of compression in petrol engines.

Practically the whole history of the development of the gas engine and of its great improvement is due to the increasing compression pressures and consequent thermal efficiency increase.

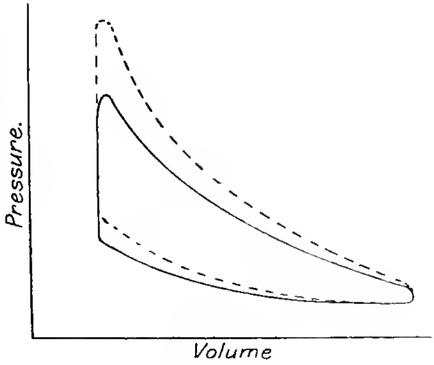

Fig. 62.

Thus the earlier types of gas engine utilized compressions of about 50 lbs. per sq. inch, absolute, and the thermal efficiencies were about 16 per cent. in the best cases.

Through intermediate stages of increase in compression we come to the case of the modern gas engine employing compression pressures of 150 lbs. per sq. inch and over, and yielding thermal efficiencies of about 37 per cent.

Some interesting results were obtained by the Gas Engine Research Committee of the Institution of Mechanical Engineers in their experiments in 1908 upon a Premier gas engine of 16 inches diameter and 24 inches stroke, using different compression pressures.

Measurements of the fuel consumption and of the indicated horse power at various compressions, very conclusively showed that commencing with a compression ratio of 4·0 and gradually increasing this ratio, the thermal efficiency increased practically at the same rate at first, and at a slightly lower rate near the highest compression ratio of 8·0 employed.

The lowest measured efficiency was about 30 per cent. with the lower compression ratio, and about 40 per cent. with the higher ratio.

The general relation between the thermal efficiency and compression ratio can be seen in Fig. 63, which shows the results, given by Clerk, of the above tests.

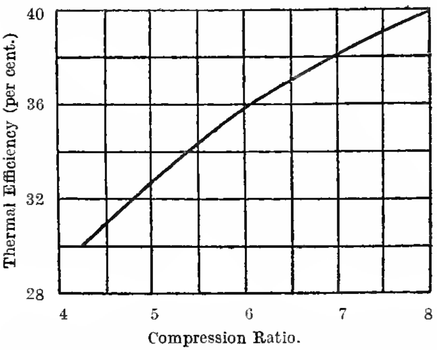

FIG. 63.—VARIATION OF THERMAL EFFICIENCY WITH COMPRESSION IN A GAS ENGINE.

Unfortunately, with the exception of Dr. Watson's results already mentioned, there do not appear to be any other reliable parallel experiments, at the time of writing, of the above nature, in the case of petrol engines. The results of Dr. Watson's tests are interesting in this connexion, but as the range of compression ratios employed, namely, from 3·9 to 4·7, is rather limited, no general deductions can be made.

In the case of petrol engines, the range of compression ratios available in practice is much smaller than in the case of the gas engine, and so the maximum possible efficiencies are necessarily lower.

Limits of Compression in Petrol Engines.

As in the case of the gas engine, the compression ratios were also low in the petrol engine in its early stages of development, and consequently efficiencies and fuel consumptions were thereby adversely effected.

The modern engine has a relatively much higher compression ratio, lying between 3·5 and 5·0 for ordinary touring engines, and at 5·0 and above for racing engines.

The limits of compression are fixed by several factors in practice, such as by the tendency to preignition of the charge, the strength of the working parts, heat losses to the walls, etc.

The higher the compression the higher will be the explosion pressure and temperature developed, and the effects of high explosion temperatures are to increase the heat losses during explosion and expansion to the walls, and to increase the mean temperature of the piston head and cylinder walls, so that the incoming charge is heated more; and this heating, in addition to the heat of compression, tends to bring the temperature of the mixture to its ignition point before the end of the compression stroke is reached, and before the spark occurs. In this manner the charge tends to self-ignite if the compression pressure is too high, and once this premature ignition occurs it will go on occurring earlier and earlier in the compression stroke, until the engine ceases to work at all.

Another adverse effect of ultra-high compression is that the total charge weight of inducted mixture is lower, owing to the higher mean temperature of the cylinder walls, which heats the incoming charge and thereby reduces its density, so that a small weight of the mixture remains at the end of the suction stroke.

Apart from the explosion temperature being higher, the exhaust gases are usually hotter, and the effect of their dilution of the fresh charge is also to increase its initial temperature; further, the higher the compression pressure, the greater will be the loss of heat to the walls during the compression period.

The design of the valves, their periods of opening and closing, and the cooling arrangements require considerably more attention when the higher compressions are used.

Another disadvantageous effect of increased cylinder and piston temperatures is that due to temperature stresses set up, owing to the necessary and unavoidable irregularities in the disposition of the material of the cylinder and piston due to its design.

In the case of larger oil and gas engines these temperature stresses and distortions reach more serious proportions, and it may be said that the sizes of these engines are practically limited by these considerations.

Obviously the effect of increased maximum explosion pressures is to require stronger parts, and therefore, in general, heavier engines, for the chief stresses set up in an ordinary engine are due to the explosion pressure, and as this is for all practical purposes proportional to the compression pressure, it might be broadly stated that the strength and weight of an engine is proportional to the compression pressure.

Modern petrol engine practice is generally towards moderately high compressions, (but not ultra-high), and higher engine speeds, and the exigencies of this practice are met with by utilizing better, more durable and stronger material, and lubrication arrangements, to keep the working parts as light and strong as possible, and to wear fairly well at the same time. There are of course other factors, which are beyond the scope of the present chapter.

It will be seen from the above considerations that the upper limit of compression pressure is fixed by several factors, such as the temeprature of the cylinder walls, the explosion stresses, and the tendency towards detonation and premature ignition, heat losses to the walls, etc.

Experiments which have been made upon relatively large gas engines by Clerk,* Hopkinson, Burstall,† and others, in which high compressions have been employed, tend to·show that the limit of useful compression is somewhere in the region of 200 lbs. per sq. inch, and which corresponds with a compression ratio of about 7·5.

In connexion with the limit of useful compression, it may be of interest to quote Dugald Clerk, who states that: "Such a compression as 210 lbs. per sq. inch above atmosphere would produce, with an explosion temperature of 1,600° C., a maximum pressure of 675 lbs. per sq. inch above atmosphere, and this would involve an engine of nearly double the weight of working parts as compared with the engine tested by himself (Clerk) at Messrs. Crossley's, with but a small increase in power for the great increase in weight.

"He (Clerk) accordingly considers a compression of 200 lbs. per sq. inch as considerably above the limit likely to be useful in a simple gas engine; to render such compressions possible he considers that compound engines will be required.

"The gas engine, in his (Clerk's) opinion, is now rapidly nearing the limit of advantageously increased compression, so that no further economy is to be expected—this view was shared by many gas engine designers."

In the case of the petrol engine of car practice, the limit of possible compression pressure increase is no doubt lower, chiefly on account of its size and speed of working, and also for other reasons.

Compression ratios of 5·5 have been used in exceptional cases of racing-car engines, corresponding to pressures of about 140 lbs. per sq. inch, and all of the evidence available at the present moment would tend to show that the maximum possible limit to the compression ratio with ordinary petrol air mixtures and with the present type of petrol engine is about 6·0, corresponding with a compression pressure of about 160 lbs. per sq. inch.

Effect of Cylinder Dimensions and Shapes upon Efficiency.

Dealing firstly with the effect of the actual design of combustion chamber upon the thermal efficiency, it is evident that as the average explosion pressure developed from a given quantity of explosive mixture depends a great deal upon the amount of heat abstracted during the period of ignition by the walls of the surrounding surfaces, that the actual shape of the combustion chamber will exercise an appreciable influence upon the thermal efficiency.

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* *The Gas, Petrol and Oil Engine*, Dugald Clerk, 1909.

† *Proc. Inst. of Mech. Engineers*, 1908 (Gas Engine Research).

Broadly speaking, if the walls of the combustion head are of large area compared with the clearance volume, then the amount of the heat loss to the walls during the combustion period will be greater and the pressure developed will be lower in consequence.

Some comparison figures taken from Dr. Watson's tests are given in Table XXIV ; these figures will give some idea of the ratios of combustion chamber area to clearance, or combustion chamber volume in the case of a motor-car engine with the valves upon opposite sides of the combustion chamber, or what is commonly known as a **T**-headed engine.

TABLE XXIV

		Ratio of Surface to Volume.	Volume of Clearance.
Actual Engine **T**-headed	A	2·01	184 c.c.
	B	1·86	203 ,,
	C	1·69	233 ,,
Hemispherical Combustion Head	A	0·99	184 ,,
	B	0·94	203 ,,
	C	0·88	233 ,,

The engine in case A was of higher compression than in the normal case B ; this result was obtained by fitting aluminium caps upon the piston heads, whereas in case C a lower compression was obtained by the insertion of liners between the cylinder flanges and the crank case.

In Table XXIV, the corresponding ratios for a hemispherical combustion chamber and flat piston head are also given for comparison purposes.

It will be observed that in the latter case the ratio of surface to volume is only about 50 per cent. of that occurring in the actual engine tested for each of the compressions used.

Several successful racing-car engines and many present-day aeroplane engines have combustion chambers approximating to the hemispherical shape, but it should here be noted that in order to obtain any comparative results of thermal efficiencies for different engines several important points have to be considered.

In the same engine, the thermal efficiency is dependent upon such factors as compression ratio, mixture strength, volumetric efficiency, speed, etc., so that for a comparison of the relative values of combustion chambers of varying shapes the conditions of comparison should be identical, if they are to be of any scientific or real value at all.

These all-important factors appear to be ignored in many comparisons, as the results of practical tests, put forward from time to time in contention of the relative unimportance of combustion chamber design.

There can be little doubt that this factor of combustion chamber shape does exercise an influence upon the thermal efficiency apart from other considerations of charge induction, and heating, exhaust products ejection, etc., from the heat-loss point of view.

It should not be forgotten, however, that in the same combustion chamber the loss of heat from the exploding charge to the walls depends also upon the *nature* of the wall surfaces.

Thus if the walls are highly polished, the heat radiated from the charge will be less readily absorbed by the walls, and a higher thermal efficiency will result, as mentioned in Chapter III.

If the walls are left in the rough state as cast, the absorption of heat will be greater than in the preceding case, as Hopkinson's tests with explosion vessels prove. Moreover, these results are borne out by the results of actual tests upon a gas engine with a polished combustion chamber, in which an appreciable increase in the thermal efficiency (and mean effective pressure) have been measured.*

Again, in petrol engines in which a coating of carbon deposit has occurred in the combustion chamber, this layer of carbon, having a conductivity of about $\frac{1}{50}$ that of iron, will be at a relatively higher temperature, owing to its bad conductivity, than the plain iron walls would be, and so the rate of heat loss to these carbon-coated walls would be reduced upon this account, and less heat would be abstracted from the combusting charge.

It will thus be evident that in comparing the performances of different engines having differently shaped combustion chambers, account should certainly be taken of the above-mentioned factors.

Coming next to te consideration of the effect of cylinder dimensions upon thermal efficiency, it would assist the reader to a clearer understanding of this important factor to refer to Callendar's original Paper entitled "The Effect of Size on the Thermal Efficiency of Motors," † upon which the following deductions are based.

The heat loss to the walls in an engine is a function of the volume of the cylinder, and of the area of the internal walls.

Since the volume varies as the cube of the linear dimensions, and the area as the square, it follows that the heat loss in different sized engines will vary as the ratio $\frac{\text{surface}}{\text{volume}}$, that is, inversely as their diameters.

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* *Gaseous Explosions Committee's Report,* 1909.

† *Proc. Inst. Aut. Engineers,* May 8, 1907.

This is taken into account in Callendar's expression for the relative efficiency of different engines, namely :—

$$\text{Relative Efficiency} = \mathrm{E}\left(1 - \frac{\mathrm{A}}{\mathrm{D}} - \frac{\mathrm{B}}{n\mathrm{D}}\right)$$

where E is the theoretical limit of efficiency which is attainable in a petrol engine, taking into account the varying specific heat of the gases, and assuming no thermal losses to the jacket by radiation, etc. (this is about 80 per cent. of the air standard efficiency).

The second term in the bracket represents the ratio of surface to volume, A being a constant, and which represents the cooling correction.

The last term takes into account the effect of speed upon the thermal efficiency; this will be considered hereafter more fully.

It is assumed that the strength of mixture, nature of surface, etc., are the same in each case, or at least that due allowance in the value of the theoretical limit E is made.

The above formula is based upon the assumption that the whole loss of efficiency $\frac{\mathrm{A}}{\mathrm{D}} + \frac{\mathrm{B}}{n\mathrm{D}}$ expressed as a fraction of the theoretical limit E (which may be taken as ·80 for petrol engines) is proportional to the ratio of surface to volume $\left(\frac{1}{\mathrm{D}}\right)$ for similar motors.

In order that the values of the constants A and B should be the same for similar engines, it is necessary that the stroke bore ratio $\frac{\mathrm{L}}{\mathrm{D}}$ should be proportional to $(r-1)$, where r is the compression ratio.

In Callendar's formula, the effect of valve pockets is not taken into account, and it is considered as possible that either the valve pockets are more liable to become coated with carbon deposit or that the turbulent motion of the induction stroke is more quickly damped out, so that they may be disregarded as influencing the thermal efficiency.

It therefore follows from the above formula that the thermal or relative efficiency of an internal combustion engine becomes greater as the diameter of the cylinder increases for engines of similar design and condition, working at the same piston speeds.

Upon this result, the efficiency of the smaller sizes of petrol engines would appear to be correspondingly low, that is to say, their relative fuel consumptions per horse-power hour should be higher.

It is of interest to note that the results yielded by Callendar's formula when applied to such diverse cases as small air-cooled petrol engines, motor-car engines and large gas engines, give values for the efficiencies corresponding closely with the observed values, when the conditions under which the observations are made are the same, or when due allowances are made for varying conditions.

Relative Thermal Efficiencies of Long and Short Stroke Engines.

It is interesting to compare theoretically the thermal efficiencies of two engines of the same capacity but with different stroke-bore ratios.

The shorter stroke engine being designated by the letter A and the longer stroke engine by the letter B, then, if we assume the same compression ratio and piston speed in each case, B will have a smaller ratio of combustion space area to volume than A, so that during the explosion period the heat loss to the walls (assumed at constant volume) will be less for the longer stroke.

Upon the other hand, the piston speeds being the same, B will run at a lower rate of revolution than A, so that the period of the explosion stroke will be relatively longer in B's case (in fact the periods of the explosion or expansion strokes will be directly as the length of stroke), so that the expanding gases will remain in contact with the cooler walls longer in B's case ; therefore the long stroke scores in efficiency over the short stroke for the first reason, but this effect is opposed by that mentioned in the second case.

It is probable, therefore, that at the same piston speeds and compression ratios that the shorter stroke will show the better efficiency, owing to a predomination of the second factor mentioned. Actual tests go to show that at low piston speeds the short stroke engine has a somewhat better thermal efficiency, and hence for a given fuel consumption yields a higher power, due chiefly to the shorter expansion stroke interval. At higher speeds, however, the long stroke engine is probably the more efficient, as it can run at higher piston speeds, for it is the rate of engine revolutions which limit the power of an engine, other things being the same.

Another important influence also demands consideration here, namely, the compression ratio ; this has been taken as being the same in each case, but it is quite possible (and practical) to employ a higher compression with the longer stroke engine, since the cooling effect of the walls during the compression stroke will be more marked in reducing the compression temperature, and it is the temperature at the end of the compression stroke which limits the compression pressure, for beyond a certain final compression temperature preignition occurs. This increased compression results in an increased thermal efficiency, as already mentioned.

From the point of view of thermal efficiency, therefore, it is quite probable, and tests bear out this supposition, that the smaller combustion area, higher piston speed, and increased compression ratio of the longer stroke engine, give it some advantage over the shorter stroke engine, in spite of the greater cooling effect upon the expanding gases, but that at moderate piston speeds the shorter stroke engine will not differ appreciably from the longer stroke.

The differences between the two types of engine can be represented upon the indicator diagram as shown in Fig. 64, in which the cooling effect of expansion in the case of the longer stroke, the higher compression (and consequent higher explosion pressure) are shown by the dotted lines, whilst the diagram for the shorter stroke type is shown shaded.

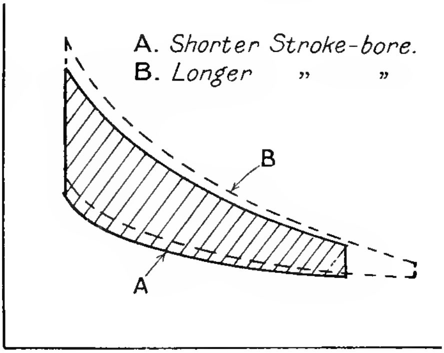

FIG. 64.

It can be shown by actual numerical examples that unless there is a fairly marked difference between the two compression ratios, the results obtained, so far as thermal efficiency are concerned, will be very much the same.

Thus for any real advantage, the stroke-bore ratio should be at least 1·5 ; below this ratio the difference in thermal efficiency is practically negligible.

The Offset Cylinder.

Whilst dealing with the effect of cylinder dimensions and proportions upon thermal efficiency, it is of interest to here mention the "desaxe" or offset cylinder.

As will be shown later, the "desaxe" cylinder gives a somewhat longer period to the explosion stroke, as compared with the normal type, so that the cooling effect will be greater, and this will adversely affect the efficiency.

On the other hand the piston travels more slowly on the induction stroke, so that a greater charge volume is inducted, and a somewhat cooler charge results.

The explosion pressure will therefore be higher, and the maximum temperature higher, with a corresponding increase in the thermal efficiency. The two effects are in the opposite direction, but it is probable that the latter mentioned effect outweighs the former.

Unless, however, the offsetting of the cylinders is appreciable, that is, greater than the few millimetres usually allowed in motor engineering practice, no appreciable difference in efficiency will result.

It should be mentioned, in conclusion, that the "desaxe" cylinder is adopted, not for thermal efficiency reasons, but in connexion with reduced piston thrust upon the cylinder walls during the firing stroke.

The Effect of Speed upon Thermal Efficiency.

It is generally known that as the speed of a petrol engine is increased up to a certain limit, the thermal efficiency increases.

It is not very difficult to understand why this should be so, for considering an extreme case when the engine is running very slowly, the time of the suction stroke will enable the inducted charge to attain the temperature of the cylinder more readily, and will thus tend to reduce the charge density. During the compression stroke the charge will part with a greater fraction of its internal heat to the cylinder walls, and more importantly still, the heat loss to the cylinder walls during the explosion stroke will be relatively much greater, and the efficiency will consequently be lower on this account. Raising the compression, theoretically, should be advantageous to slow-running petrol engines in reducing part of the heat losses.

On the other hand, when the engine is running at a high speed, the duration of the periods of charging, compressing, and exploding the mixture will be relatively much smaller, and the quantitative values of the heat exchange between the charge (either unburnt or when combusted) and the cylinder walls will be smaller.

During the explosion stroke the heat loss to the cylinder walls will be smaller, so that one would reasonably anticipate a somewhat better efficiency at higher speeds from these considerations.

It was expected, upon theoretical grounds, that the thermal efficiency of the relatively small-sized petrol engines would be low as compared with those of large gas engines, but the results of actual experiments, such as those of Watson, Hopkinson, Neumann, and others, prove that the relative efficiencies of these little engines compare very favourably with those of large gas engines ; this has been shown to be chiefly on account of the very much higher speeds at which they run.

Fig. 65 illustrates the results of some measurements of thermal efficiencies at different engine speeds, made by the author, upon a four-cylinder Talbot engine.

An examination of either of the curves given in the figure shows that the efficiency increases fairly uniformly up to a certain limiting speed, beyond which it tends to fall off again ; this is probably because of the limiting efficient cylinder temperatures having been reached. The curves also show that the efficiency of the partially throttled engine [running at $\frac{3}{4}$ full power] is higher than when running at full power. It has been stated by competent authorities, that from the fuel economy point of view, a large engine running normally upon part throttle, is better than a high speed engine working at full power,

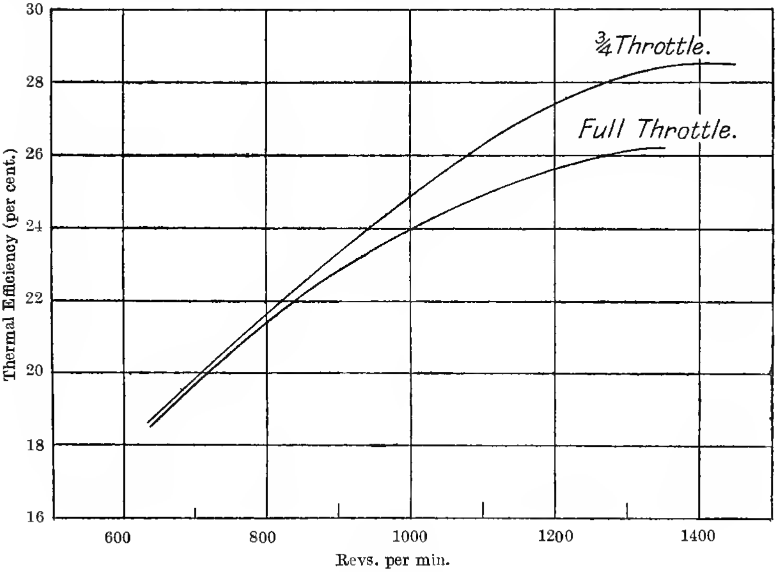

FIG. 65.

Any higher temperatures would probably result in successive heating of the incoming charge, causing a falling off in charge weight, and tendency towards preignition.

Apart from temperature considerations, practical points connected inherently with the design of the valves and gas passages, weight of the moving parts, etc., fix an upper limit to the speed.

Generally speaking, however, the best thermal efficiency is attained at some point before the maximum speed of the engine is reached, so that at this speed the thermal efficiency, and therefore the fuel consumption, per brake-horse-power hour will be lower.

Many transmission inventions, that have been brought out at different times in the history of the motor-car, have aimed at running the engine at its maximum thermally efficient speed permanently, and obtaining various gear ratios by suitable mechanical means, so that the constant torque of the engine could be reduced to any degree at the road wheels.

Turning again to the theoretical aspect, in Callendar's formula for the relative efficiency of a petrol engine of cylinder diameter D, there is a term to include the effect of speed.

This formula is usually given as :—

$$\text{Relative Efficiency} = \cdot 80\left[1 - \frac{A}{D} - \frac{B}{nD}\right]$$

The term $\frac{B}{nD}$, where B is a constant, represents the speed factor, n being the number of revolutions per minute.

It will be seen that as the speed increases, this term becomes relatively smaller, and the efficiency relatively higher on this account.

According to this formula, which was evolved from Callendar's experiments upon a small air-cooled petrol motor, the limit of efficiency in any given engine is reached when the speed is infinite ; the limiting efficiency then becomes $\cdot 80\left(1 - \frac{A}{D}\right)$, which is dependent upon the cylinder dimensions only.

In actual practice, as previously mentioned, the design of the engine, with its influences upon gas inertia, and lag, fixes the upper limit of efficiency.

Thus in Callendar's experiments it was found that the maximum efficiency of 0·52 occurred at 2,200 revolutions per minute.

It has further been shown that the variation of efficiency with speed is nearly the same for different compression ratios, and different mixture strengths over a wide range, as an examination of the published results of Watson and others will show.

Thermal Efficiencies of the Four-Cycle and Two-Cycle Engines.

It has been pointed out that the greatest possible thermal efficiency of a petrol engine working under ideal conditions (that is, with no thermal losses to the water jacket, etc.) is 20 to 25 per cent. less than the air standard efficiency, which in itself depends upon the compression ratio employed.

In the case of petrol engines of the touring-car type, the compression ratio lies between 3½ and 5, and corresponding to this upper limit, the highest efficiency that could possibly be attained would be about 38 per cent.

When one considers that the thermal efficiency of modern four-cycle engines is in the region of 28 to 30 per cent., as the results of numerous scientific tests have shown, it will be realized how extremely efficient is this type of internal combustion engine, and what a small range of possible improvement is left—indeed, it would seemingly appear futile to attempt to improve in this respect the modern petrol engine, working upon the Otto cycle.

Considering the elementary type of two-cycle engine in which the piston uncovers the exhaust and inlet ports respectively, measurements of the thermal efficiency of this type of engine show that the maximum efficiency obtainable under the best conditions of mixture strength, speed, etc., is about 23 per cent. for the same compression ratio as in the four-cycle motor-car engine.

This relatively lower efficiency of the two-cycle engine is no doubt due to the charge wastage which is associated with the design of the engine, and in the better designs of engine, where overlapping of the ports cannot occur, or is at least reduced, higher efficiencies may be expected.

When due allowance is made for the amount of fuel which passes out with the exhaust gases unconsumed, so far as the engine is concerned, the net thermal efficiency attains the value of about 26 per cent.

It is therefore probable that the relatively higher cylinder temperatures will tend towards higher thermal efficiencies, and that generally speaking there is not much to choose between the two- and four-cycle engines in the matter of thermal efficiency in well-designed types.

It might be pointed out here that the usual shape of the combustion chamber in two-stroke engines is hemispherical, on account of the absence of valve pockets, etc., so that theoretically some advantage in efficiency should be expected.

The Heat Balance in Petrol Engines.

In Chapter IV it is shown mathematically that if the temperatures of the working substance be known at the four points ABCD of the indicator diagram (Fig. 54), and the quantities of heat energy present at these points be also known, then the whole heat energy of the inducted charge can be analysed in its distribution for the complete cycle of operations.

This heat balancing, as it is termed, involves a fairly accurate knowledge of the variation of the specific heat of the working substance, and of the nature of the combustion process, two more or less uncertain factors.

It is possible, however, to examine the way in which the heat energy of the charge is utilized in the various operations in the working cycle by actual experiment, and generally speaking the results are more definite and reliable, if the tests are properly carried out.

Thus by measuring the quantity of mixture used in a given time by an engine, the total heat energy of the induced charge can be ascertained, knowing the calorific value of the fuel.

Measurements of the indicated horse power give data for calculating the actual percentage of this total heat energy usefully employed, or used up in doing useful mechanical work. By measuring the temperature rise of the cooling water, and the quantity of same employed, an idea of the actual amount of heat lost to the jackets by conduction and radiation can be obtained.

By means of an exhaust calorimeter the quantity of heat leaving the engine in the exhaust gases can be ascertained.

It is even possible to measure approximately the quantity of heat radiated from the engine, and one method of arriving at this quantity is given in *Report of the Committee on the Efficiency of Internal Combustion Engines*, 1904–6.

From the results of these different measurements some idea can be formed of the actual manner in which the heat energy of the charge is allotted to the different processes.

In the case of one of the gas engines tested by the above Committee, the following table represents the average heat balance obtained—the engine working under full load :—

TABLE XXV

Heat usefully employed (I.H.P.)	31·8
,, lost by exhaust gases	34·1
,, lost to cooling water	26·5
,, lost by radiation	7·6
	100·0

From these results it will be noticed firstly that the thermal efficiency as represented by the figure 31·8 per cent. is rather higher than in the case of petrol engines, and this is accounted for partly by the higher compression employed, and partly by the larger cylinder diameter, upon which, as previously shown, the efficiency depends; in many engines of modern design efficiencies of 38 per cent. are fairly general, in gas engine practice. Secondly, it will be noticed that more heat is carried off by the exhaust gases than is given out as useful work, and that a large percentage of heat is given to the cooling water.

The above results for a gas engine have been quoted inasmuch as similar figures for a petrol engine are difficult to obtain, but the analogy between the two types is very close as the working conditions are about the same.

An average heat balance chart for a modern petrol engine is given herewith, based upon the results of numerous tests made by the author, but it should be understood that this table is approximate only, in so far as the exhaust and jacket losses are concerned, as will be explained later.

Heat Balance Chart.

TABLE XXVI

Heat available as B.H.P.	24·0
,, utilized in overcoming engine friction . .	4·0
,, carried off in exhaust gases	40·0
,, given to jacket water, and . . .	28·0
, lost by radiation	4·0
	100·0

In the above table, the value given for the heat carried off by the exhaust gas is intended to represent the actual quantity of heat remaining in these gases and measured just under the exhaust valve.

It will be evident that the exhaust gases are losing heat to the cylinder walls and jacket during the whole of the exhaust stroke, and also to the metal around the valve and its ports, so that it is very difficult to obtain an exact idea as to the real quantity of heat of the charge lost to the jackets during the explosion stroke, and to the amount in the gases at the completion of the expansion stroke. It is practically impossible to separate these two quantities, although an idea of their relative magnitudes can be obtained by a study of the action occurring in the case of the ideal engine —for it is possible to estimate the quantity of heat contained off in the exhaust, and by differences that lost by conduction through the cylinder walls can be obtained.

As an example, consider the ideal diagram corresponding to an actual indicator diagram, as shown in Fig. 66, in which the shaded

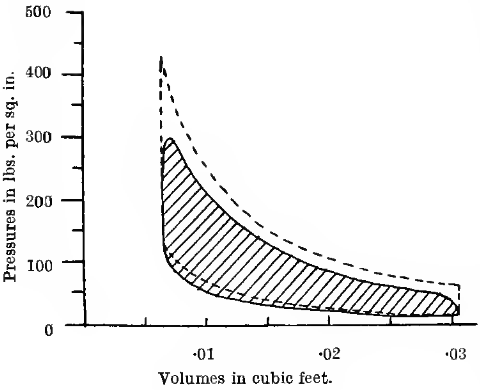

FIG. 66.—IDEAL AND REAL INDICATOR DIAGRAMS.

diagram represents the indicator diagram taken from a petrol engine, and the dotted diagram the ideal one.

The heat supply per cycle in this case is 1,000 foot-pounds.

The ideal diagram is drawn for the case of a variable specific heat cycle, and from these two diagrams the following results have been obtained :—

	Heat Converted into Work.		Heat Rejected.
Ideal cycle . . .	440	..	560 foot-pounds
Actual cycle .	250	..	750 ,,

Now in the ideal cycle no thermal losses to the walls are assumed, so that the ideal engine would reject 560 foot-pounds to the exhaust.

The actual engine rejects 750 foot-pounds, which is made up of both exhaust and conduction losses, so that the amount of heat probably lost to the cylinder walls is 750−560, or 190 foot-pounds.

Hence one may now estimate the probable percentage heat accounts for the actual engine, considering the true quantities of heat given up to the walls alone, and to the jacket alone.

The following is then the probable heat account or balance :—

TABLE XXVII

Heat usefully employed (I.H.P.)	25·0
,, given to exhaust gases	56·0
,, given to jacket water and lost by radiation .	19·0
	100·0

From what has already been mentioned in reference to the maximum possible efficiency of the Otto cycle, it is evident that the apparently large losses of heat energy of the charge to the jacket water and to the exhaust gases cannot be appreciably reduced, so long as the engine works upon this cycle of operations. Generally, the attempts which have been made to reduce, say, the cooling water and radiation losses have resulted in increased exhaust losses, and vice versa.

The only general method of slightly reducing these losses is of course to aim at higher thermal efficiencies.

Volumetric Efficiency.

The quantity of charge drawn into the cylinder of a petrol or other high speed internal combustion engine is always less than the theoretical quantity of charge which would fill the working volume of the cylinder at atmospheric pressure and temperature, and the ratio of the actual to the theoretical quantity is called the "Volumetric Efficiency."

The quantity of fresh charge induced depends upon several factors, namely, upon the rate of flow, the amount of obstruction encountered, and to a minor extent upon the skin friction, and eddying resistance, temperature and shape of the induction system, etc.

Considering the velocity of flow of the gases, first, for small differences of pressure, such as those existing in practice, the mean velocity of the gases is given by $v^2 = \frac{2g(P - P_o)}{\sigma}$

where P is the atmospheric, and P_o the pressure within the cylinder, σ being the density of the gases, that is, the weight per cubic foot ; σ is usually taken as being 0·081 lbs. per cub. foot at 0°C.

If P and P_o be in lbs. per cub. foot, the velocity will be given in feet per second.

The general energy equation for streamline flow for any gas is

$$\frac{v^2}{2g} + \int\frac{dP}{\sigma} + H = \text{constant} \quad . \quad . \quad . \quad . \quad (A)$$

where σ = the density, g the acceleration due to gravity, and H the datum "head" of the gas.

For adiabatic flow we can combine equation (A) with the adiabatic law expressed as $\sigma = C.P^{\frac{1}{\gamma}}$ . . . . (B) since the density varies inversely as the volume.

By substitution of this value for σ in equation (A) we obtain for the velocity of flow between pressures P and P_o

$$\frac{v^2 - v_o^2}{2g} = \frac{\gamma P_o^{\frac{1}{\gamma}}}{(\gamma - 1)\sigma.} \cdot \left(P^{1-\frac{1}{\gamma}} - P_o^{1-\frac{1}{\gamma}}\right) \quad . \quad . \quad . \quad (C)$$

The value of γ for air is 1·408.

If the terminal velocity $v_o = 0$ when the pressure is P_o we have $v^2 = \frac{2g}{\sigma.} \cdot \frac{\gamma}{\gamma - 1}\left(P^{1-\frac{1}{\gamma}} - P_o^{1-\frac{1}{\gamma}}\right)$ . . . . (D)

By using the approximation $(1 + P)^{1-\frac{1}{\gamma}} = 1 + \left(1 - \frac{1}{\gamma}\right)P$ where P_o is very little less than P we can reduce (D) to

$$v^2 = \frac{2g}{\sigma.}\left(P - P_o\right).$$

This is the approximate equation previously given. From this equation, which can be applied to the case of the exhaust and inlet gas flow in a petrol engine, the velocity of the gases can be computed.

As an illustration of the value of this equation, consider the case of the inlet gases in a petrol engine. Assuming that there is a difference of pressure of 1 lb. per sq. inch between the cylinder (during the induction stroke) and the outside atmosphere around carburettor. Assuming further that the density of a mixture of 15 parts air to 1 of petrol is ·090 lb. per cub. foot, we have

$$P = 14{\cdot}7 \times 144 \text{ lbs. per sq. foot}$$

$$P_o = 13{\cdot}7 \times 144 \quad ,, \quad ,,$$

$$\text{Then } v^2 = \frac{2 \times 32{\cdot}2}{{\cdot}09}(14{\cdot}7 - 13{\cdot}7) \times 144$$

$$\text{whence } v = \sqrt{\frac{64{\cdot}4 \times 144}{{\cdot}09}}$$

$$= \underline{320 \text{ feet per second, very nearly}}$$

This theoretical velocity, needless to say, is higher than would be realized in practice, as the effects of eddies, obstructions and skin friction have not been taken into account; the flow being considered as streamline.

Weight of Charge.

It is often useful to be able to compute the total weight of gas flowing in any given case.

If the area of the orifice or port be denoted by A, then the volume of gas Q flowing per second (if v is in feet per second) is given by

$$Q = Av.$$

The weight of gas $W = Q\sigma = Av\sigma$.

The value of the velocity v is known from the general equation of adiabatic flow equation (D), so that by substitution and simplification the weight of gas flowing per second can be expressed as

$$W = A.\left(\frac{P_o}{P.}\right)^{\frac{1}{\gamma}}.P\sqrt{\frac{2g\gamma}{\gamma-1} \cdot \frac{\sigma.}{P}\left[1 - \left(\frac{P_o}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad . \quad . \quad . \quad . \quad (E)$$

As all of the terms on the right-hand side of equation are known, W can be calculated.

It is of interest to mention that for the maximum quantity of a gas to flow between two places at pressures P and P_0 respectively, in an adiabatic manner P_0 must equal ·527 P.

This result can readily be obtained by differentiating equation (E) with regard to P_0, and equating to zero in order to find the maximum weight W.

Returning to the approximate relation $v = \frac{2g}{\sigma}(P - P^o)$, it follows that the value of v will depend upon the difference between the pressures existing inside and outside the cylinder.

It will be shown later that the frictional resistance due to the skin friction, eddying and bend effects varies approximately as the square (and probably to higher powers of the velocity) of the velocity, and hence that it is imperative to keep this charge velocity as low as possible, and therefore the pressure difference must also be small, in order to ensure a high volumetric efficiency, assuming the density to remain about constant.

In regard to the question of keeping the inlet velocity as small as possible, this is attained in practice by the employment of valves of large area, or by a multiplicity of valves, combined with fairly straight and smooth pipes and passages.

In practice, the inlet velocity varies from 120 to 250 feet per second, according as the engine is a normal or racing type.

Computation of the inlet velocity of several successful racing-car engines indicates that the most suitable velocity is between 240 and 280 feet per second.

The mean velocity of the inlet gases can be computed, approximately, from the engine speed, piston area and induction pipe area (assumed uniform).

Thus the mean velocity $V = \left(\frac{D}{d}\right)^2 V_P$

where D and d are the respective areas of the piston and inlet pipe, and V_P the mean piston velocity, which may vary from 1,000 to 1,500 feet per minute in touring engines, and up to 3,000 feet per minute for racing engines.

It should be remembered, however, that the maximum velocity of the piston, which occurs in the vicinity of the centre of its stroke, is about twice the mean, and that the velocity is variable throughout the stroke, so that the frictional losses will vary with this velocity to some power. It may be pointed out, here, that the results of aerodynamic tests show that at the high velocities attained in the above cases, the resistances to gaseous motion may vary to a higher power of the velocity than the square.

Effect of Friction.

The effect of skin friction, due to the nature of the surface of the inlet system, can be taken into account, and an illustration of the method is here given.

The frictional resistance of a given amount of gas flowing in a pipe of any section is given by

$$F = f.\frac{l}{m}v^2$$

where f is a coefficient dependent upon the nature of the surface of the pipe

l = the length of pipe

m = the hydraulic mean depth or the ratio $\frac{\text{area of section}}{\text{perimeter}}$ for the pipe

v = the velocity when below about 150 feet per second.

For a circular pipe

$$F = f.l\ \frac{4}{d}\ v^2$$

This expression is only approximate, and applies to cases of streamline flow below the critical velocity. It serves, however, to show how the frictional resistance (expressed as a loss of " head " or pressure) varies directly as the length, and inversely as diameter of the pipe, and directly as the square of the velocity.

A more exact expression for the velocity of flow under a given head of pressure is obtained from Biel's empirical formula,* in which frictional effects are included :—

$$H = \left(0{\cdot}12 + \frac{0{\cdot}072}{\sqrt{d}}\right)\frac{4lV^2}{1000\ d}$$

where l is the length in metres

and d is the diameter in metres

and H is the head in millimetres of water, the surfaces being taken as similar to that of galvanized iron.

The loss of head will be seen to vary inversely as the diameter to the power of three halves, and directly as the length.

The more recent experiments of Stanton † upon the subject of the flow of gases and solids in pipes of varying diameter show that these obey the same laws of flow ; this result is of interest as indicating the law of flow for a mixture of gases and vapours.

The experiments were made upon pipes varying in diameter from about one-eighth of an inch to about five inches, and the velocities of flow varied between the limits of about 5 to 115 feet per second.

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* *Zeitschriftverein Deutcher Ingenieure,* 1908, p. 1035.

† *Phil. Trans. Roy. Soc.*, A, Vol. ccxiv. *Engineering,* May 22, 1914.

The law of resistance which all fluids or gases were found to obey is given by

$$R = \varrho v^2 f\left(\frac{v.d}{\gamma}\right)$$

where R is the resistance per unit area, ϱ the density of the fluid or gas, v the velocity, and the expression $f\left(\frac{v.d}{\gamma}\right)$ a function of $\frac{v.d}{\gamma}$, where d is the diameter and γ the kinematical viscosity of the fluid, that is the ordinary viscosity divided by the density.

If this law be expressed in the form $R = k.vn$, it has been found that the value of the index n rises with the speed of flow of the fluid, or gas, as previously mentioned.

Further, the ratio of the maximum velocity to the mean velocity of flow in the pipe varies from 0·5 to 0·8 for higher values of the function $\left(\frac{vd}{\gamma}\right)$.

The curve A given in Fig. 67 illustrates the results obtained by Stanton for rough and smooth pipes, respectively, the diameters of the pipes used in the experiments being $1\frac{1}{8}$ inches.

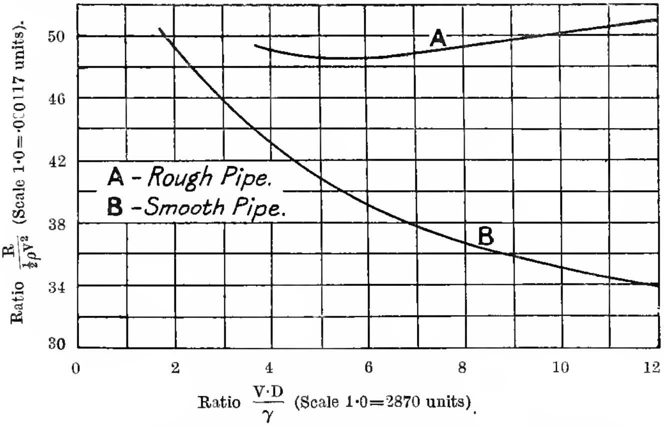

FIG. 67.—FLUID FRICTION IN ROUGH AND SMOOTH PIPES.

In these curves R is the frictional force per unit area, D the diameter of the pipe, and v the fluid velocity at the axis of the pipe.

The expression $\frac{R}{\frac{1}{2}\varrho v^2}$ has for comparison purposes been plotted against $\frac{vd}{\gamma}$, and it is interesting to note that the curves given are true for both air and water.

It may be mentioned that the kinematical viscosity of air is thirteen times that of water.

The reader who is desirous of studying this subject at fuller length is recommended to the original papers given in the foot-notes.

In order to apply the previous formulae and expressions to the case of the inlet system, it must be remembered that the effect of loss of head in the case of gas moving through a pipe is to cause a loss in charge weight, more especially as the period of induction is very small in the case of high speed internal combustion engines.

This may be better understood if the difference of pressure causing gas flow be regarded as constant, and also the equivalent area of the inlet pipe (or system); then the effect of friction will be to cause a reduction in the mean velocity, and therefore in the quantity, since quantity = velocity × area.

Another useful analogy is to regard frictional loss as being equivalent to reduction of pipe area, the velocity being constant; the quantity of gas flowing will then be seen to be reduced as the frictional losses become greater.

Effect of Bends.

The effect of bends in the inlet system is equivalent to increased frictional resistance, for the gases, having their path of flow diverted, impinge upon the outer part of the bend and eddies are caused to occur; these eddies are directly responsible for a loss of head.

The more acute the bend, the greater the equivalent frictional loss of head due to the bend.

The manner in which the loss of head varies with the radius of the bend will be better seen from the following table, giving some experimental results upon head losses in bends :—

TABLE XXVIII

Radius of Bend . .	11	7	5	2	1	·75	·5
Equivalent resistance expressed in length of straight pipe .	4·1	6·1	7·7	9·0	17·3	34·1	118

It will be seen that for a given diameter pipe the frictional loss of head varies inversely as the radius of the bend, up to the limits of bending of the pipe.

Other Sources of Loss of Volumetric Efficiency.

There are other causes of volumetric inefficiency quite apart from the effect of skin friction and bends, the chief amongst which include loss of head (and therefore volumetric efficiency) due to more or less sudden variations of section in the inlet pipe, as for instance where the inlet valve port joins the cylinder.

Every change of section is a source of eddy disturbance, and therefore of loss of head.

The effect of obstructions in the inlet system, such as the carburettor jet, valve stem, ledges, etc., is to set up " head " resistance, and thereby to cause a loss of head in proportion to the head resistance.

Other sources of loss of volumetric efficiency are those due to the heating of the gases by the hot walls of the inlet system, combustion chamber and piston, and to effect of the work done upon the gases by the piston.

So far as frictional losses are concerned, the " back pressure " effect of the exhaust gas flow will follow the same conditions as the inlet flow.

The only reliable information as yet published, in connexion with the resistance of surfaces, or skin friction, is that published by Prof. Zahm. His experiments were made upon boards varying in length from 1 foot to 16 feet, and at velocities from 5 to 20 miles per hour.

The resistance which such surfaces offer at different velocities are given as follows :—

$$R = \cdot 0000316\, l^{\cdot 93}\, V^{1 \cdot 85}$$

where R is the resistance in lbs. for the two surfaces of the board, of width 1 foot

l = the length of board in the direction of the wind
and V = the velocity in miles per hour.

A more accurate expression is that given by the National Physical Laboratory, namely—

$$F = \cdot 0000082\, A^{0 \cdot 93}\, V^{1 \cdot 86}$$

where F is the force in lbs., A the area in sq. ft. of a single surface in the case of a board, or the interior surface of a pipe, and V the velocity in feet per second. For a speed of 100 miles per hour the skin frictional force is about $\frac{1}{10}$ lb. per sq. ft.

It is quite probable that in the case of gases moving at comparatively high velocities, such as those in the inlet and exhaust systems, the resistance encountered, or " head " lost by skin friction, will vary as the velocity to a much higher index, such as 2·0 or 3·0, and that consequently the skin friction losses will be very much higher, comparatively.

Further it should be remarked that the surfaces in the inlet system and exhaust are, relatively speaking, much rougher than that of the smooth boards of Zahm.

The effect of friction, especially at the high velocities of flow encountered in practice, becomes quite serious, and were this factor negligible it would be quite possible to obtain the same volumetric efficiency with only half the valve area employed in practice.

It can readily be shown by the foregoing methods that whereas the theoretical valve diameter for a given gas velocity is 1·2 inches, actually it has to be made 1·8 inches, to allow for frictional effects through the valve, etc., for the same charge efficiency.

Heating of the Charge.

Although the fresh charge enters the induction pipe at atmospheric temperature, yet, by contact with the hot ports, valves, combustion chamber and piston top, it rapidly becomes heated up.

This heating of the charge depends largely upon the design of the engine, that is, upon whether the charge has to pass over any appreciable amount of the hot engine surfaces before it reaches the cylinder proper.

The effect of premature heating of the charge is to cause the gases (the petrol vapour and air) to expand.

Further, if the exhaust products are not effectively got rid of during the previous stroke, the charge will also be heated by direct mixing and contact with the residual products.

In order to afford some idea as to the effect of a hot engine in reducing the charge volume, Fig. 68, which illustrates the results of some charge measurement tests upon a four-cylinder car engine, is shown.

Curve A represents the quantity of charge sucked in when the engine was motored around cold, with the valves, of course, working in the usual manner, for various engine speeds.

Curve B represents the quantity of charge inducted when the engine was firing in the usual way, measured by means of the "throttle plate" method * at the carburettor.

It will be evident from these curves that although in both cases the quantity of charge drawn in falls off fairly rapidly with increase in the engine speed, yet about 12 per cent. more charge is drawn in when the cylinder is cold than when hot at 800 revolutions.

Hopkinson † mentions the case of a Siddeley engine in which

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* "The Measurement of Air Supply to Internal Combustion Engines by means of a Throttle Plate," Watson and Schofield, *Proc. I.M.E.*, May, 1912.

† The Effect of Size and Speed upon the Performance of an Internal Combustion Engine, *Proc. I.A.E.*, 1909.

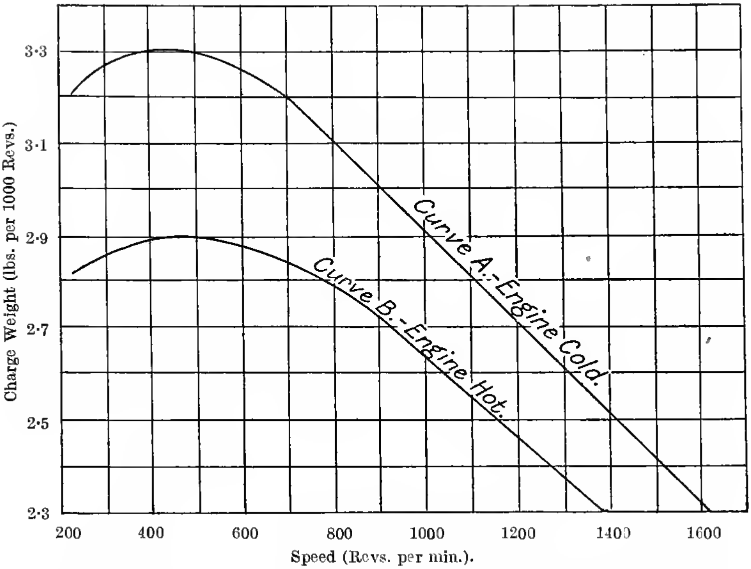

Fig. 68.—The Effect of Temperature upon Volumetric Efficiency.

about 15 per cent. increase in power is possible if the heat absorption effect is reduced during the suction period.

The volumetric efficiency of the petrol engine decreases as the engine speed increases, in all ordinary petrol engines, to a greater or lesser extent according to the design.

Fig. 69 indicates the usual volumetric efficiencies obtained in car engines of touring pattern, and also how this varies with the engine speed.

In the case of the sleeve valve engine, where the port areas, etc., are more conducive to charge

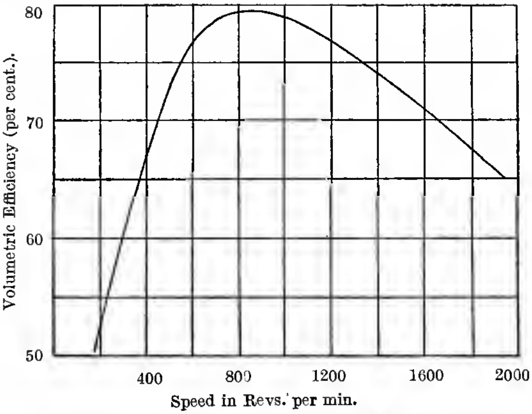

Fig. 69.—Volumetric Efficiency (Moline-Knight Engine).

efficiency, fairly high volumetric efficiencies are usually obtained.

Generally speaking, it can be taken that at 1,000 revolutions per minute a charge efficiency of from 80 to 85 per cent. is obtained in modern car practice, and this figure falls regularly with increased speed, on account of increased frictional resistance and the other effects previously mentioned.

In slow-running engines, such as gas engines, volumetric efficiencies of 90 per cent. are quite common.

Two-Stroke Engines.

One of the defects of the ordinary two- or three-port type of two-cycle engine is the low volumetric efficiencies obtained with the crankcase compression method.

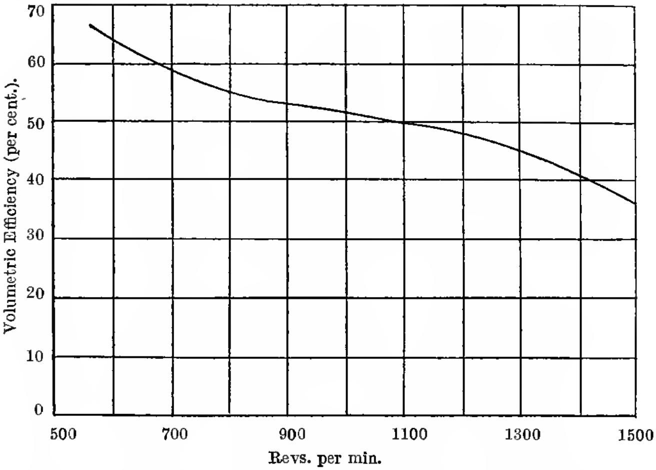

FIG. 70.—VOLUMETRIC EFFICIENCY OF TWO-CYCLE ENGINE.

The results of tests made by Dr. Watson * upon a two-port, two-stroke engine are shown graphically in Fig. 70.

It will be observed that the volumetric efficiency falls from about 64 per cent. at 600 revolutions to 47 per cent. at 1,500 revolutions, the maximum speed.

Obviously these low efficiencies are quite detrimental to high powers for a given cylinder capacity, and economical running, and the development of this type of engine is more a matter of design and experiment, in the attainment of higher volumetric efficiencies.

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* "The Thermal Efficiency of a Two-Cycle Engine," *Proc. I.A.E.*, 1908–9.

Effect of Charge Density.

The effect of running an engine partially throttled is somewhat analogous to that of an engine showing poor volumetric efficiency at high speeds.

The maximum power which an ordinary explosion engine can develop is determined by the quantity of oxygen which can enter into combustion in a given time, and this in its turn depends upon the maximum amount of charge which can be drawn in.

Obviously if one could fill the whole of the clearance and working volumes of the cylinder with pure mixture under the most suitable cylinder wall temperature conditions, and the compression were suitable, the greatest explosion pressure would occur.

The effect of poor volumetric efficiency at higher speeds is to lower the compression pressure, as a relatively smaller amount of charge remains at the beginning of the compression stroke, and the effective pressure upon the piston is therefore less.

This compression effect can be represented on the indicator diagrams, as shown in Fig. 71.

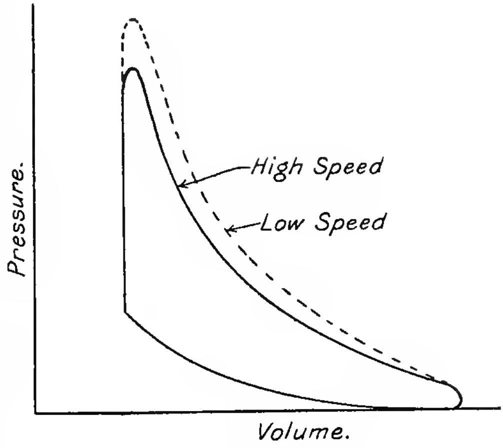

FIG. 71.

The full line diagram shows the pressures obtained at high engine speeds, and the dotted line diagram those at low engine speeds for the same engine, the difference in area between the two diagrams, representing the loss in effective pressure by wire-drawing, etc., at the higher speeds.

It must be remembered, however, that when an engine is designed under low volumetric efficiency conditions, either by accident or intent, any efforts that are made (either by enlargement of carburettor and port passage, valve lift and valve timing, etc. to increase the volumetric efficiency, and which result in still higher speeds being attained and higher explosion pressures, may not be beneficial to the strength of the working parts, which in certain cases are designed only to resist the lower pressures and temperatures.

It is common practice in high speed engines to adjust the valve area and valve timing, disposition of the valves, etc., so that the volumetric efficiency is high at the working speeds.

Mechanical Losses in the Engine.

Under this heading will be considered that part of the engine losses which represents the difference between the true indicated horse power and the brake, or effective horse power at the flywheel or clutch.

The mechanical efficiency, being the ratio of the latter to the former horse power, it follows that the smaller the mechanical losses are, the higher will this ratio be—a condition which should be aimed at by every designer of engines.

These mechanical losses, as we shall for convenience term them, may be divided into two parts :—

(*a*) Frictional losses.

(*b*) Pumping losses.

The total mechanical losses being the sum of these two respective losses, it is of interest to know which is the more serious in an engine.

The actual relation between these separate factors, in any engine, will depend largely upon the design and speed of the engine.

Thus in an engine with heavy reciprocating parts, badly machined rubbing surfaces, inadequate lubrication arrangements, etc., the frictional losses will constitute practically the whole of the net mechanical losses, whilst the pumping loss may relatively be negligible.

Upon the other hand, an engine with restricted port areas, badly designed carburettor, unsuitable inlet and exhaust valve timing, etc., will have relatively large "pumping losses," although the mechanical working parts may be perfect.

To give an idea of the average values of these two losses, the values obtained from a Talbot engine may be quoted, in which the frictional losses were about 75 per cent. of the whole, and the pumping losses about 25 per cent.

In the Daimler engine tested by Hopkinson, the former losses amounted to 82 per cent. and the latter to 18 per cent.

The Talbot engine mentioned, is at the time of writing, about four years old, and there is no doubt that in most modern car engines in which the design of port areas and valve timing is much improved the pumping losses will not amount to more than from 10 to 15 per cent. of the whole mechanical losses.

Dealing with these two losses in order, the frictional losses which occur within the engine are, of course, due to the frictional resistance offered to motion by the various working parts ; thus there exists frictional resistance (partly solid and partly fluid) between the piston, with its rings, and the cylinder wall, the amount of which is dependent upon the connecting-rod thrust, cylinder pressure, and state of the interposing surfaces. At each of the connecting-rod and crankshaft bearings, also, there will be frictional losses (usually of a fluid nature where adequate lubrication is maintained).

Minor losses which come under heading (*a*) include "air" frictional and churning losses in the crank chamber due to the skin friction of the rapidly moving parts, valve gear and auxiliary losses, vibrational, and out-of-balance effects, etc.

Power absorbed in driving the auxiliary mechanisms such as the magneto, circulating pump, air pump, etc., should be included under the mechanical frictional losses of the engine as a whole.

These frictional losses will of course depend somewhat upon the "design" of any particular engine ; the term "design" is here meant to include the weight, material, general design and lubrication arrangements, which are more or less inherent features of the engine.

The frictional losses in any given engine depend upon two other factors, namely, the speed and the temperature of the circulating water.

The effect of increased speed upon the frictional losses is to cause an increase in these losses, and the nature of the variation of these losses with speed can best be understood by reference to the curves shown in Figs. 72 and 73 which illustrate the results of measurements of mechanical losses.

Fig. 72 represents the losses at different speeds in the case of a four-cylinder poppet valve engine, and Fig. 73 the same losses in the case of a single-cylinder sleeve valve engine.

In the former case it will be noticed that at 600 revolutions about 1·6 H.P. is required to overcome the frictional losses, whilst at 1,200 revolutions about 3·9 H.P. is necessary, so that these losses do not vary directly as the speed, but as some higher power of the speed ; this appears to indicate that the resistance is more in the nature of a fluid resistance.

In the case of the single-cylinder sleeve valve engine, these losses increase from about 0·5 H.P. at 600 revolutions to 2·25 H.P. at 1,200 revolutions, but in this case it should be pointed out that the cylinder temperatures (and that of the lubricant) are appreciably higher at the higher speed.

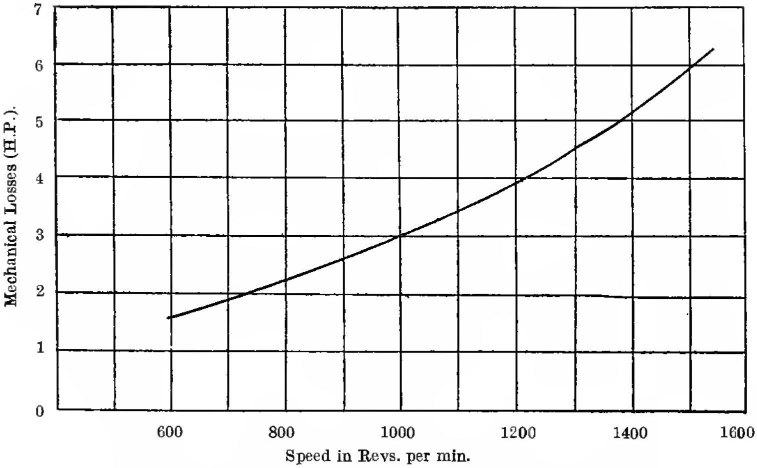

Fig. 72.—Losses in a 30 H.P. Four-Cylr. Car Engine.

It should be mentioned that the losses illustrated in the figures include the pumping losses, which do not however vary very much with the speed, so that the nature of the variation of the frictional losses with the speed is not sensibly unaltered, and therefore follows a fluid resistance law of the type

$$R = fV^{1\cdot5 \text{ to } 2}$$

where R = the frictional resistance
f = a constant
V = the speed of engine.

This law is, however, only approximate, for the variations in the cylinder temperatures, and explosion pressures at different speeds, have also a direct influence upon these losses.

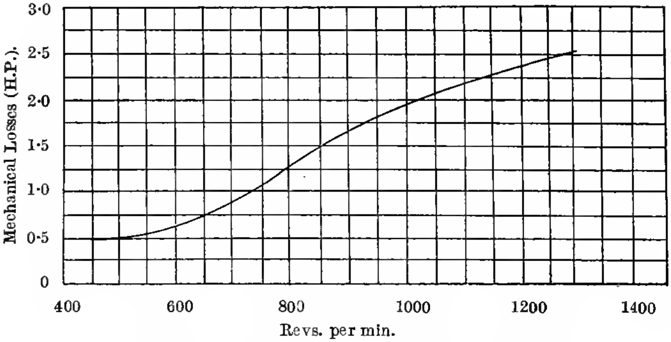

Fig. 73.—Losses in a Sleeve Valve Engine.

As would be expected in the case of the sleeve valve engine with its relatively greater surfaces of sliding, and resistance, the effect of speed is more marked, as the curve indicates.

Generally speaking, the frictional losses increase at a greater rate than the speed ; it will perhaps be interesting to inquire a little further into this variation.

The total frictional losses in any engine, temperature and lubrication remaining constant, depend upon

(*a*) Frictional losses due to the static pressure of the exploding gases (and compressing),

(*b*) Frictional losses due to rotating parts.

Losses (*a*) will remain constant with speed increase unless, as more often than not happens, the mean effective pressure drops with speed ; these losses will then be slightly lower.

Losses (*b*) depend upon the pressures between the rotating parts, and in the case of the piston, connecting rod and main bearings, the forces due to the inertia of the reciprocating parts, and unbalanced masses increase in proportion to the square of the velocity (or engine speed), and the losses by friction will go up proportionally.

Apart from the engine itself, the power required to drive the water, oil and air pumps will be found to increase at a greater rate than with the speed.

Analysis of Losses.

As the frictional loss includes that portion due to the piston, connecting rod and main bearings, etc., it may be of interest to consider how these various parts absorb power.

Unfortunately the available data in connexion with this subject is very limited, but some tests by Hopkinson tend to throw some light upon these points.

He determined the power required to drive an engine around by employing an electric motor to drive the same. The circulating water was maintained at its normal working temperature, and the valve caps were removed so that no appreciable work was done in drawing in and expelling the air.

He found that it took 4·0 H.P. to drive the engine around under these circumstances at 180 revolutions per minute, whilst with the piston and connecting rod removed it required 1·4 H.P. at the same speed.

From these figures it will be evident that about 2·6 H.P. or 65 per cent. of the frictional losses is due to piston and crank-pin friction, and that about 1·4 H.P. or 35 per cent. is due to the main bearings, valve gear and minor sources.

The author has recently made some measurements of the power absorbed in driving the various auxiliaries of a car engine.

The engine in question was driven around electrically, the losses in the motor having been accurately determined for various speeds and current values, and the power required to drive the engine with the valves and caps removed measured.

The magneto was then disconnected, and the engine motored around; the power necessary to drive the magneto being thus obtained by the method of differences. The gearing, air and water pumps were next disconnected in turn, the power required to drive the remaining parts being measured, respectively, in each case.

The temperature of the circulating water and lubrication were maintained constant throughout, at about the working temperatures.

The following table shows the results of such measurements, and serves as an indication of the nature of the losses incurred.

TABLE XXIX

Part of Engine.	Speed.	H.P. to Drive.
Whole engine with caps removed	1,000 revs. per minute	8·8
Magneto alone	,,	0·25
Circulating pump	,,	0·25
Pump and magneto skew gearing	,,	0·40
Air pump	,,	0·30

It will be seen that the H.P. absorbed by the auxiliary parts represents about 14 per cent. of the total H.P. absorbed in the mechanical losses.

Effect of Circulating Water Temperature.

Part of the frictional losses vary with the cylinder wall temperature, as previously stated; this variation being due to the reduced viscosity of the lubricant between the rubbing surfaces at the higher temperatures.

Thus in some experiments made upon a Daimler engine, Hopkinson found that with the circulating water (in the jacket) at a mean temperature of 65° F., the frictional losses were 4·0 H.P.; whilst at 150° F. they were 2·6 H.P., and finally at 212° F. with the water just boiling they were only 2·0 H.P., the speed in each case being 800 revolutions per minute. The total output of the engine at 800 revolutions was about 16 H.P.

In the case of a sleeve valve engine tested by Dr. Watson * it was found that the B.H.P. increased rapidly as the inlet temperature of the circulating water rose from 70° F., at which the B.H.P. was 4·6, to 180° F., where it became 5·95; the I.H.P. remaining constant over this range of temperature at about 8·1.

These examples will serve to show that the temperature of the lubricant has an important influence upon the frictional resistances; it is the common experience of motorists that their engines pull much better when thoroughly warmed up than at starting.

One or two other interesting points brought out by Hopkinson's experiments are given in the accompanying table, which embodies the results of tests made with normal lubrication, excess of lubrication, and with water injection.

The engine, as before, was motored around at 180 revolutions with the valve caps removed, so that air losses were minimized, and the temperature of the circulating water was varied as indicated.

TABLE XXX

Conditions.		Temperature of Water.	H.P. Absorbed.
Engine hot.	Normal lubrication . .	180° F.	4·0
Engine cold.	Normal lubrication . . .	70° F.	6·5
,, ,,	Excessive lubrication . .	70° F.	4·7
,, ,,	Water injected . .	70° F.	2·7

These results serve to bring out the effects of circulating water temperature and of lubrication upon horse power. The marked drop in the power absorbed when water is injected in the cylinder at the lower temperature is interesting, and is no doubt due to the very much lower viscosity of the water and to its effect upon the fluid losses.

In some experiments of Morse (one of Hopkinson's students) upon a Siddeley engine, careful tests were made at 720 revolutions per minute to find out whether the amount of oil had any effect upon the mechanical losses or the I.H.P., and it was found that whether the exhaust was smoky or quite clear there was no appreciable effect at this speed.

However, at 1,220 revolutions it was found that the quantity of oil employed appeared to have a distinct effect upon the mechanical (friction) losses, and it was therefore necessary to maintain an ample supply of lubricant.

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* "Test of a Daimler Sleeve Valve Engine," *Proc. I.A.E.*, 1912.

The Pumping Losses.

By the pumping losses in any engine is meant the work which is done (at the expense of the indicated horse power) in the induction or drawing in of the fresh charge and in the expelling of the exhaust products.

Generally, the pressure within the cylinder during the suction stroke is below atmospheric, whilst the pressure of the exhaust products is above atmospheric.

The amount of "negative work" which has to be performed in the exhaust and induction operations is a measure of the pumping losses, and is represented by the area of the suction-exhaust loop on the indicator diagram, as shown shaded in Fig. 74.

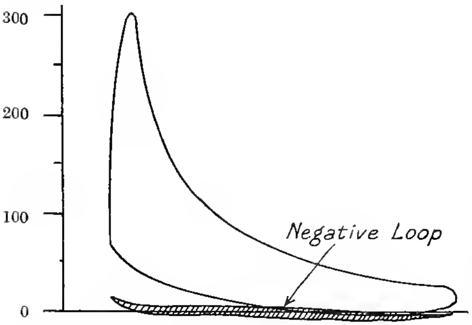

FIG. 74.—PUMPING LOSSES UPON THE INDICATOR DIAGRAM.

As was previously pointed out, these losses in a car engine amount to between 10 to 20 per cent. of the total losses between the indicated and brake horse power.

It has also been shown that the temperature existing within the cylinder has an influence upon the quantity of air and petrol drawn in per suction stroke, and here it might be added that the resistance to motion of heated air, or exhaust gases, is less than in the case of cool air.

Considering the influence of speed upon the pumping losses, these losses will increase with increased speed, owing to the greater wire-drawing or throttling of the gases flowing at higher speeds.

This also follows from a consideration of the mean pressures existing in the inlet and exhaust pipes near the valves.

Fig. 75 illustrates the results of measurements of the inlet manifold mean negative pressures at different speeds for a four-cylinder car engine. The measurements were made by means of a mercury manometer with suitable arrangements to damp out the periodical variations of pressure due to the intermittent suctions. The suction or negative pressure, as the figure indicates, increases fairly rapidly with speed.

In the case of the exhaust gases the "back pressure," as it is termed, also increases with speed, in a very similar manner.

It will be seen, therefore, that the net difference between the inlet and exhaust pressures increases very appreciably as the speed goes up, and hence that at the higher speeds, the " negative " loop, might reasonably be expected to be greater than at the lower speeds.

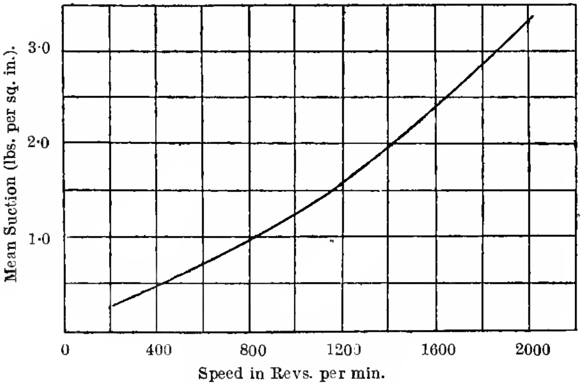

FIG. 75.

There is, as experiment shows, an increase in the magnitude of the pumping losses at higher speeds, but by no means so great as one would expect, probably on account of the lower volumetric efficiencies at the higher speeds, and also due to the higher temperatures existing.

The curve given in Fig. 76 illustrates the magnitude and variation of the pumping losses in the Daimler engine tested by Hopkinson ;* this figure also shows the frictional losses, and the total mechanical losses, at various speeds.

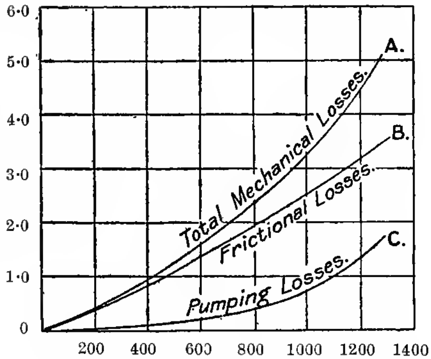

FIG. 76.—LOSSES IN 20 H.P. DAIMLER ENGINE (HOPKINSON).

The method of obtaining the magnitude of the losses under consideration is interesting, and will also serve as an example of the utility of the indicator diagram in these and similar investigations.

In finding the mechanical losses of power in an engine, as a means of obtaining or checking the brake horse-power measurements, one common method employed is to couple the engine up to an electric motor, which, by means of suitable connexions and switches, can be converted into a dynamo for absorbing the power output of the engine—that is, as a means for varying the external load upon the engine.

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* From *The Gas, Petrol and Oil Engine*, D. Clerk.

When the engine is started up, it is first motored around electrically, and when the proper speed is attained the ignition is switched on. Directly the engine fires, the motor connexions are cut out, leaving the engine temporarily without external load and giving it an opportunity to pick up in speed, until at the suitable moment the dynamo connexions are made, and the load gradually applied until the conditions are quite steady; the engine will now be firing regularly under load, with the circulating water at its proper temperature, as indicated by suitably placed thermometers.

The ignition is next switched off, and the dynamo and motor connexions exchanged by a suitable switch; the engine is then speeded up (but not firing) to the desired speed at which the determination of the losses is required by means of the motor.

Measurements of the current and voltage provide the necessary data required for obtaining the power taken to drive the engine at the given speed (the efficiency of the motor being accurately known under all conditions of speed, and armature and field current strengths).

The power thus measured, as a little consideration will show, will be found to consist of :—

(1) The power necessary to rotate and overcome the frictional resistance of the moving parts.

(2) The power required to draw in and expel the air from the cylinder.

(3) The power necessary to compress and expand the air drawn in, upon the equivalent compression and firing strokes.

Item (1) gives a very near approximation to the frictional losses actually existing within the engine when working normally at the given speed, as the temperature of the jacket water is the same in each case.

This method, however, is open to criticism, as the pressures on the bearings and cylinder walls will be different, and reversed in most instances; and further, instead of *hot* exploding and exhaust gases being within the cylinder, cool air takes their place, and this may have an appreciable influence upon the viscosity and constitution of the lubricant, etc.

Measurements of losses made by this method have been found by Hopkinson, Watson and others to give results approximating closely to those obtained by direct measurements of the I.H.P. and B.H.P., so that the above objections cannot be very serious.

Item (2) is very nearly the same for the engine firing, as the mean temperature of the metal in the ports and walls will not have had time to cool down appreciably.

Item (3) is the factor which does not occur in the actual engine, and so a correction must be applied to the power measurements thus obtained.

It might at first be thought that when the air is compressed and expanded no net work would be done upon the air, but actually, the air loses heat to the cylinder walls during compression, and does not regain all of this during expansion. External work is therefore done upon the air as a net result.

In order to estimate the magnitude of this "loss," indicator diagrams have been taken with the engine not firing, during the period of motoring around directly after switching off the ignition in the above-mentioned method of measuring the mechanical losses.

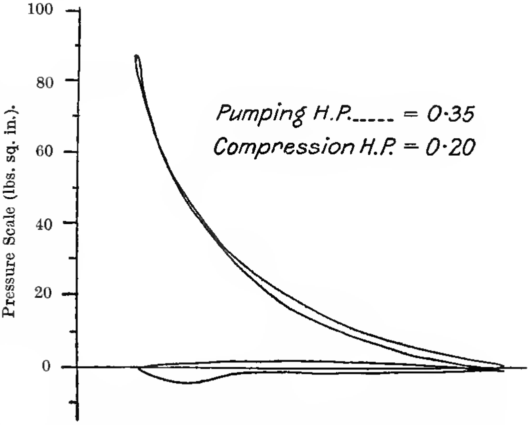

FIG. 77.—INDICATOR DIAGRAM TAKEN AT 600 REVS. PER MIN. FOR PUMPING LOSSES.

Examples of indicator diagrams obtained by the author are shown in Figs. 77 and 78 for engine speeds of 600 and 1,100 revolutions per minute respectively.

It will be observed that the expansion line is below the compression line; the area of the "loop" thus made represents the amount of work done upon the air in the cylinder.

At 600 revolutions, in the four-cylinder engine tested, the power to be debited from the measured power to motor engine around at this speed is ·20 H.P., or about one-fifth of a horse power as measured from the diagrams taken at this speed.

At 1,100, the power under the same conditions is ·47 H.P.

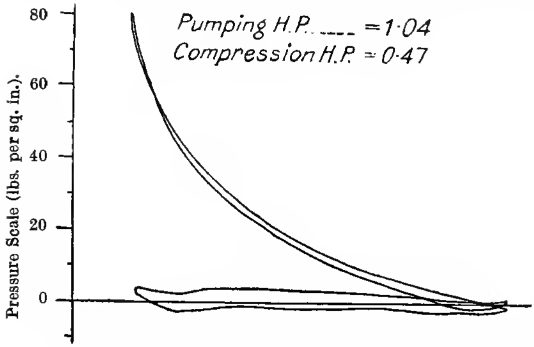

Fig. 78.—Indicator Diagram taken at 1100 revs. per min. for Pumping Losses.

These corrections can be applied to the total power measured, and the difference between the two will give the mechanical losses in the engine at the given speeds.

From the examples of indicator diagrams given, the areas of the negative or "suction" loops can be measured off; these give an idea of the magnitude of the pumping losses.

In the cases under consideration, at 600 revolutions per minute, the pumping losses amounted to ·35 H.P., and at 1,100 revolutions to 1·04 H.P.

From measurements of similar diagrams taken over the indicated range of speed, and with the engine hot in each case, the curve given in Fig. 79 has been obtained; the throttle was set full open throughout the tests.

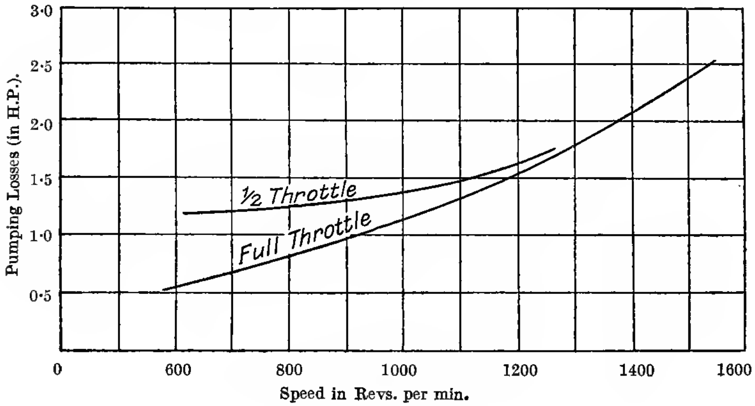

Fig. 79.

The upper curve gives the pumping losses for the engine half-throttled, that is, with the throttle set so that half the maximum amount of mixture was used; these will be seen to be somewhat higher for the range of measurements made.

Light Spring Diagrams.

The pumping losses can be accurately measured and studied by taking indicator diagrams, using a weak spring in the indicator instrument, so that the pressure scale is much exaggerated.

Generally a stop is provided in the indicator to prevent the effects of the explosion pressure deflecting the spring further than a small predetermined amount.

An example of such a weak spring diagram from a four-cylinder engine is given in Fig. 80.

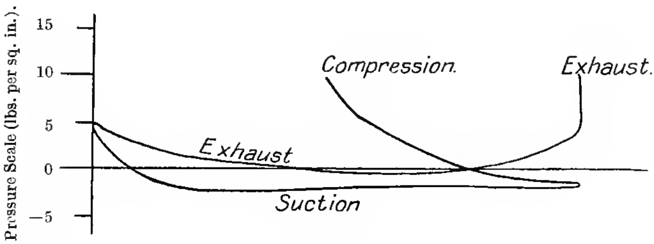

FIG. 80.—MAGNIFIED EXHAUST AND SUCTION STROKES.

It will be noticed that the exhaust line, generally speaking, lies above the atmospheric line, whilst the suction line is below.

The rise in pressure at the end of the exhaust stroke is interesting and suggests a throttling of the exhaust gases, thus causing a back pressure of about 5 lbs. per sq. inch to exist at the moment just before the inlet opens on top centre.

Incidentally, it might be mentioned that light spring diagrams can be taken of pressure variations in the exhaust pipe, inlet pipe, crank chamber of a two-cycle engine, etc., and afford a valuable means for studying the exact conditions existing in these cases.

Mechanical Efficiency.

Although the earlier consideration of the engine losses is complete in itself, yet it is often convenient to compare the performances of different engines, not by their total mechanical losses, but by the relation of the B.H.P. to the I.H.P. as measured directly.

If the mechanical losses are known at different speeds, and the I.H.P. can also be determined, we have then a basis of comparison of engine performances.

The ratio of the B.H.P. to I.H.P. of any engine is termed the mechanical efficiency.

This may be written as :—

$$\text{Mechanical Efficiency} = \frac{\text{I.H.P.} - \text{mechanical losses}}{\text{I.H.P.}}$$

If the mechanical losses were constant for all speeds the mechanical efficiency would also be constant, but it has been seen that the former increase with speed, so that, from the above relation, the mechanical efficient should decrease with the speed.

In good gas engines the mechanical efficiency varies from 85 to 90 per cent., and in ordinary touring-car engines of good design the value of this ratio may be as high as 90 per cent. at low speeds, falling off to about 80 per cent. at maximum speeds.

For sleeve valve engines the mechanical efficiency varies from 90 per cent. at low speeds to about 75 per cent. at high.

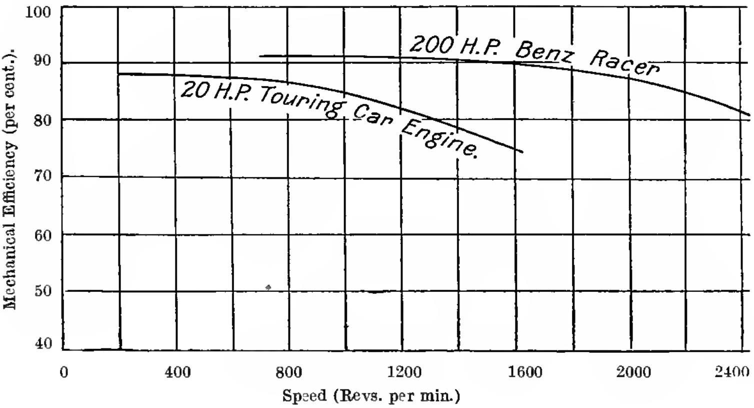

FIG. 81.

In two-stroke engines of the two- or three-port type the mechanical efficiency falls from about 83 per cent. down to as low a value as 70 per cent. and less at the maximum speeds.

Much of course depends upon the actual design of the engine, and it is the aim of the designer to obtain as high a value as possible, at all speeds, for the mechanical efficiency.

The success of many racing type engines is largely due to the efforts which have been made in the design and construction, to minimize the mechanical losses, and so obtain as high a value for the mechanical efficiency as possible at high speeds.

The curves shown in Fig. 81, for the cases of touring- and racing-car engines, illustrate the nature of the variation of the mechanical efficiency with the speed.

The falling off in the efficiency as the engine speed increases is additional evidence in support of the predicted and observed increment in the mechanical losses with the speed, and the deductions made therefrom. From theoretical considerations, based, however, upon the results of accurate tests, Callendar has shown that the mechanical efficiency of an engine is a function of the cylinder diameter in similar engines working under similar conditions, and that the mechanical efficiency increases with the diameter of the cylinder according to the relation :—

$$\text{Mech. Effic.} = k\left(1 - \frac{0{\cdot}30}{\text{D}}\right) \text{ per cent.}$$

where k is a constant, the value of which for a petrol engine varies from 0·87 to 0·93, and D is the cylinder diameter in inches.

Overall Efficiency.

Before proceeding with the consideration of this subject, it will be necessary to recapitulate a little, and to briefly distinguish between the various kinds of "efficiency" dealt with in internal combustion engine work, as a great deal of confusion has arisen in the past due to the improper use of these terms ; one often hears of one efficiency being compared with another efficiency of an entirely different character.

It has been seen that when one wishes to compare the thermodynamic performances of different engines of the same type, or of a given engine under varying conditions, the "thermal efficiency" is used as a basis for comparison, and further furnishes information in respect to the fuel consumption reckoned on the actual indicated horse power.

In order to compare the performances of engines differing widely in size and type, working under different conditions of speed and with various fuels, etc., the ratio of the thermal efficiency to the ideal standard efficiency, and which is called the "relative efficiency," provides a valuable basis for the thermodynamic comparisons to be made.

Coming next to the more practical aspect of the subject of internal combustion engines, it is of course necessary to have as high a thermal efficiency as possible, for fuel economy, but it is also necessary for the "mechanical efficiency" to be as high as possible for real practical economy, for it will be evident that although an engine may develop a high indicated horse power, yet if an appreciable percentage of this power is used up in overcoming the mechanical losses, then the net power (or the brake horse power) may be small compared with the indicated, and fuel economy may be very much less marked.

Hence in comparing the practical performances of engines, the thermal efficiency reckoned upon the brake horse power is often employed, and is termed the "brake efficiency."

As this term includes the mechanical efficiency it gives a useful comparison for different engines under similar conditions of running.

It will be evident from what has been mentioned concerning the mechanical losses that since these losses usually increase at a greater rate than the speed, the mechanical efficiency falls off as the speed increases, and hence the "brake efficiency" will fall off at a greater rate than the "thermal efficiency" reckoned upon the indicated horse power.

Thus if (as is sometimes the case) the thermal efficiency remains nearly constant over an appreciable range of speed, then the brake efficiency will decrease with speed.

There is another method of comparing the performances employed by designers of racing and high speed engines of different types, which is irrespective of fuel consumption, and depends upon the brake horse power developed for a given cylinder capacity.

This is here termed the "overall efficiency," and is the ratio of the brake horse power developed (measured at the flywheel) to the capacity of the engine cylinder swept out by the piston during each stroke, or to the sum of the working volumes of the cylinders, usually expressed in litres (1 litre equalling 1000 cu. cms., or 61 cu. ins.).

The overall efficiency will be more or less independent of the fuel consumption, and will depend upon how much useful power can be obtained from a given size of engine, by various mechanical devices or means.

It is well known that the most thermally efficient mixture strength is a weak mixture consisting of about seventeen parts of air to one part petrol by weight, whereas the mixture which gives the greatest mean effective pressure and flame velocity is a rich mixture of about twelve, so that from this point of view thermal and overall efficiency are opposed.

The question of the thermal efficiency of overall efficient engines is a complicated one, and depends much upon the individual design.

The tendency at present is towards a high speed engine whereby overall efficiency is obtained by increased engine revolutions.

As the power obtainable from a given engine varies directly as the mean pressure and the revolutions per minute, it is obvious that if the revolutions can increase at a greater rate than the pressure decreases as the speed is increased, then the product of the two will increase with the speed, and therefore the horse power will also increase with the speed.

In well designed high speed engines it is possible to maintain the mean pressure nearly constant up to speeds of 2,000 or 2,500 revolutions per minute, so that the net effective power on the brake is roughly proportional to the speed up to this limiting speed, when it begins to decrease with further speed increase above this limit.

The employment of high compressions and well designed combustion chambers may have a good influence upon the thermal efficiency of the high overall efficient engine, but it is generally known that these engines are not so thermally efficient as the moderate speed engine running upon suitable mixtures and maintaining suitable cylinder wall temperatures.

Effect of Throttling upon Efficiency.

In further support of this might be mentioned the results of some tests of thermal efficiency made upon a four-cylinder car engine with various throttle positions. It was found that the thermal efficiency at full throttle and full speed was somewhat lower than that at three-quarter throttle position ; and further, when the engine was running at its best, in the half-throttle position the thermal efficiency was practically the same as at three-quarter throttle.

Below this, diminishing the throttle opening caused the thermal efficiency to diminish almost directly as the speed.

As the mixture strengths and temperature of the cooling water were kept constant in all cases, and the ignition adjusted to give the best power output, it appears as though the design of the large engine working normally upon part throttle would be more economical in fuel than the extra-high speed engine.

This view is further supported by the results of some tests upon a sleeve valve engine * in which it was found that the thermal efficiency upon the brake horse power, for a mixture strength of about 15 to 1, on full throttle and for speeds between 900 and 1,200 revolutions per minute, was 18·2 per cent. For the same mixture conditions, and when throttled down to three-quarter power, the value of the thermal efficiency was 18·3 per cent., and so did not fall off.

The explanation of these results may be similar to that advanced for the maximum thermal efficiency occurring at poor mixture strengths.

Apart from the question of thermal efficiency, it is often desirable from considerations of engine rating, power for weight, and

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* "Test of a Sleeve Valve Engine," *Proc. Inst. Aut. Engineers,* 1912.

racing purposes, to be able to obtain the maximum power output on the brake from a given size of cylinder.

H.P. Data for Car Engines.

In order to give a quantitative idea of the possible power obtainable from a given cylinder capacity, it may be stated that for the average slow speed design of motor-car engines about 6 brake horse power is obtainable for every 1,000 cubic centimetres, or per litre, working volume of the cylinder at about 1,000 revolutions per minute.

Under this heading would be included the earlier designs of engine, the cheaper American car engines, etc.

For good designs of engine, such as generally found in modern moderate speed touring-car engines, about 7 to 8 horse power per litre at 1,000 to 1,300 revolutions are obtained.

High speed racing engines of the high overall efficiency type will yield between 8 and 9 horse power per litre at speeds of about 1,000 revolutions, and from 16 to 18 at 2,000 revolutions.

If higher speeds are designed for, it is even possible to get as much as 25 to 35 H.P. per litre at the maximum speeds.

Thus in the case of the Talbot engine fitted to the car first maintaining a speed of 100 miles for one hour, the power curve showed that a maximum overall efficiency of 27 H.P. per litre was obtained at a maximum speed of 3,200 revolutions per minute, as tested upon a water dynamometer.

The ordinary touring type of Talbot engine of the same size, namely, 101·5 mm. bore by 140 mm. stroke, gives a maximum overall efficiency of 15·5 at a speed of 2,300, and of 9 at a speed of 1,200 revolutions.

In the case of one of the Sunbeam Graded Prix racers of 1913, about 28 H.P. per litre at 3,000 revolutions was obtained in bench tests, with 9 H.P. per litre at 1,000 revolutions.

Coming next to the case of sleeve valve engines, the Daimler Knight touring-car engine has been shown by tests to give about 9 to 10 H.P. per litre at 1,200 revolutions, and a maximum of from 14 to 16 at 2,000 revolutions.

For air-cooled types of engine of the motor-cycle type the results of numerous bench tests show that for speeds of 1,000 revolutions about 8 H.P. per litre, and at 2,000 revolutions 15 H.P. per litre.

For extreme types of racing air-cooled engine it is possible to obtain as much as 22 to 26 H.P. per litre for appreciable periods.

Before concluding this section of the subject, it might be of some interest to mention the general principles whereby maximum overall efficiency is attained.

Methods of Obtaining High Overall Efficiencies.

Although, strictly speaking, this subject should be treated in the design portion, it will be useful to briefly enumerate here the chief factors concerned from the theoretical standpoint.

The essentials for maximum volumetric output are :—

(1) Suitable mixture strength for obtaining the maximum effective pressures.
(2) Rapid ignition, adequate in intensity.
(3) High volumetric efficiency, by suitable valve and port, carburettor, areas and design.
(4) High compression pressures.
(5) Proper inlet and exhaust valve timing, and suitable operating means for working the valves.
(6) Lightness of all reciprocating and working parts.
(7) Proper balance of all rotating and reciprocating parts.
(8) Good lubrication.
(9) Adequate jacket water cooling.
(10) Minimum of frictional and pumping losses.
(11) Suitable quality of materials employed for all working parts.

Apart from the above methods generally employed, it is further probably possible to obtain maximum power output by such means as scavenging, forced induction, separate compression (in order to obtain higher compression pressures without preignition, by distracting the heat of compression by suitable means), water injection, adoption of the two-cycle principle, etc.

Although these methods are put forward as possible means for obtaining higher overall efficiencies, there does not appear to be much information yet available as to the real benefits obtained, and very few authoritative tests have been published, in so far as petrol engines are concerned.

Water Injection.

In gas engine practice marked improvement has been shown to result from water injection into the cylinder, the water being injected as a fine spray in part of the air supply during induction, so as to avoid the formation of a film upon the cylinder walls.

It was shown in the case of a 50 H.P. engine that with water injection at the rate of about 7 lbs. per hour the thermal efficiency was 37 per cent., and the relative efficiency nearly 64 per cent. of the "air standard."

The important point about these figures is that they represented an appreciably higher value than any hitherto obtained without such means.

It is possible that the real benefit accrues from the internal cooling which is obtained at the expense of the latent heat of the water—the formation of steam bringing about this result. The steam pressure resulting is comparatively small, as the quantity of moisture injected is very small, so that the volume of steam would be comparatively large, and the pressure therefore low. It is also believed that reduced radiation results during combustion and expansion, due to the presence of the steam formed.

In Hopkinson's system of water injection,* the water jacket proper was done away with, and the whole of the cooling obtained by means of the arrangement illustrated in Fig. 82.

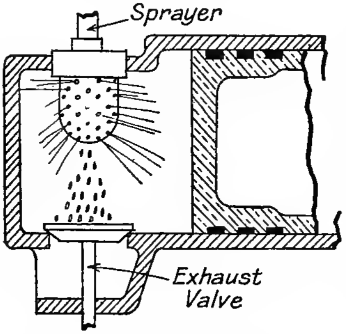

FIG. 82.—HOPKINSON'S METHOD OF COOLING BY WATER INJECTION.

Water is introduced during the process of combustion and expansion in the form of drops, in distinction to a spray, so that the greater portion reaches the combustion chamber walls before being converted into steam.

It was further found that the thermal efficiency of the engine was increased with rich mixtures, whereas with weak mixtures it was actually reduced.

The explanation of these results is that the cylinder temperatures are much higher with the richer mixtures, and the heat losses to the walls and exhaust are consequently also higher, whereas with the weaker mixtures the water injection would reduce the mean wall temperatures below that corresponding with the maximum efficiency.

It is probable, in the light of the results of Hopkinson's tests, that the limiting compression pressures might be raised by means of suitable water-injection, and that the thermal efficiency would be thereby increased.

Apart from any gain directly due to better cooling, it is obvious that local cooling of the gases and of the hottest parts of the combustion chamber and exhaust valve, etc., will be beneficial in tending towards better volumetric efficiency, and less preignition tendencies.

Super Compression Methods.

Another successful method utilized in gas engine practice is that by means of which higher compressions and higher volumetric efficiencies have been obtained without excessive cylinder temperatures resulting.

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* Vide *Internal Combustion Engineering*, Aug. 6, 1913, and June 10, 1914.

The high temperatures and the rapid rate at which surplus heat has to be carried off by the cylinder walls depreciate from the efficiency of an internal combustion engine.

If, therefore, the mean temperature of the cylinder can be reduced without reducing the mean pressure, benefit will result in several ways.

In one method employed by Dugald Clerk, he introduced at the end of the suction stroke a further charge of air (or cooled exhaust gases) to raise the suction " pressure " to about 8 lbs. per sq. inch above atmospheric.

In this way the charge weight was increased by about 40 per cent., and the compression pressure increased, without increasing the temperature of compression.

It was found that the maximum temperature of explosion was lowered, and that the mean pressure was increased by about 20 per cent.

It suggests itself, from these results, that by introducing a cool inert gas such as air or cooled exhaust gases, or by the injection of moisture before the compression stroke is completed, that a greater thermal and overall efficiency might be obtained in the case of petrol engines without the ill-effects of abnormal cylinder wall and piston temperatures.

It should further be added that high cylinder temperatures also lead to cylinder and piston metal distortion, and inadequate lubrication results, both of which detract seriously from the mechanical efficiency and power output of the engine.

One of the greatest problems met with in Diesel and high compression engine design is that of overcoming the effects of high cylinder temperatures, that is, to prevent serious metal distortions occurring which may either crack the walls of the combustion chamber or cause piston binding, and put severe stresses upon the parts.

Many elaborate systems of piston cooling have been devised and adopted in oil and gas engine practice.

In concluding this Chapter, reference should be made to the fact that the future development of high speed internal combustion engines may not necessarily be along the present lines of the reciprocating type of engine, but quite possibly towards the rotary or gas-turbine type working upon the principle of continuous communication of angular momentum resulting from a process of continuous combustion of the explosive mixture of gases, or gases and vapours.

CHAPTER IV

PRESSURES AND TEMPERATURES IN INTERNAL COMBUSTION ENGINES

For the purposes of calculation and of design it is of importance to know, and to be able to predict in certain cases, the pressures and temperatures existing at different moments in different parts of the engine system.

The difficulty of calculating the probable pressures existing at different parts of the cycle is great, and involves a knowledge of the variation in the specific heats of the gases, the nature and amount of the cooling losses, the volumetric efficiency, and other more or less indeterminate factors.

For the purposes of comparison of the performance of any engine, thermodynamically, with the ideal engines, it is necessary to be able to predict the theoretical indicator diagram, by an application of the principles enunciated in the earlier portion of this book.

It is proposed, therefore, to give an actual example of the method of calculating the ideal diagrams and to then deal with the results of experiments upon pressures and temperatures in petrol engines.

Method of Obtaining Ideal Pressures and Temperatures.

It is required to obtain the pressures and temperatures at the four points ABCD of the ideal diagram (Fig. 83), assuming the working substance to be air; this ideal cycle can be modified afterwards for the variations in the specific heats of the gases and for the real expansion curve law.

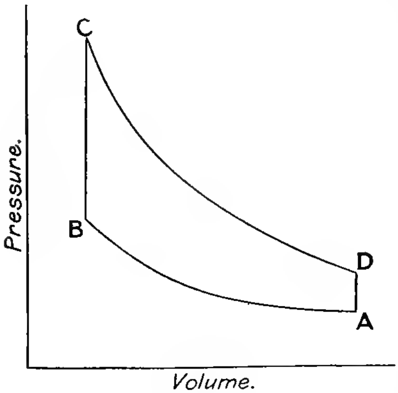

FIG. 83.

The ratio of the specific heats at constant pressure and volume for air, $\gamma = 1{\cdot}408$.

Taking the compression ratio as 4·7, and the initial temperature at A as 100° C. or 373° absolute, We have the pressure at B from the relation

$$P_bV_b^{\gamma} = 14{\cdot}7\ (4{\cdot}7)^{1{\cdot}4} = P_b.1, \text{ since } V_b = 1$$

Whence $P_b = 128$ lbs. per sq. inch absolute.

For the temperature at B we have

$$T_b = T_a r^{\gamma-1} = 373\ (4{\cdot}7)^{{\cdot}4} = 693^{\circ} \text{ absolute.}$$

If now we assume that heat is given to the compressed air at B until its temperature is, say, 2,000° C. absolute (in the actual case, we should require to know the specific heat of the mixture and the heat of combustion to determine this maximum temperature),

Then we have from $PV = C{\cdot}T$, since V is constant, that

$$P_c = \frac{T_c}{T_b}.P_b = \frac{2{,}000}{693}\ 128 = 361 \text{ lbs. per sq. inch.}$$

Also $P_d.(4{\cdot}7)^{1{\cdot}4} = P_c.1$, whence $P_d = 41{\cdot}3$ lbs. per sq. inch.

Also we have

$$T_c = T_d\ r^{\gamma-1} \text{ or } 2{,}000 = T_d.(4{\cdot}7)^{{\cdot}4}$$

Whence $T_d = 1{,}076^{\circ}$ C. absolute.

So that enumerating the pressures and temperatures calculated for the ideal air-standard cycle we have

$P_a = 14{\cdot}7$ lbs. per sq. inch $T_a = 373^{\circ}$ absolute or 100° C.
$P_b = 128$,, ,, $T_b = 693^{\circ}$,, 420° C.
$P_c = 361$,, ,, $T_c = 2{,}000^{\circ}$,, 1727° C.
$P_d = 41{\cdot}3$,, ,, $T_d = 1{,}076^{\circ}$,, 803° C.

Constant Specific Heat Conditions.

In order to calculate the pressures and temperatures for the case of an ideal engine employing a gas with specific heats which do not vary with temperature, and which obey an adiabatic law of compression and expansion, consider the case of an engine of working capacity ·025 cub. feet, and assume that the heat supply during combustion, along BC (in Fig. 83), be 1,150 ft. lbs. (in mechanical units).

Then as before we have $P_a = 14{\cdot}7$ $T_a = 373^{\circ}$ absolute
$P_b = 128$ $T_b = 693^{\circ}$ absolute

Now from Table V we see that the specific heat of a gaseous mixture possessing the same physical properties as air is 19·4 feet lbs. per cub. foot.

Now the heat supplied along BC = 1,150 feet lbs.

$$= C_v(T_c - T_b).\times{\cdot}025.$$

That is, $1{,}150 = 19{\cdot}4\ (T_c - 693)\ {\cdot}025$.

From which $T_c = 3{,}068^{\circ}$ absolute.

$$\text{Also}\quad P_c = \frac{T_c}{T_b}.P_b = \frac{3{,}068}{693}.128 = 577 \text{ lbs. per sq. inch}$$

Again, $P_d = \frac{T_c}{T_b}.P_a = \frac{3{,}068}{693}.14{\cdot}7 =$ 65·1 lbs. per sq. inch

And $T_d = \frac{P_d}{P_a}.T_a = \frac{65{\cdot}1}{14{\cdot}7}.373 = 1{,}650^\circ$ absolute

Enumerating these pressures and temperatures we have

$P_a = 14{\cdot}7$ lbs. per sq. inch $T_a =$ 373° absolute or 100° C.
$P_b =$ 128 ,, ,, $T_b =$ 693° ,, 420° C.
$P_c =$ 577 ,, ,, $T_c =$ 3,068° ,, 2,795° C.
$P_d =$ 65·1 ,, ,, $T_d =$ 1,650° ,, 1,377° C.

It is further of interest to note that by finding the heat rejected into the exhaust, we can directly obtain the efficiency of the cycle.

Thus the heat rejected, $H_E = C_v^1 (T_d - T_a) \times {\cdot}025$ feet lbs.
$= 19{\cdot}4\ (1{,}650 - 373) \times {\cdot}025$ ft. lbs.
$= 619$ feet lbs.

Then the efficiency $= \frac{H - H_E}{H} = \frac{1{,}150 - 619}{1{,}150} = {\cdot}462$, which is of course the same as when calculated from the relation $E = 1 - \left(\frac{1}{r}\right)^{\gamma - 1} = 1 - \left(\frac{1}{4{\cdot}7}\right)^{{\cdot}408}$.

Variable Specific Heat Conditions.

Consider next the ideal cycle, but with the specific heats varying according to the experimentally determined law of variation, treated in detail in Chapter I.

The expansion curve will not follow the adiabatic law, but a somewhat similar law $pv^n =$ constant, where the index n has to be determined.

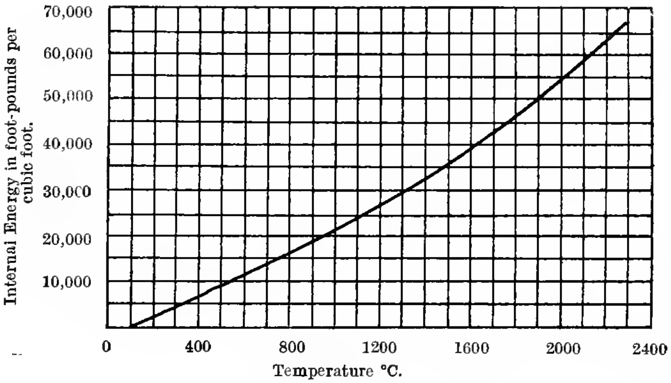

FIG. 84.

Hopkinson,* as the result of many experiments, has obtained the internal energy curve shown in Fig. 84 for certain gaseous mixtures similar in constitution to those employed in actual engines ; other curves and tables showing the internal energy variation with temperature for different gases and mixtures will be found in Chapter I. These curves illustrate a convenient and general method of expressing the specific heat variation with temperature. Table XXXI is also given for petrol-air mixtures.

TABLE XXXI

Energy in Foot-pounds per Cubic Foot

Temperature, Centigrade.	Ratio $\frac{\text{Air}}{\text{Petrol}}=18$	Ratio $\frac{\text{Air}}{\text{Petrol}}=14$	Ratio $\frac{\text{Air}}{\text{Petrol}}=10$
200°	$2{\cdot}2\times10^5$	$2{\cdot}2\times10^5$	$2{\cdot}2\times10^5$
400°	6·3 ,,	6·2 ,,	6·1 ,,
600°	10·9 ,,	10·7 ,,	10·5 ,,
800°	15·7 ,,	15·4 ,,	15·2 ,,
1,000°	21·0 ,,	20·5 ,,	20·3 ,,
1,200°	26·9 ,,	26·1 ,,	25·7 ,,
1,400°	33·3 ,,	32·2 ,,	31·7 ,,
1,600°	40·2 ,,	38·6 ,,	38·3 ,,
1,800°	47·2 ,,	45·7 ,,	45·3 ,,
2,000°	55·5 ,,	53·3 ,,	52·7 ,,
2,200°	63·7 ,,	61·2 ,,	60·7 ,,
2,400°	72·7 ,,	69·7 ,,	69·2 ,,

As explained in Chapter I, if the internal energy of the mixture is known, its corresponding temperature can be read off the curve expressing the results of experiments. In the proceeding calculations the values given in Fig. 84 are employed.

In the cycle of operations under consideration, the initial temperature and pressure are $P_a = 14{\cdot}7$ and $T_a = 373°$ absolute respectively, and the compression is assumed to be adiabatic, so that $P_b = 128$ lbs. per sq. inch and $T_b = 693°$ absolute.

If the compression be not adiabatic, but follow the law $PV^n =$ constant, the pressures and temperatures can be estimated in a similar manner.

The value of n for petrol engines lies between 1·2 and 1·4, and is usually about 1·35 for the compression line.

The pressure at the end of the compression stroke can be estimated from the relation $P_aV_a^n = P_bV_b^n$.

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* "On the Limits of Thermal Efficiency in Internal Combustion Engines" (Hopkinson), *Proc. Inst. Civil Engineers,* vol. clxix.

The temperature at the end of compression stroke is deducible from the relation $T_b = \frac{T_a . r^n}{V_a^n}$

Considering the work done by the piston upon the gas we have

$$\begin{aligned} \text{Heat energy added along AB (Fig. 83)} &= \cdot 025\ (T_b - T_a).C_v. \\ &= \cdot 025 \times 320 \times 19\cdot 4 \\ &= 154 \text{ foot lbs.} \\ \text{Also heat added along BC} &= 1{,}150 \\ \text{So that internal energy at C} &= 1{,}150 + 154 \text{ foot lbs.} \\ &= 1{,}304 \text{ foot lbs.} \end{aligned}$$

Knowing the internal energy at C, we can refer to the curve shown in Fig. 84, and read off the corresponding temperature which is the abscissa of the point $\frac{1{,}304}{\cdot 025}$ (foot lbs. per cub. foot), that is, 52,150 foot lbs. per cub. foot.

This temperature will be found to be 1,960° C. or 2,233° absolute as the temperature at C.

From the relation $\frac{P_c}{P_b} = \frac{T_c}{T_b}$ we have $P_c = \frac{128 \times 2{,}233}{693}$, that is $P_c = 418$ lbs. per sq. inch.

From C to D the specific heat is constantly varying, owing, of course, to the falling temperature during expansion. Assuming the expansion curve to be of the form $pv^n =$ constant, it is a process of trial and error to find the value of the constant n.

Thus if $n = 1\cdot 23$ we have $T_d = \frac{T_c}{4\cdot 7^{\cdot 232}}$

$= \frac{2{,}233}{4\cdot 7^{\cdot 232}} = 1{,}563°$ absolute, or 1,290° C.

The internal energy corresponding to this temperature 1,290° C., from the curve, is 29,000 foot lbs.

So that the change of internal energy from C to D will be 52,150 – 29,000, or 23,150 foot lbs.

Now this should be the same as the external work done by the gas in expanding from C to D, that is

$$C_v\,(T_c - T_d) = 33\cdot 7 \times 670 = 22{,}530.$$

It will be seen that these two quantities do not quite correspond, so that it will be necessary to choose another value of n, such as $n = 1\cdot 22$, when a very near approximation will result between these two quantities.

Having found $n = 1{\cdot}22$, say: this corresponds with the equation $pv^{1{\cdot}22}$ of an adiabatic shown dotted in Fig. 85, and magnified.

It will be seen that this adiabatic lies below the true expansion curve during the first half of the expansion stroke, and above it during the latter half, so that the temperature estimated at D^1 is slightly greater than the real temperature D.

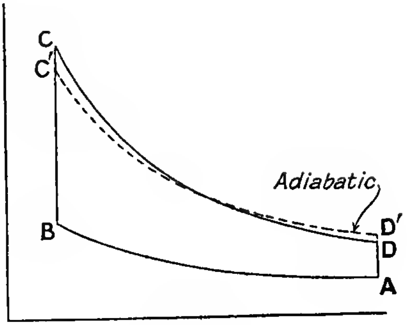

FIG. 85.

We have also $P_d = \frac{T_d}{T_c} \; \frac{P_c}{4{\cdot}7}$

And for $n = 1{\cdot}22$ $T_d = 1{,}575°$ absolute.

$$\text{so that } P_d = \frac{1{,}575}{2{,}233} \cdot \frac{418}{4{\cdot}7} = 63 \text{ lbs. per sq. inch.}$$

Hence for the case of a variable specific heat we have—

$P_a = 14{\cdot}7$ lbs. per sq. inch	$T_a = 373°$ absolute or	100° C.
$P_b = 128$,, ,,	$T_b = 693°$,,	420° C.
$P_c = 418$,, ,,	$T_c = 2{,}233°$,,	1,960° C.
$P_d = 63$,, ,,	$T_d = 1{,}575°$,,	1,302° C.

$$\text{Further, the efficiency} = \frac{\text{Available work}}{\text{Total heat added}}$$

Now the work done along AB on the gas = 154 foot-lbs.

And the work done along CD by the gas = 23,150 × ·025 = 579 foot-lbs.

So that the available work = 579 − 154 = 425 foot lbs.

$$\text{Hence the efficiency} = \frac{425}{1{,}150} = {\cdot}369$$

It will be seen that this value is much lower than that obtained upon the assumption of a constant specific heat.

It is this lower value of the efficiency of the ideal engine with which the real performance of any engine should be compared, as it represents the maximum possible efficiency that an actual engine can attain, and takes account of the properties of the working substance.

There are other methods for approximately calculating the pressures and temperatures within the cylinder of an internal combustion engine which take into account the cooling effect of the walls, and the increase in volume of the gases after combustion, which may amount to 5 per cent.

The problem of estimating temperatures and pressures from the indicator diagram, knowing approximately the properties of the gases, and the actions occurring within the cylinder, is complicated, as the actual state of affairs existing is not yet agreed upon by all authorities, and only imperfectly known, at least in degree.

A knowledge of the duration of the period of combustion, the nature and magnitude of the wall action throughout the cycle, the radiation factor, etc., is necessary before the true temperatures and pressures are calculable.

For an approximate method of estimating the quantities under consideration the reader will do well to consult *The Automobile Engineer*, from December,* 1911, onwards.

The assumptions there made are based upon the results of closed explosion vessel experiments, in which the resulting pressures are nearly always found to be about half the pressures calculated, assuming no wall actions, and an example of the pressures and temperatures obtained by the author of the above article is reproduced in Figs. 86 and 87.

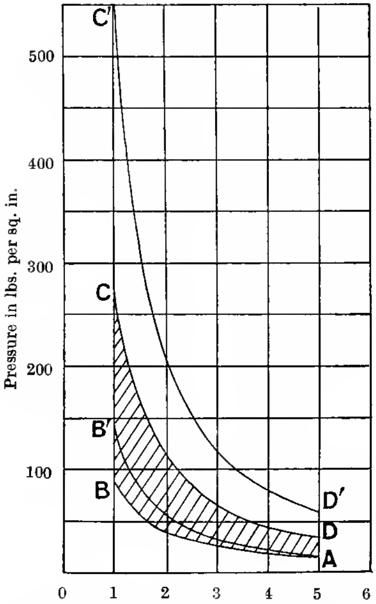

FIG. 86.—CALCULATED PRESSURES DURING A WORKING CYCLE (NAPIER).

The diagram shown by the full lines ABCD in Fig. 86 represents the ideal theoretical diagram assuming adiabatic processes and no cooling effects.

The shaded diagram, Fig. 86, shows the calculated results, taking into account the assumed cooling effect of the walls upon the charge, the external cooling from the walls by radiation and conduction, and the heating effect of the remaining exhaust gases upon the fresh charge.

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* "Pressure and Temperature in the Internal Combustion Engine," by J. L. Napier.

Similar assumptions are made in estimating the temperatures at various points of the working cycle, the results of which are reproduced in Fig. 87.

The shaded diagram represents the calculated temperatures at different points in the cycle, making due allowance for the wall and external cooling action.

It should be mentioned that in the above cases the compression ratio is taken as 5, and that the "heating value" of the mixture is assumed to be 2,000 centigrade units.

The full line diagram in Fig. 87 represents temperatures calculated upon the assumption that no wall action or external cooling occurs.

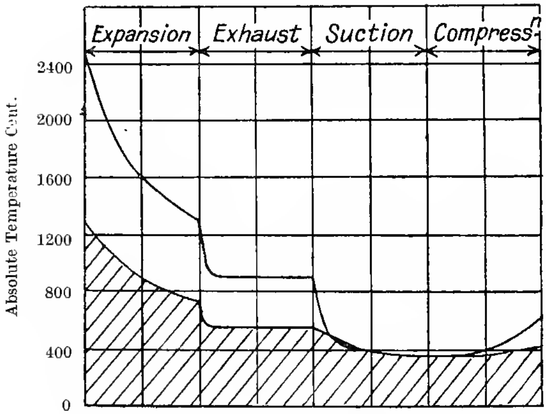

FIG. 87.—CALCULATED TEMPERATURES DURING A WORKING CYCLE (NAPIER).

In a Paper [*] upon the properties of internal combustion engine fuels a method is mentioned of estimating the temperatures during the expansion and compression strokes. In the examples considered the data was obtained from a petrol engine to which two indicators were fitted, a three-way cock enabling either to be independently connected to the cylinder. One indicator had the usual diaphragm for recording compression and expansion pressures, whilst the other was fitted with a thin diaphragm for recording suction and exhaust stroke pressures.

The exhaust temperature was first calculated upon the assumption of a suction temperature of 147° C., and then the suction temperature resulting from admixture of the measured fresh charge with the residual exhaust gases at the calculated temperature was estimated.

Using this new calculated suction temperature as the correct one, the temperature corresponding to any other point on the indicator diagram could be estimated.

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[*] "Benzol, Alcohol and Mixtures, as Fuels for Internal Combustion Engines" (Dr. Watson), *Proc. Inst. Aut. Engineers*, 1914.

The results were corrected for the increase in volume which occurs during combustion and which affects the expansion stroke.

The results of calculations made in the above manner and with the aid of the indicator diagrams, are illustrated in Fig. 88 for both benzole and petrol mixtures with air.

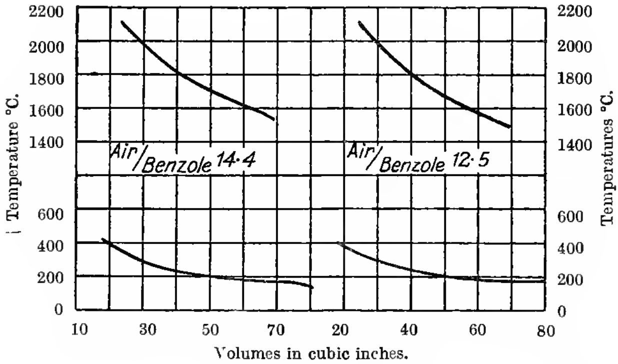

FIG. 88.—EXHAUST AND COMPRESSION TEMPERATURES (WATSON).

It will be noticed that the temperatures attained with these fuels are very similar.

Effect of Residual Exhaust Gases upon Suction Temperature.

For the purposes of more accurately estimating the suction and other temperatures, it is necessary to take into consideration the temperature and amount of exhaust gases remaining within the cylinder at the end of the induction stroke.

The effect of the combustion chamber volume of hot exhaust gases at atmospheric pressure mixing with the inducted fresh charge at atmospheric pressure and temperature will be to increase the suction temperature.

If the temperature of the exhaust gases be T° C., and atmospheric temperature be t° C., and if V be the volume of the combustion chamber, we have—

Volume of fresh charge at atmospheric pressure and temperature t° C. $= (n - 1)$ V, where n is the compression ratio.

If the density and specific heats of the gases remain constant we get—

$$\frac{nV}{T_s} = \frac{(n-1)V}{t} + \frac{nV}{T} \text{ by Charles's law.}$$

Whence the suction temperature—

$$T_s = \frac{n}{\frac{n-1}{t} + \frac{n}{T}} = \frac{n.t.T}{(n-1)T + nt.}$$

The temperature of the exhaust gases can be obtained by direct measurement with a platinum thermometer.

Temperature Measurements.

It has long since been recognized that any theoretical estimate of the temperatures existing within the cylinder can be approximate only, on account of the complicated nature of the actions occurring within, and so attempts at direct measurement have been made in the cases of the gas and petrol engine.

The direct measurement of temperature within the cylinder during a working cycle is a difficult process, on account of the high explosion temperatures attained, and which have been found to affect the working of the measuring apparatus, for the temperatures are usually above that of the melting point of platinum, so that it is impossible to employ electrical thermometers of this type to measure explosion temperatures.

Another difficulty lies in the rapid variation in temperature of the charge during one complete cycle, and in the case of the petrol engine, where about 1,000 cycles per minute may happen, such rapid temperature changes are exceedingly difficult to follow.

Most of the measurements so far attempted have been made upon comparatively slow-running gas engines, although a few temperature measurements have been made in connexion with the petrol engine.

It will be of interest to briefly consider here the methods and results of gas engine measurements first, and to deal with the petrol engine later.

In studying the behaviour of the gases from the indicator diagram, it is very essential to know with some degree of accuracy the temperature at one point of the cycle, from which the temperatures and pressures at other points can be deduced, as explained in the earlier portion of this chapter.

Earlier Attempts.

The earliest attempts at single temperature measurements at the commencement of compression (called the "suction temperature") were made by measuring the pressure, volume and temperature of the fresh charge entering the cylinder and simultaneously measuring the temperature of the discharging exhaust gases, from which it was assumed that the temperature of the gases retained in the combustion chamber at the commencement of the suction stroke could be obtained.

This method was employed by Dugald Clerk, Slaby and Thurston, and gave approximate values.

The value for the suction temperature in the low compression gas engines of the period 1880 to 1885 was found to be about 120° C. at full load, and the exhaust temperature measured in the exhaust pipe to be about 400° C.

Burstall's measurements for the Gas Engine Research Committee in 1895 gave the suction temperature as lying between 84° C. and 99° C., depending upon the compression ratio employed.

The results obtained by this method can only be regarded as approximate, as the exhaust gas temperature within the cylinder cannot be accurately inferred from measurements within the exhaust pipe, although in modern gas engines where the compression pressures are high and the combustion chamber volume small the error introduced by wrong estimation of the exhaust temperature is small.

This more or less unsatisfactory method has led to attempts at direct measurement of the temperature, the earliest being made by Burstall,* with the engine running upon weak charges under light load and at low speeds, since the temperature attained during explosion, under normal conditions of running, melted the platinum wire of his electrical thermometers. He was able to obtain a few temperature values under the above-mentioned conditions, from which he concluded that the highest explosion temperature was at least 1,250° C. In some later experiments made for the Gas Engine Research Committee, temperatures as high as 1,570° C. were observed.

The suction and compression stroke temperatures have been measured accurately by Profs. Callendar and Dalby † by means of the platinum resistance thermometer, by the "jack-in-the-box" method.

This method consisted in introducing a fine platinum loop of $\frac{1}{1000}$ inch diameter (to take up the temperature quickly) into the cylinder by an arrangement shown diagrammatically in Fig. 89. From this diagram it will be seen that the stem of the admission valve is hollow, whilst the "temperature valve" which was worked quite independently, and by a separate cam, could be made to expose the delicate platinum loop during the compression and suction strokes, but during the explosion stroke the temperature valve was closed, thus shielding the platinum loop.

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* *Phil. Mag.*, vol. xl. p. 282.

† *Proc. Roy. Soc.*, 1907.

In conjunction with this ingenious temperature valve, which could be opened at any given part of the stroke, a contact breaker was employed to make an instantaneous connexion between the thermometer and the measuring instrument just at the moment the temperature was required to be measured.

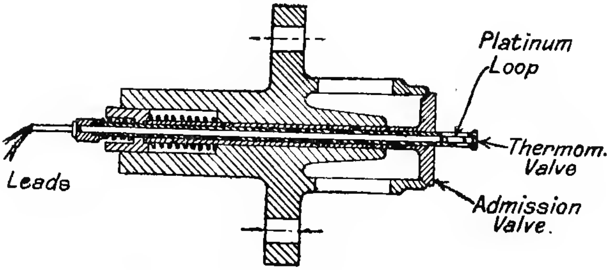

Fig. 89.—Temperature Valve [Callendar and Dalby.]

The resistance of the platinum loop could thus be measured for a very small time interval by means of the galvanometer.

At the predetermined point in each cycle the galvanometer received a small current, so that the average of several working cycles could be obtained.

In conjunction with indicator diagrams taken at the same time, the temperatures at different points of the diagrams could be calculated.

A diagrammatic sketch of the arrangement employed is illustrated in Fig. 90, in which the connexions between the contact breaker C and the electrical " bridge " B are shown.

The spring plates *a* and *b* were made to press upon two cams worked off the lay shaft, and could be adjusted relative to each other.

Contact was made by the point *p*, which completed the battery circuit when the cam was in its lowest position.

The graduated disk D could be adjusted to make the battery circuit at any given moment, or for any piston position.

The duration of the contact corresponded to about 20° crank angle, and the strength of the current through the thermometer was ·005 ampere.

It will be evident that with rapidly varying temperatures there will always be a certain time lag between the occurrence of the true temperature and the recorded temperature. The amount of this lag will depend upon the thickness of wire used in the thermometer, and upon the sensitiveness of the measuring instruments.

In Callendar and Dalby's apparatus the time lag was reduced to a small value; at a speed of 130 revolutions per minute, and with a temperature variation of about 200° in half a revolution, the thermometer lag was ·06 second, corresponding to about 10° of crank angle.

In order to determine the thermometer lag, the engine was motored around, with the platinum thermometer in place, and pure air only was compressed and expanded, in one case with the valves working normally, and in another case with the valves closed.

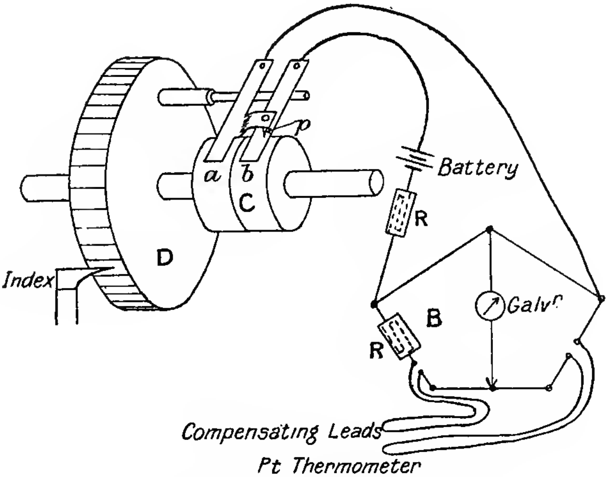

FIG. 90.—DIAGRAMMATIC ARRANGEMENT OF CALLENDAR AND DALBY'S TEMPERATURE APPARATUS.

Indicator diagrams were taken during these tests, and also thermometer readings.

The temperatures calculated from the indicator diagrams were compared with those directly measured, and the lag effect correction thus obtained.

Fig. 91 illustrates approximately the temperatures obtained by the two methods in these tests, and the amount of the lag at different points in the stroke can be at once obtained.

It was found by this method that the suction temperature varied from 95° C. at light load to 125° C. at full load.

The mixture employed was rather rich, consisting of air and gas in the proportions 5·8 to 1.

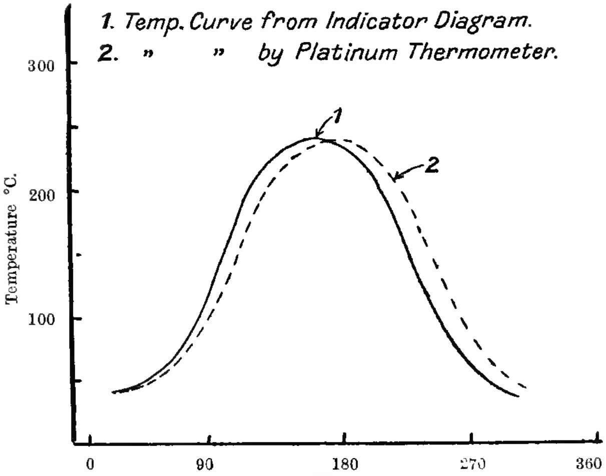

FIG. 91.—TIME TEMPERATURE CURVES.

From these suction and compression temperatures it was calculated that with the rich mixture employed the explosion temperature was about 2,500° C., whilst with a less rich mixture of 7·1 air to 1 of gas the maximum temperature attained was about 2,250° C.

Further, the exhaust temperature in the case of the rich mixture was 870° C., and in the case of the poorer mixture was 800° C.

Prof. Coker's Method.

The method employed in Prof. Coker's earlier tests consisted in using thermo-couples of platinum rhodium and platinum iridium; with such couples he found the explosion temperature with a weak charge to be about 1,700° C.

The results of later experiments in 1912 led to the conclusion that the maximum temperature with a mixture of 7·35 to 1 was somewhat higher, namely, about 1,800° C.

In some recent experiments by Coker and Scoble * upon a 12 H.P. gas engine running at 240 revolutions per minute, thermocouples consisting of cast-iron plugs and wrought-iron wires were used to measure directly the temperatures at or near the surfaces of the cylinder, a typical arrangement being to insert such a couple in the head of one of the valves, as shown in Fig. 92.

Both cyclical and mean temperatures could be measured by means of these couples.

For measuring instantaneous temperatures a percussion contact

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* *Proc. Inst. Civil Engineers,* 1913–14.

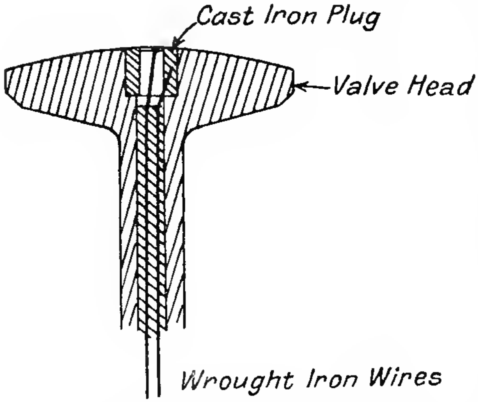

FIG. 92.—VALVE TEMPERATURE MEASUREMENT.

breaker similar to Callendar and Dalby's was employed, and the authors state that with their arrangement a complete cycle of twelve equidistant readings can be obtained in about two minutes, the engine, of course, running quite consistently during this period.

In measurements of the explosion temperatures the platinum rhodium and platinum iridium couples were found to withstand the temperatures attained for fairly long periods.

The general arrangement of the explosion temperature couple is indicated in Fig. 93.

The two platinum alloy wires of about ·0005 inch thickness, AB and BC, are electrically fused together at B, and insulated from the metal body of the valve by fireclay tubes E, plaster-of-paris being employed to keep the whole in position.

The outer steel sheath V containing the couple was inserted in the combustion chamber of the cylinder, the conical seating making the necessary gas-tight joint with the metal of the cylinder walls.

The couple itself in some of the tests made projected by about ½ inch into the cylinder.

The results of measurements made at different points of the working strokes are shown in Fig. 94 for an air-gas ratio of 7·35 to 1.

The maximum temperature measured was about 1,840° C. and the temperature at the commencement of the exhaust stroke about 850° C.

It was found that within limits the richer the mixture of gas and air, the higher was the maximum temperature measured.

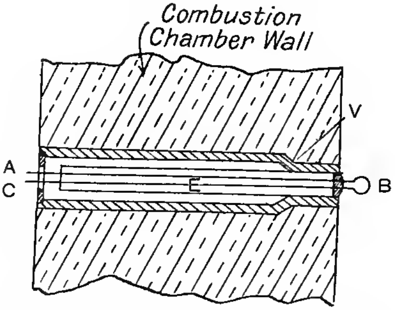

FIG 93.—ARRANGEMENT FOR MEASURING TEMPERATURE OF COMBUSTION CHAMBER.

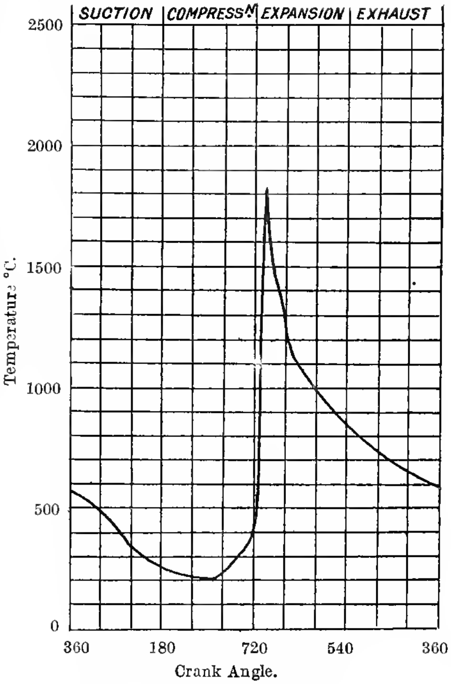

FIG. 94.—TEMPERATURE CYCLE FOR GAS ENGINE.

Thus using a mixture of 5·66 to 1 the maximum pressure (from the indicator diagram) was 433 lbs. per sq. inch, and the maximum temperature was 2,250° C.

It was also noticed that the temperature of the jacket outlet did not affect the temperature of the gas during expansion, but that the compression temperatures were higher with warmer jackets.

As regards the temperature of the gases in the exhaust pipe, which were measured at different points of the working cycle, these temperatures were found to vary between about 300° C. and 550° C., and to follow definite cyclical variations, as shown in Fig. 95.

It will be rather interesting to compare the temperature variations in the exhaust pipe with the corresponding pressure variations given in the latter part of this chapter and to notice the drop in temperature after the exhaust valve opens, and sudden rise as the valve opens, and the effect of the pressure pulsations upon the temperatures.

The mean temperature of the exhaust gases for different mixture strengths in the case of a petrol engine is shown in Fig. 96, and obtained by the author by direct measurements, electrically, of these temperatures in the case of a motor-car engine running at 1,100 revolutions per minute.

In connexion with the results of measurements of the metal temperatures within the cylinder, etc., the authors of the above-mentioned paper found that the average temperature of the exhaust valve head was 400° C., this being the highest recorded metal temperature.

The inlet valve head temperature was 300° C., and at the centre of the piston its maximum mean value was 340° C.

Hopkinson found in the case of a Crossley engine running upon a rich mixture and with boiling jacket water that the exhaust valve temperature was 530° C., and the inlet valve 330° C., and concluded that at the centre of the piston head the temperature was probably 500° C. It is also of interest to note that he gives 700° C. as the preignition temperature, above which valve the incoming charge would preignite.

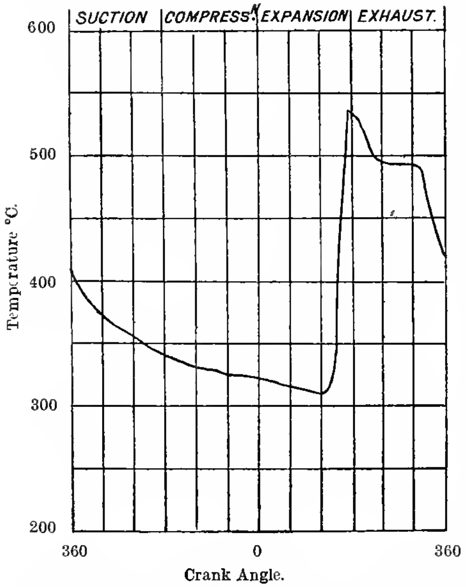

Fig. 95.—Temperature of Gas in Exhaust Pipe.

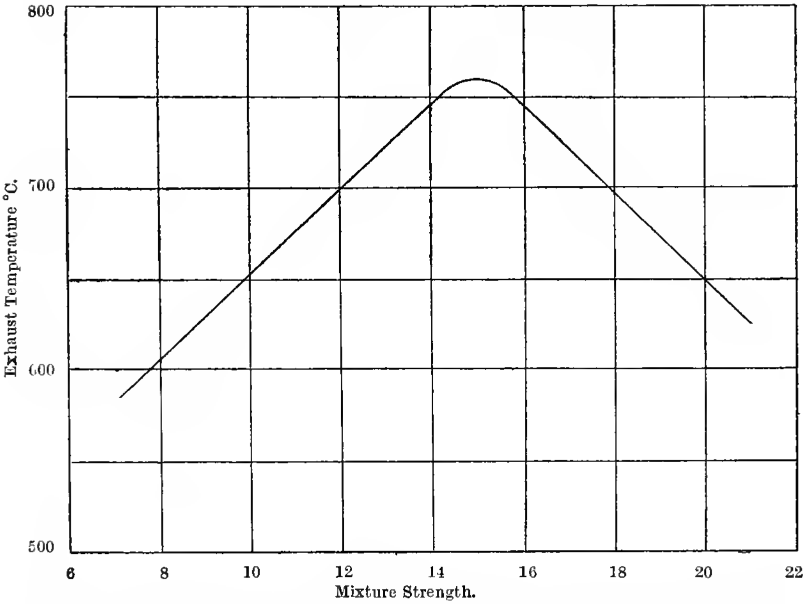

Fig. 96.—Variation of Exhaust Temperature with Mixture Strength.

Under medium conditions Hopkinson obtained a mean exhaust valve temperature of 400° C., and of 250° C. for the inlet valve, accompanied by a cyclical variation of 15° C., which latter variation agrees very well with Coker and Scoble's results.

Petrol Engine Temperatures.

Unfortunately, the available data in the case of the petrol engine is very limited, owing no doubt to the much greater difficulty of measurement and to the complications introduced by the relatively higher speeds and conditions of running of these engines.

Prof. Callendar made some temperature measurements upon an air-cooled petrol engine in 1904, the results of which are interesting.

Using thermo-couples of iron and nickel wires about one millimetre in diameter in conjunction with a sensitive galvanometer, he measured the temperatures of the walls and valves under road-running conditions. The temperatures of the cylinder head above the inlet and exhaust valves were measured by a pair of couples screwed into the metal just over the valves.

At 2,000 revolutions, with retarded spark and half-throttle the temperature on the exhaust side rose to 570° C. in a short time, whilst the inlet side remained comparatively cool, with the engine not running. The actual temperature of the exhaust side under road conditions with a cooling draught of air did not exceed 400° C.

He found that the highest exhaust valve temperatures were obtained at full throttle with the spark retarded, the greater part of the heating effect being due to the sudden outrush of the hot gases at the moment of exhaust, and not to the explosion temperature.

With the spark advanced, and running under load the temperatures observed were lower for the same throttle setting. The inlet valve was found to be kept comparatively cool at about 70° C. with the cooling air blast, and also partially due to the vaporized petrol-cooling effect.

As regards the combustion chamber temperature, it was found that without artificial or external air-cooling the exhaust side of the cylinder head was about 80° C. hotter than the inlet side after a few minutes running.

Employing fans he found that the temperature of the combustion chamber never rose above 200° C. on the exposed side and 260° C. upon the sheltered side, and that the cylinder barrel mean temperature varied between 150° C. and 200° C.

" Overheating " was found to commence when the temperature of 300° C. was reached at the upper part of the piston stroke.

Cylinder barrel temperatures were measured by means of thermo-couples placed at the top, middle and bottom of the piston stroke.

Although one would expect to find the top of the barrel hotter than the bottom, from conduction of heat considerations, yet actual measurements showed practically no difference of temperature, the top of the cylinder barrel being at 200° C., and the lowest part corresponding with the lowest position of the piston top being only 20° C. lower in temperature.

This effect is attributed to " piston convection " ; similar effects have been found in high speed steam engines, where the piston at the top of its stroke absorbs heat and carries it quickly to the lower end, where it is absorbed by the cooler walls.

The difference of temperature between the two ends is smaller as the speed increases ; this effect is beneficial in equalizing the cylinder barrel expansion and cooling effects.

With retarded spark, and therefore a slower explosion, it was found that more heat was communicated to the cylinder head, owing to the outrush of the hot exhaust gases.

With advanced spark more heat was found to be communicated to the barrel surface, due to the prolonged exposure, and this heat was found to be distributed uniformly along the barrel by piston convection.

From Hopkinson's experiments it would appear that the temperature of the exhaust gases at the end of the exhaust stroke is from 600° to 700° C. ; this agrees fairly well with the measurements of temperatures near the exhaust valve made by the author upon a car engine, which were found to vary with the mixture strength (the speed being constant at 1,100 revolutions per minute), from 600° C. for very weak and very strong mixtures, to 750° C. for the correct strength of mixture.

The actual exhaust temperature in any given engine depends upon the compression ratio, engine speed, and general design, so that it may be expected to vary, for these reasons, in different engines.

The temperature at the centre of the piston in a Siddeley four-cylinder car engine Hopkinson tested is given as about 500° C.

The inlet valve mean temperature probably lies between 100° C. and 200° C. in normal car engines, and that of the exhaust valve between 350° C. and 450° C. when working at medium speeds under normal loads ; these figures must be taken as provisional only, and in the absence of further experimental evidence.

Pressures in Internal Combustion Engines.

It has already been shown how the pressures at different points in the working cycle may be calculated upon certain given assumptions ; it may here be of some interest to consider the results of measurements of the pressures within the cylinder of a petrol engine and in closed explosion vessel experiments—the actual methods of measurement employed, however, will be considered in detail in Chapter V.

Experiments made at the Massachusetts Institute of Technology upon the pressures attained by exploding mixtures of air and petrol vapour, in a closed explosion vessel, show some interesting results.

These experiments were performed upon mixtures, in different proportions, of air and petrol vapour, the liquid density of which was 0·648 at 86° Fah., and the initial pressures were atmospheric in all cases.

Two grades of petrol were tried, but the general results were the same with each, the maximum pressures being nearly identical.

The results of some of these experiments are given in Table XXXII.

TABLE XXXII

Closed Explosion Vessel Experiments upon Air-Petrol Mixtures

Percentage of Petrol Vapour in Mixture.	Time of Explosion in seconds.	Maximum Pressure in lbs. per sq. inch above Atmosphere.	Mean Pressure in lbs. per sq. inch.	$\frac{\text{Mean Pressure}}{\text{Vapour Ratio.}}$
1·51	·083	70	49·4	3,260
1·64	·100	73	51·0	3,110
1·79	·090	71	47·7	2,670
1·96	·083	76	51·7	2,634
2·17	·058	70	48·4	2,225
2·44	·067	80	53·4	2,190
2·56	·075	84	56·4	2,200
2·63	·055	86	57·0	2,164
2·78	·083	78	54·0	1,945
3·03	·091	76	53·4	1,760
3·23	·083	77	54·0	1,675
3·45	·083	77	54·7	1,587
3 85	075	66	50·0	1,300
4·17	·066	60	46·0	1,104
4·76	·066	50	44·0	925

From a study of these values it will be clearly seen that the air-petrol mixture giving the maximum pressure consists of 2·63 per cent. of petrol vapour, the pressure reached being 86 lbs. per sq. inch above atmospheric.

The figures given in the third column are of some interest, indicating the mixture yielding the greatest pressure per unit mass of petrol vapour.

It will be seen that the mixture containing 1·51 per cent. of petrol vapour is the most efficient in this respect, and this result is in accordance with the well-known fact that the weaker, in petrol, mixtures employed in petrol engines are the most thermal efficient ones.

From experiments made upon coal gas and air in the same apparatus it was further shown that whereas with petrol air mixtures at atmospheric pressure initially the best explosive pressure was 86 lbs. per sq. inch, yet with the best coal gas and air mixture in the proportions of 1 to 5, the maximum pressure reached was 96 lbs. per sq. inch. This result would appear to indicate the inferiority of the petrol engine so far as power is concerned in comparison with the gas engine of similar dimensions, but the effects of the high speed of the petrol engine and other conditions existing in reality result in higher mean pressures being attained for the same conditions of compression.

As a point of interest, mention will be here made of other types of internal combustion engine, in order that the relation of the petrol engine type to these may be understood.

Dealing with the modern gas engine, we find that compression pressures up to 200 lbs. per sq. inch can be employed for general use, corresponding with compression ratios varying between 4·5 and 8·0, the usual compression pressure employed being about 150 lbs. to the square inch above atmospheric, and corresponding with a compression ratio of about 6·4.

It has been found in gas engine work that with such compressions the best fuel economies are obtained, and that the volumetric efficiencies are correspondingly suitable.

The explosion pressures measured in gas engines vary from 400 to 600 lbs. per sq. inch under the above conditions; thus the average explosion pressure measured in the case of a large Crossley gas engine with an indicated compression pressure of 150 lbs. per sq. inch and with a rich gas to air mixture was 460 lbs. per sq. inch.

In the case of the usual heavy oil type of Diesel engine the compression pressure is usually between 450 and 600 lbs. per sq. inch, being about 500 lbs. per sq. inch upon the average, corresponding with a temperature of compression of 600° C.

The explosion or combustion pressures are generally somewhat higher, and with high compressions may reach values as high as 1,000 lbs. per sq. inch.

An engine of this type tested by Prof. Junkers with a compression of 600 lbs. per sq. inch absolute, when working normally upon Californian heavy oil, gave a maximum pressure of 690 lbs. per sq. inch and a mean indicated pressure of 151 lbs. per sq. inch; further, when this same engine was overloaded, a maximum pressure of 1,070 lbs. per sq. inch with a mean pressure of 194 lbs. per sq. inch at 200 revolutions per minute were attained.

The above results refer to an experimental engine; with the usual marine type Diesel engines, the maximum explosion pressures will not generally exceed 650 lbs. per sq. inch, whilst the mean pressures are usually not greater than 100 lbs. per sq. inch in well-designed types.

The crude oil semi-Diesel type of engine used upon motor vessels usually employ compressions from 150 up to 250 lbs. per inch, whilst with slow-running paraffin engines of the motor-boat and stationary lighting set the compressions are fairly low, and usually lie between about 40 and 90 lbs. per sq. inch.

Pressures occurring in Petrol Type Engines.

With engines primarily designed for running upon petrol, the compression ratios vary from 3 to $5\frac{1}{2}$, according to the type.

The slower speed commercial type of car engine and most of the earlier car engines seldom have compression ratios exceeding 4·0 and corresponding with compression pressures of about 90 lbs. per sq. inch.

The average car engine of the present time is given a compression ratio lying between 4·0 and 5·0, being about 4·5 upon the average, which corresponds with a compression pressure of about 105 lbs. per sq. inch.

The maximum compression pressures which appear to have been used at present are about 135 lbs. per sq. inch, and correspond with compression ratios of 5·3.

The explosion pressures for the range of compressions used in motor engine practice vary from about 200 lbs. per sq. inch to 350 lbs. per sq. inch.

It is of course possible to obtain even higher explosion pressures under abnormal conditions up to 450 lbs. per sq. inch; the author has observed indicated maximum pressures of over 400 lbs. per sq. inch, following a period of non-firing in one cylinder.

The usual explosion pressures in good designs of car engine lie between 250 and 350 lbs. per sq. inch.

The mean effective pressures developed in car engines at the maximum power output vary from 70 to 110 lbs. per sq. inch, reckoned on the brake horse power; this mean effective pressure is generally denoted by the symbol $\eta.p$, where η is the mechanical efficiency, and p the indicated mean effective pressure.

The maximum value for ηp obtained in the Rating of Petrol

Engines Committee's tests * was 104 lbs. per sq. inch at about 1,000 revolutions per minute.

The compression pressures of air-cooled petrol engines are lower than with the water-cooled types, and vary from 70 to 95 lbs. per sq. inch in modern types, being chiefly limited by cooling considerations, since high compressions are associated with high temperatures.

In connexion with two-stroke petrol engines of the three-port type, the allowable compression pressures with both air and water-cooled types are generally appreciably lower than with corresponding sizes of four-cycle engines; the compressions realized are also affected by the poorer volumetric efficiencies, which correspond in effect with throttling of the charge, poorer exhaust scavenging and loss of charge, etc.

With the improvement of the two-stroke engine, the compression pressure has risen, and mean effective pressures varying from 70 to 80 lbs. per sq. inch have been indicated.

Some tests upon exceptionally good two-cycle engines † have given measured values of ηp varying from 80 to 90 lbs. per sq. inch.

In connexion with aeroplane engines, in which it is essential to obtain the maximum power output from a given cylinder capacity and engine weight, the values of ηp for air-cooled engines vary from 70 to 95 lbs. per sq. inch, being upon the average about 80 lbs. per sq. inch, whilst with the water-cooled types in which higher compressions are used, the values of ηp vary from 80 to 110 lbs. per sq. inch, a good average figure from test results ‡ being 90 lbs. per sq. inch. It might be here mentioned that most types of aeroplane engines usually run at about half of the maximum speed of car engines, owing to the fact that aerial propellers work more efficiently at lower speeds. The speeds of aeronautical engines, in which the propeller is directly coupled to the crankshaft, vary from 1,100 to 1,400 revolutions per minute; it has been found possible, however, in some engines to drive the propeller off the cam-shaft, at half the engine speed, in which case the engine revolutions per minute vary from 1,700 to 2,500, and better weight economy results. Engines of the second type are usually on the Vee cylinder principle, examples of which are to be found in the 90 H.P. Renault running at 1,800 revolutions per minute, and the Sunbeam aeroplane engines running at 2,000 revolutions per minute.

* *Proc. Inst. of Aut. Engineers*, 1910–11, vol. v.

† "Two-Cycle Engines" (Brewer), *Proc. Soc. Engineers*, 1912.

‡ "Aeronautical Engines" (Graham Clark), *Proc. Inst. Aut. Engineers*, 1912.

Aeroplane engines are usually of larger horse powers and cylinder dimensions.

Up to a certain limit the effect of speed increase is to raise the mean pressure, owing to the reduction in the cooling losses. Above this limit the opposing effects of poorer volumetric efficiency, charge heating, etc., causes a reduction in the mean pressure.

The effect of cylinder dimensions upon the mean pressures is considered later, and it will be shown that the larger diameters of cylinders are more favourable to the development of high mean pressures.

From the point of view of the cooling losses, therefore, the aeroplane engine would appear to be at a disadvantage owing to the relatively lower speeds, but from its larger cylinder dimensions, in many cases, and from the higher volumetric efficiencies realized at the lower speeds, the mean pressures should be higher than with the car engine.

Generally speaking, the mean pressure realized in practice depends largely upon the design of engine (that is, upon the cylinder dimensions, combustion head design, valve settings, and design of the working parts), the compression pressure employed, the speed of the engine, temperature of the cooling system, nature of the fuel employed, and the mixture proportions.

Variation of Mean Pressure.

The values previously given represent the best pressures obtained under the most favourable conditions, and it does not follow that a given compression pressure will correspond with a certain mean pressure, for the latter varies very considerably with mixture strength, engine design, and to a lesser extent with the speed and throttle opening.

As an example might be mentioned the case of a car engine tested by the author with a compression pressure of 80 lbs. per sq. inch, in which the mean pressure varied at 1,100 revolutions per minute from 60 lbs. per sq. inch with weak mixtures to 86 lbs. per sq. inch with fairly rich mixtures, and from 82 lbs. per sq. inch down to 75 lbs. per sq. inch as the speed increased from 1,000 revolutions per minute up to 1,700 revolutions per minute with the same mixture strength—this latter effect being due to throttling of the charge owing to the design of the engine tested.

In well designed engines the volumetric efficiencies do not fall off very much with speed over the working speed range, and the mean effective pressures are fairly well maintained over this range.

Variation of Mean Pressure with Mixture Strength.

The manner in which the mean pressure is influenced by the quality of the mixture is shown graphically in Fig. 97 for the case of a four-cylinder car engine tested by Dr. Watson, and running upon petrol as fuel.

It will be observed that with poor mixtures (in petrol) the mean pressures (which in this case are indicated pressures) are low, on account of the low heating value of the quantity of mixture present; as the mixture is enriched in petrol the mean pressure increases, and reaches a maximum value for a mixture proportion appreciably richer than the optimum mixture for theoretically perfect combustion at about 14·7, on the curve shown.

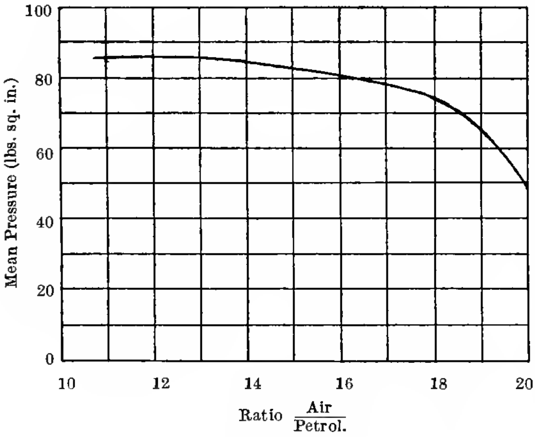

FIG. 97.—VARIATION OF M.E.P. WITH MIXTURE STRENGTH.

There is no doubt that the residual exhaust products at the commencement of the suction stroke have an influence upon the fresh charge, and possibly the richer mixtures supplied by the carburettor, are necessary to counteract the effects of exhaust dilution for the maximum pressures to be realized.

From the curve given in Fig. 97, it appears that the mixtures giving the maximum power, but not maximum fuel economy, or thermal efficiency, are rich mixtures; this fact is borne out by the experiences of carburettor experts and racing men.

Some interesting experiments * have been made upon a sleeve valve engine using other fuels than petrol, the results of which are illustrated in Fig. 98, the pressures given being indicated ones.

The fuels employed were petrol, benzole and alcohol, and the same conditions of compression and speed were realized in each case.

It is interesting to notice that the maximum pressures with the

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* *Proc. Inst. Aut. Engineers*, 1914–15 (Watson).

two latter fuels were slightly higher than with petrol, although the compression pressures were the same in each case.

It is known that different fuels when mixed with air require different compression pressures in order to yield their maximum working pressures; this fact in itself would suggest different designs of engine for different types of fuel.

The compression pressures required for the best results with paraffin as a fuel are about 75 to 85 per cent. of those with petrol, whilst with benzole compression pressures of 110 to 120 per cent. of those used in petrol engines are necessary for higher mean pressures to be realized.

Alcohol works most satisfactorily with compression pressures varying from 150 to 200 lbs. per sq. inch. These compression pressures are about 100 per cent. greater than are employed in petrol engines, and yield high mean pressures under these conditions.

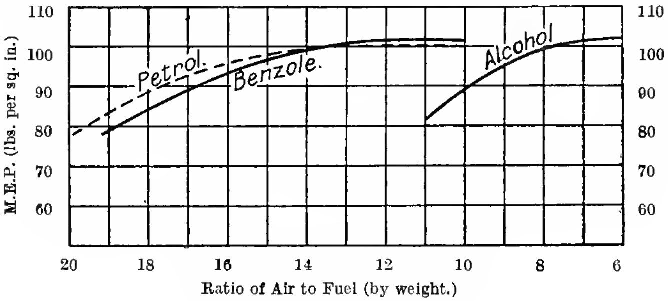

FIG. 98.

Effect of Cylinder Dimensions upon Mean Pressures.

In connexion with the subject of thermal efficiency (page 143), some reference was made to Prof. Callendar's expression for the efficiency of internal combustion engines of varying sizes running under similar conditions.

The expression there given for the efficiency was deduced from the results of experiments upon an air-cooled petrol engine, and gas engines, and included terms taking into account the effect of cylinder diameter and engine speed.

It was shown that the cooling losses during combustion and expansion were relatively greater in smaller engines than in larger engines running at the same speed and mixture conditions.

It will be seen that for combustion chambers of similar design, the surface of the chamber which is directly exposed to the hot gases varies as the $(\text{diameter})^2$, while the volume of the chamber varies as the $(\text{diameter})^3$, so that the ratio of surface to volume varies as $\frac{1}{\text{diameter}}$ and therefore increases as the bore diminishes, and vice-versa; the larger diameters will accordingly give smaller cooling losses.

The effect of the relatively greater cooling losses in the smaller engine would be to reduce the mean effective pressure by lowering the expansion curve, and therefore the area of the indicator diagram.

The variation of the mean pressure with cylinder diameter is given by Callendar * as $\eta p = k\left(1 - \frac{1}{d}\right)$, where k is a constant, ηp being the brake mean pressure and d the cylinder diameter.

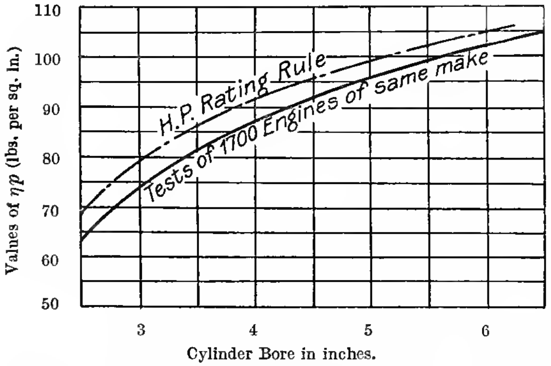

FIG. 99.—EFFECT OF CYLINDER BORE UPON MEAN EFFECTIVE PRESSURE.

The information given in the Horse Power Ratings Committee's Report † throws an important light upon the effect of the cylinder diameter upon mean pressure.

The results of some 124 tests upon various makes and sizes of engines did not yield any conclusive evidence, but when the results of tests made upon 1,700 engines of one particular make, but of different sizes, were analysed, it was found that the value of ηp increased with the cylinder diameter, as shown in Fig. 99; the experience of many other makers of engines also proves that in testing engines under similar conditions, ηp increases with the cylinder dimensions.

It is also known that the mechanical efficiency of large engines is greater than in small engines, apart from the smaller relative cooling losses.

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* "The Effect of Size on the Thermal Efficiency of Motors" (Callendar), *Proc. Inst. Aut. Engineers*, 1906–7.

† *Proc. Inst. Aut. Engineers*, 1910–11.

The H.P. Ratings Committee's Results.

The Ratings Committee, as the result of a careful analysis of the results of numerous engine tests, and upon theoretical grounds were led to adopt an expression for the brake mean pressure which took account of cylinder dimensions. To quote from the Report: "Taking into consideration all these matters and the results of many tests upon petrol engines for cars, the Committee is of opinion that it is necessary to allow for an increase of ηp with the cylinder diameter, and they accept $68\frac{1}{2}$ lbs. per sq. inch for ηp when $d = 2\frac{1}{2}$ inches and $99\frac{1}{2}$ when $d = 5\frac{1}{2}$ inches." The best current practice as to ηp is therefore taken as—

$$\eta p = 130\left(1 - \frac{1{\cdot}18}{d}\right) \text{ lbs. per sq. inch.}$$

This expression corresponds with the maximum output of the engine. The values for ηp corresponding with different cylinder bores is also shown in Fig. 99, and is marked in distinction to the previously mentioned single set of experimental results.

It may be of interest to here mention that the piston speeds were found to increase with the ratio of stroke to bore from 1,200 feet per minute with a stroke bore ratio of unity, up to 1,600 feet per minute for a ratio of 1·6.

The rate of increase of maximum practical piston speed with stroke bore ratio is given by

$$\sigma = 600\ (r + 1),$$

where r is the ratio of stroke to bore.

It should be remembered in making comparisons of values of mean pressures for engines of divergent makes and sizes that the conditions of comparison must be the same. It has been shown that the values of ηp vary with the design, speed, mixture strength, and size of the engine, and that with so many variables direct comparisons are rendered a matter of difficulty; these points are often lost sight of in this connexion.

In connexion with the relatively greater cooling losses in smaller diameter cylinders, it follows that during the compression stroke the losses to the walls will be relatively greater, and therefore the temperature at the termination of the compression stroke will be relatively lower than in larger engines.

As this temperature, which is fixed from preignition considerations, limits the degree of compression in those engines which are designed for maximum overall efficiency, it also follows that higher safe maximum compression ratios can be employed in small engines than in large ones, and higher compression pressures may be realized;

whether the increase in area of the indicator diagram for a small engine, due to the higher compression ratio, more than compensates for its diminution on account of the greater cooling losses on the expansion stroke is still an open question.

It is interesting, in support of the above contention, to quote some figures given by Lanchester for the compression ratios of different sizes of sleeve valve engines giving about the same mean pressures on the brake, as shown in the following table :—

TABLE XXXIII

Diameter of Cylinder in millimetres.	Compression Ratio.
70	6·00
80	5·60
96	5·11
100	5·00
124	4·60

The range of diameters is fairly limited, and it does not therefore follow that the same conditions will hold beyond this range, or that these results will hold in similar poppet valve engines in which the combustion chamber design is necessarily different.

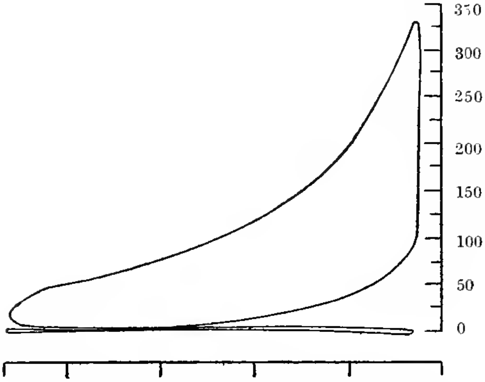


Fig. 100.—Typical Diagram of Working Pressures for a Petrol Engine.

Data from the Indicator Diagram.

The relations existing between the pressures and volumes in a petrol engine can best be understood by reference to the indicator diagram shown in Fig. 100, and which is reproduced from an actual car engine.

It has been found from measurements of a number of petrol engine indicator diagrams that the equation of the expansion curve is given by PV^n = constant, where the constant n varies between 1·15 and 1·35.

For the compression curve the same type of equation holds good, and the value of the constant n lies between 1·20 and 1·30.

In the case of a sleeve valve engine tested by Dr. Watson, the values of n for petrol-air mixtures for the expansion line are given as $n = 1{\cdot}18$, and for the compression line as $n = 1{\cdot}29$; whilst with benzol-air mixtures $n = 1{\cdot}25$ in the former case and 1·28 in the latter, for rich mixtures in both cases.

For a two-stroke engine having a compression pressure of about 75 lbs. per sq. inch, the values of n for the expansion and compression strokes are given as 1·35 and 1·25 respectively.

It will be evident that the value of n will vary with the design of engine, the speed and mixture strength, since the shapes of the expansion and compression curves will be modified by the cooling effects of the walls, etc.

The smaller the cooling losses, and the higher the engine speed, the higher will be the value of n, for the adiabatic conditions are represented by high values for n.

In slow-running engines, and where the cooling losses are great, the expansion and compression lines will result under conditions approaching isothermal ones, and n will therefore approach unity.

It should be mentioned that the value of the constant n is not the same for all points lying on the expansion line, since the cooling losses and piston speed do not remain constant throughout the stroke.

Inlet and Exhaust Pressures.

Having considered the working pressures within the cylinder during a cycle of operations, it is now proposed to examine, in some detail, the pressure changes which occur in the induction and exhaust pipes of internal combustion engines.

Dealing with the inlet pressures first, there are two quantities which it is necessary to examine: the mean pressure during any set conditions of running, and the actual pressures existing at different instances during a complete cycle of changes.

The average pressure existing within the inlet manifold, which is usually measured fairly close to the inlet valve, and which is always negative (that is to say, a suction effect), is usually obtained by connecting a mercury manometer provided with a suitable pressure damping arrangement to the induction pipe at the chosen position, and noting the difference in levels in the two limbs of the instrument.

The maximum negative pressure which can exist in any engine is dependent upon the design of the carburettor and inlet pipes and ports, and is also a function of the speed and valve timing.

In ordinary touring-car engines a negative mean pressure of 4 lbs. per sq. inch may exist in extreme cases (that is, a suction of

4 lbs. per sq. inch below atmospheric pressure), corresponding with a difference in level of the mercury columns of about 8·2 inches.

Such relatively high suctions occur at high engine speeds, and it will generally be found that where high suctions occur, low volumetric efficiencies also are in evidence.

The manner in which the suction varies with the speed is given in Fig. 75 upon page 171.

The method of measurement of this negative pressure is quite simple, and in practice it is found that the variations in pressure in the inlet pipe do not appreciably influence the readings of the mercury columns, due no doubt to the relatively great inertia of the mass of mercury present. The engine speed must, however, be kept constant during a reading.

The instantaneous variations in pressure are obtained by attaching an indicator of suitable design so as to follow the rapid changes quickly to the selected place in the inlet pipe.

The suction then actuates the pressure recording mechanism of the indicator (a very weak or " light " spring being fitted for the purpose).

The phase gear of the indicator, wherewith the pressure is recorded at the exact position along the piston stroke where it occurs, is driven off the engine in the usual manner.

Dr. Watson has obtained indicator diagrams from the induction pipe in this manner, examples of which are produced in Figs. 101 and 102.

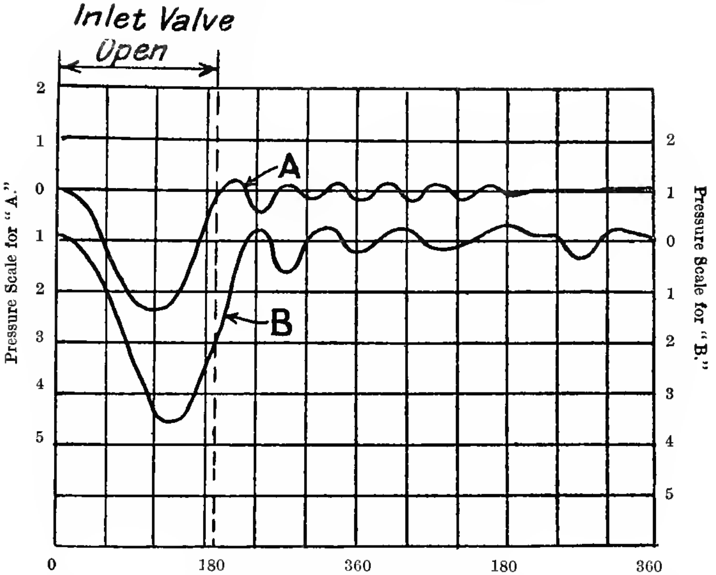
Fig. 15 represents the pressure variations in the case of a single cylinder sleeve valve engine running at about 800 revolutions per minute in curve A, and at about 1,200 revolutions in the case of curve B.

The effect of the increased speed in increasing the amplitude of the pressure variations is apparent in the diagram. The point of inlet valve closing is shown; the pressure variations following are quite marked, and indicate the presence of waves in the induction pipe.

The damping out of these waves is also evident just before inlet commences again.

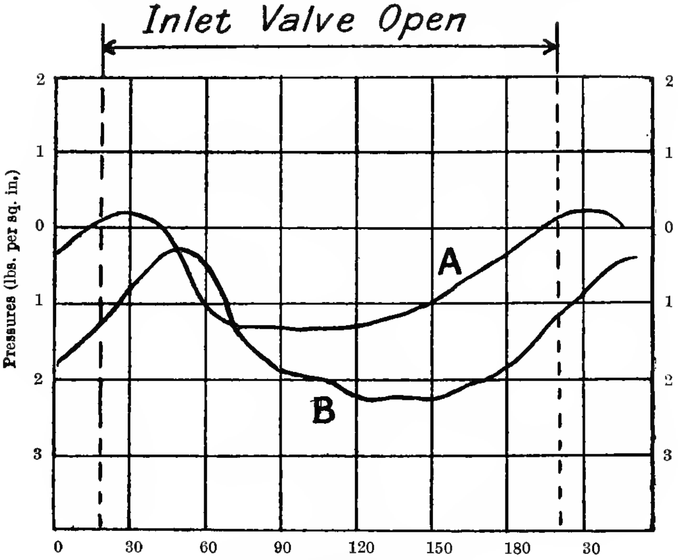
Fig. 102 represents the same type of pressure variations in the case of a four-cylinder engine; it will be seen that as before the effect of speed is very marked. Curve A corresponds with a speed of about 650 revolutions, and curve B to 1,200 revolutions per minute.

It will be noticed that the effect of cylinder multiplication is to smooth out the pressure variations, and more uniform suctions generally occur in these cases. The amplitude of the pressure variations is also considerably reduced, and the cycle of changes



A—Speed, 800 revs. per min.
B—Speed, 1,200 revs. per min.

FIG. 101.—PRESSURES IN THE INLET PIPE OF A SINGLE CYLINDER ENGINE.



A.—Speed, 650 revs. per min.
B.—Speed, 1,800 revs. per min.

FIG. 102.—PRESSURES IN THE INLET PIPE OF A 4-CYLINDER PETROL ENGINE.

has a greater frequency—the net effect of smaller amplitude and greater frequency being analogous to the " bobbing " up and down of a candle flame—the smaller the amount, and the greater the frequency of the flickerings, the more continuous the illumination effect.

In the case of the four-cylinder engine, it will be noticed that immediately after the inlet valve has commenced to open there is a pressure rise in the inlet pipe, which is partly due to the inertia of the gases in the inlet pipe (on account of the waves set up) and partly to the fact that there is a small back pressure within the cylinder due to the exhaust gases just expelled and those remaining within the clearance spaces immediately before the inlet valve opens.

It has been found that this pressure rise, which is chiefly due to the inertia of the gases in the inlet pipe, is greater with longer inlet pipes, and vice versa.

With short inlet pipes, and with the air admitted near to the jet, the rise of pressure above atmospheric does not occur.

It is interesting to note that the volumetric efficiency falls off slightly more rapidly than does the mean pressure in the inlet pipe reckoned from atmospheric. The mean pressure can also be obtained from the pressure variation curves by finding the mean height of the curves.

Coming next to the exhaust side of the engine, the mean and instantaneous pressures have been measured in a similar manner to the inlet pressures, with the exception that the mercury manometer usually requires some better form of " damper " to give steady readings, owing to the greater surging effects.

Fig. 103 illustrates a typical curve of exhaust mean pressure variation with speed for a single-cylinder poppet valve engine provided with a simple expansion chamber of generous proportions for a silencer and an exhaust pipe of some five feet length and of suitable diameter.

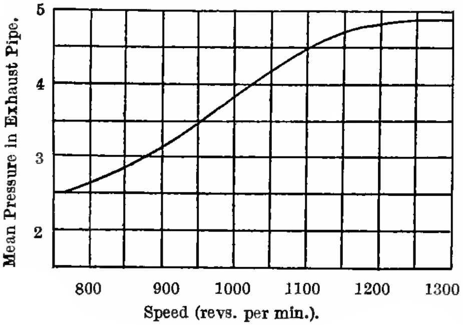
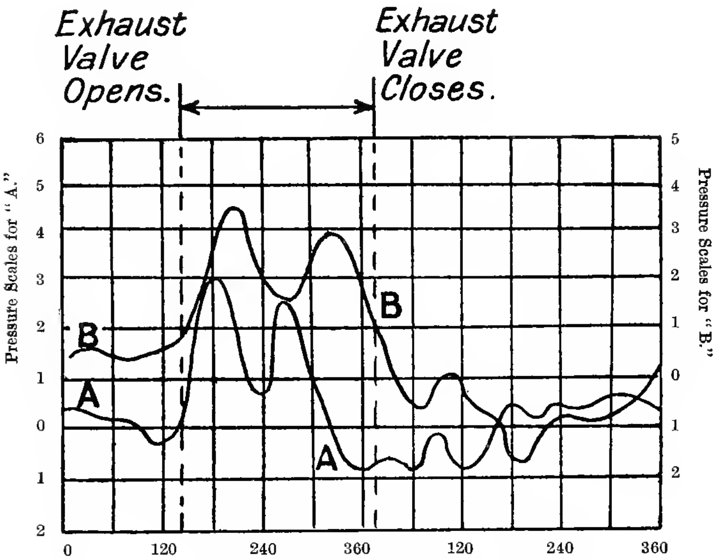


Fig. 103.

Fig. 104 illustrates the exhaust pressure variations in the case of the sleeve valve engine previously mentioned, in connexion with inlet pressures.

It is noticeable that the amplitude of the pressure variations is greatest during the period of the exhaust valve opening, and that just before the valve closes, when the piston is nearing the top of the exhaust stroke, there is a fairly rapid drop of pressure below atmospheric, followed by a rise to atmospheric immediately before the exhaust valve opens again.

This negative pressure effect is no doubt due to the fact that the flow of exhaust gases along the exhaust pipes is followed by a vacuum or negative pressure effect due to the inertia of the rapidly moving gases.

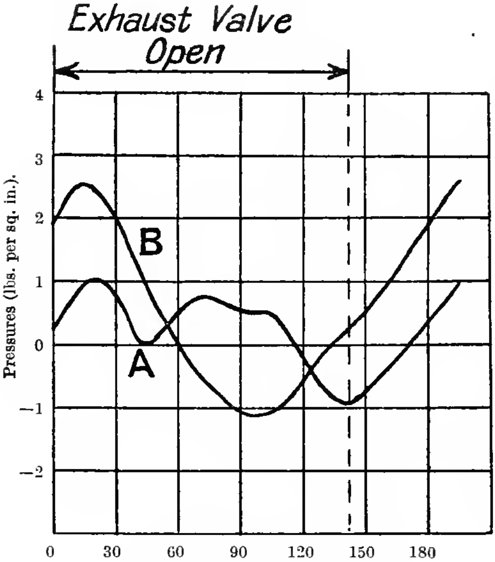


A.—Speed, 800 revs. per min.
B.—Speed, 1,200 revs. per min.

FIG. 104.—PRESSURE IN THE EXHAUST PIPE OF A SINGLE CYLINDER ENGINE.

This is the principle of the scavenging action first brought into prominence by Atkinson, in gas engine work, and in which the vacuum caused by the exhaust gases is utilized by opening the air valve just before the gas valve, thus allowing fresh air to flow into the cylinder for scavenging purposes.

The length of the exhaust pipe plays an important part in this scavenging effect, and it is possible to find a certain length of pipe for a given engine which will give pressure waves in resonance with the engine speed.



A.—Speed, 600 revs. per min.
B.—Speed, 1,240 revs. per min.

Fig. 105.—Pressures in Exhaust Pipe of Four-Cylinder Engine.

The curves given in Fig. 105 which were obtained from a four-cylinder engine illustrate this point.

The period of the surgings is smaller than in the case of the single cylinder, as in the case of the inlet pipe variations.

The nearly perfect resonance effect at the higher speed is marked. The mean height of the curves will be seen to increase with the speed.

In the case of the four-cylinder engine the exhaust pipe was about 9 feet long, and provided with two simple expansion boxes of about 1 cub. foot capacity each, in series, placed nearer to the outlet end of the exhaust.

These particulars are given, as the design of the silencer and dimensions of the exhaust piping have a great influence upon the value of the mean and instantaneous pressures occurring.

By experimenting with various sizes and lengths of exhaust pipes (apart from silencer design), it is possible to improve the volumetric efficiency and therefore the power output.

CHAPTER V

Indicators and Indicator Diagrams

In order to study the internal changes occurring in connexion with the working of the petrol engine, and other types of internal combustion engine, it is necessary to have a reliable and trustworthy form of indicator.

The ordinary form of steam engine indicator designed to work below speeds of 400 revolutions per minute is totally unsuited to moderate and high speed explosion engines.

A special form of steam engine indicator has, however, been adapted by several makers to the requirements of gas and oil engines running at speeds below 600 revolutions per minute.

The subject of indicators will, for the purposes of detailed treatment, be considered under two separate headings, namely, (*a*) the slow speed indicator, and (*b*) the high speed indicator.

Under the category of heading (*a*) is included the gas and oil engine forms of indicating apparatus, and in special cases the slow-running lighter spirit type of combustion engines.

The high speed, petrol (and lighter spirit) types of engine running up to speeds of 2,500 revolutions per minute and above, from a minimum of about 500 revolutions per minute, require special forms of indicator, which come under heading (*b*).

The Crosby Indicator.

(*a*) Probably the best known form of mechanical indicator is that of the Crosby type, which is shown in Fig. 106.

The function of all indicators is to draw, to a small scale, a pressure-volume diagram which is exactly representative of the actual changes occurring within the engine at the time, and over a working cycle of operations.

In the illustrated form, which is intended for use with gas and oil engines, a small piston P communicates upon its lower side with the combustion chamber of the engine, a three-way tap being interposed between the piston and engine, so that the indicator can be placed in communication with the engine cylinder or the atmosphere at will, for calibration purposes.

The piston is made in the form of the central zone of a sphere, and therefore is in line contact with the walls of the cylinder, to minimize friction effects.

The up and down motion of the piston is transmitted through the piston rod, which is provided with a ball-and-socket joint at the piston, to a light link mechanism terminating in an arm provided with a pencil point.

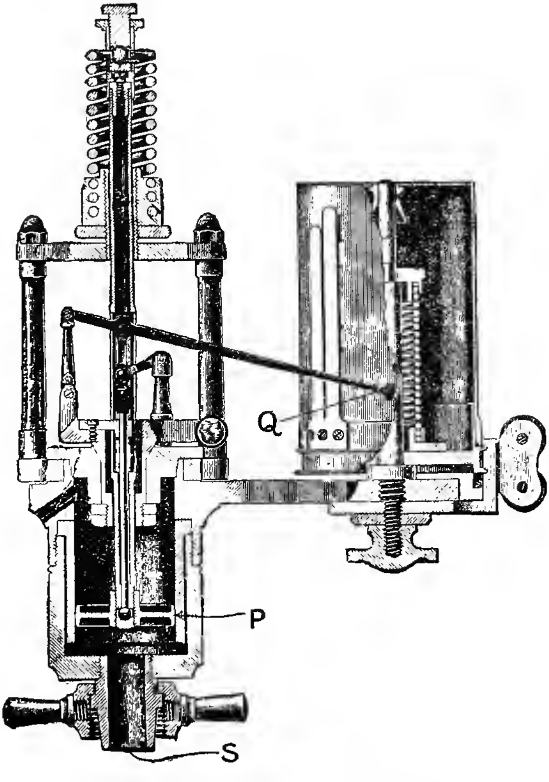


FIG. 106.—THE CROSBY INDICATOR.

The function of this mechanism is to reproduce at the pencil point a vertical straight line motion exactly proportional to the small piston's motion.

In some forms of this type of indicator the well-known Watts' parallel motion mechanism, first used in connexion with beam engines, is employed to give the straight line motion.

In the Crosby indicator, the straight line motion of the pencil Q is obtained by rocking levers whose positive and negative curvatures neutralize each other in all positions.

The motion of the piston is controlled by a calibrated helical spring placed above the link mechanism, so as to be unaffected by the heat of the engine.

A somewhat larger form of piston than in the usual inside spring types is employed with the outside spring type, in order that a greater total force may be available to overcome the increased inertia effects due to the longer piston rod necessary with this type.

It will be evident from the fact that the deflection of a spring is proportional to the load, up to a certain limit, that the vertical motion of the pencil Q is proportional to the pressure acting upon the smaller piston, that is, to the pressure existing within the cylinder of the engine to which it is connected, and the scale of pressure will be uniform.

In other words, equal vertical movements of the indicator pencil represent equal increments of pressure.

The pencil thus draws vertical lines upon the indicator drum (to which the paper, or " card," is attached) the lengths of which are proportional to the pressure.

The horizontal motion of the drum, representing piston displacements, is obtained by rotating it about its vertical axis, this motion being effected by suitably reduced reciprocating motion obtained directly from the engine piston rod, or in some cases by gearing and mechanism reproducing the piston's motion faithfully.

Usually a cord is attached to the periphery of the drum, and at its other end to the reciprocating piece, an internal spring in the indicator drum giving the return movement.

The area of the piston in the Crosby indicator for explosion engines is $\frac{1}{2}$ sq. inch.

To facilitate the taking of diagrams the whole pencil mechanism can be rotated about axis of piston so as to bring the pencil in or out of contact with the paper.

The pencil is usually a brass or metal point, and the paper is specially prepared to yield fine lines with a minimum of friction.

Disadvantages of Mechanical Indicators for High Speed Work.

The modified steam engine indicator of the pencil and paper type, although in most respects well suited for slow running requirements, is totally inadequate for high speed petrol engines for the following enumerated reasons :—

(1) The inertia of the piston and pencil mechanism due to their

weight has the effect of retarding the initial motion of the piston, and encouraging its final motion, with the result that the recorded minimum pressure is lower, and the recorded maximum pressure is higher than the actual existing pressure.

The phase of the pressure is also incorrect in cases of excessive inertia.

This inertia effect, which becomes appreciable at speeds of about 500 revolutions per second in most mechanical indicators, and which varies approximately as the square of the speed, causes the effects at higher speeds to render instruments of this type quite useless.

Further, these inertia effects give rise to violent vibrations, when the period of the explosions approaches that of the indicator and its spring.

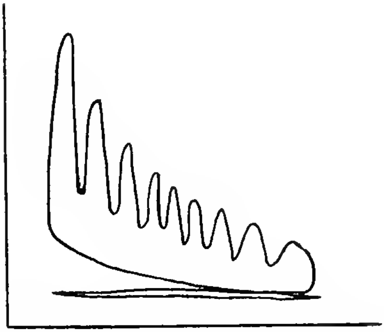


FIG. 107.—SHOWING INERTIA EFFECTS OF INDICATOR MECHANISM.

This is of the nature of a resonance effect, and will vary with the natural period of the indicator, and with the frequency of the explosions.

It is evident that in indicating over a range of engine speeds that the vibration effect will always occur sooner or later, and that over a wide range of speeds may occur two or even three times.

The natural period of the indicator can be reduced by using a stronger spring, with an accompanying reduced pressure scale; this is the principle of the micro-indicator of Dr. Mader (described in detail later).

The effect of the inertia of the pencil mechanism and indicator piston upon the indicator diagram is to give a wavy expansion line, somewhat as illustrated in the diagram shown in Fig. 107, which was obtained with a mechanical indicator from a petrol engine running at 550 revolutions per minute.

(2) The friction of the pencil upon the paper, if pressed on sufficiently to give a definite line at high speeds, has the effect of giving a false indicator diagram.

This will at once be evident from Fig. 108, in which the dotted lines represent the incorrect diagram drawn by the pencil, and the full lines the correct one.

Friction causes the pencil to lag behind its true position, so that the minimum pressures recorded are too low, and the area of diagram recorded too large; this friction can be minimized by employing very smooth paper and a fine lead pencil, so that the relative coefficient of friction is a minimum.

In some cases too much friction causes the pencil pointer to bend slightly and to thus enhance the effects.

As light a pressure upon the paper as possible consistent with an intelligible diagram is necessary to further reduce friction effects.

(3) Backlash in the pin joints, which in the best of indicators amounts to ·01 to ·02 inches, becomes serious when the vertical movement is between one and two inches, as is generally the case.

In the case of a spring with a pressure scale of 300 lbs. to the inch, this may give rise to an error of about 1 per cent.

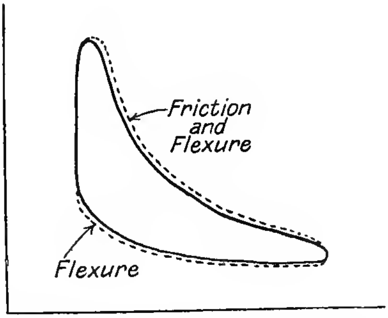


FIG. 108.—SHOWING THE EFFECTS OF FRICTION AND FLEXURE OF THE PENCIL.

Vertical backlash has the effect of increasing the recorded mean pressure, whilst horizontal backlash reduces it.

Provision for taking up an appreciable backlash is provided in the best forms of indicator.

(4) The stretching of the cord which gives the drum its motion is a frequent source of trouble in attempting to employ the mechanical indicator at high speed engines, and further, the very rapid motions of the drum enhance the effect at high speeds, giving untrustworthy diagrams; these are chiefly on account of the recorded piston phase being incorrect.

From these, and other minor defects, the ordinary indicator is quite unsuited to petrol engine work, the inertia effect being the chief source of trouble at speeds above 500 revolutions per minute.

It is quite possible at this speed to obtain a total error amounting to 10 per cent., due to backlash, inertia and friction effects.

The Clerk Optical Indicator.

Next under the heading (*a*) of the slower speed engine indicators, and worthy of mention, is the form invented by Mr. Dugald Clerk and used in his experiments, the results of which were published in 1906.*

This indicator employs a beam of light in place of the pencil mechanism in the Crosby instrument.

The principle of the Clerk indicator is shown diagrammatically in Fig. 109.

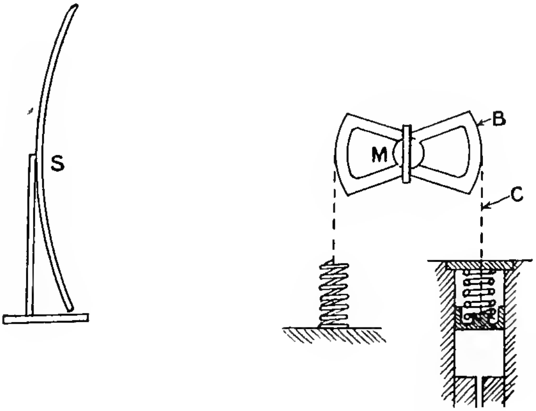


Fig. 109.—Principle of the Clerk Indicator.

The pressure, as in the previously described indicator, acts upon a piston attached to a calibrated enclosed spring, and in place of the usual piston rod there is a chain C, which is attached to one end of a beam B pivoted about its centre.

The other end of the beam arm is connected by a similar chain to a spiral spring, the function of which is to keep the connexion between the beam and the piston tight.

Upon the spindle of the beam, and rotating with it is a silvered mirror M, which is convex, and which is silvered upon the front instead of the back (as is usually the case).

A beam of light is projected upon the mirror from a pin-hole source, using an arc lamp flame as illumination.

The movement of the reflected ray of light from the mirror is recorded upon a sheet of sensitized paper held in a sliding frame.

The paper is held against a concave screen S, the degree of concavity being proportional to that of the mirror M.

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* *Proc. Roy. Soc.*, March, 1906.

The whole frame is carried along in the same direction as the axis of mirror upon small rollers, and the amount of motion of the frame carrying the sensitized paper is proportional to the main piston's motion, and in the same phase with it.

The actual detailed arrangement of the Clerk indicator is shown in Fig. 110.

For slow speed combustion engines this type of indicator is very suitable, and is an improvement upon the pencil-and-paper forms of indicator.

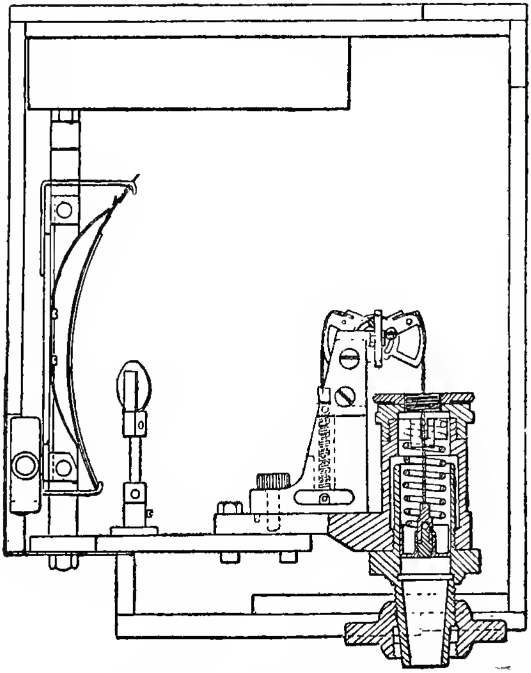

Fig. 110.—Arrangement of the Clerk Optical Indicator.

The one disadvantage of the Clerk instrument is that it is not so convenient for commercial purposes as the pencil mechanism types.

The Hopkinson Indicator.

This indicator, which is shown in section in Fig. 111, is of the sliding piston optical type, similar in principle to the Clerk instrument, but more compact in form and design; it is also better suited to higher speeds.

The plug P is screwed into the cylinder of the engine, and the tap T enables the piston P within the cylinder of the indicator to be placed in connexion either with the main cylinder of the engine or with the atmosphere (for obtaining the atmospheric datum line of pressure).

The size of the piston used can be varied (by fitting small liners and corresponding pistons), the areas being in the ratio of 1, 2 and 4.

The actual areas of the pistons are $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ sq. inches respectively, and their respective weights are 8·3, 14·3 and 21·5 grammes.

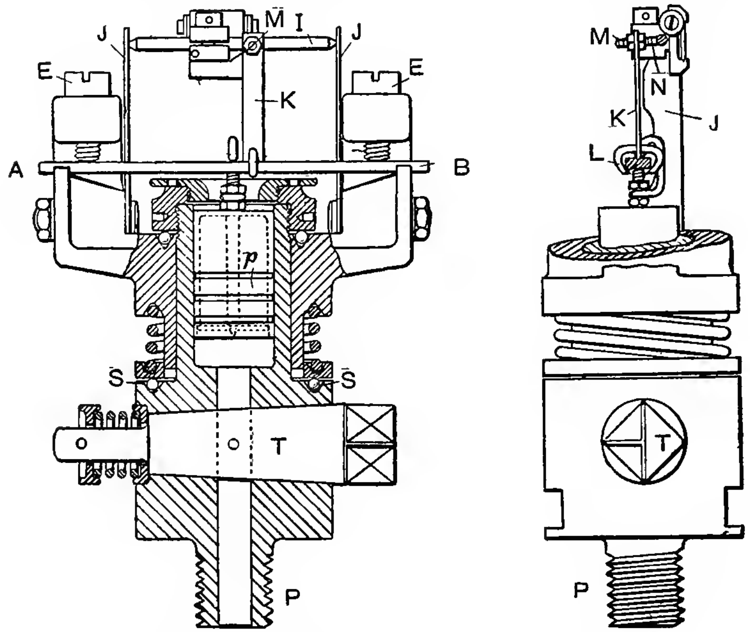

Fig. 111.—The Hopkinson Indicator [in section].

In place of the usual spiral spring is fitted a spring of the beam type, consisting of a steel strip AB resting in grooves at the end of the body of instrument.

This steel strip is held down by the two set screws EE shown, and is slightly bowed before insertion, in order that negative pressures may be measured.

The sensitiveness of the instrument may be varied by using different strengths of steel springs AB.

The motion of the piston, which is controlled by the steel spring, is communicated to the mirror M by a flat steel strip K.

The lower end of this steel strip is held firmly to the face of the main spring by means of jaws, and the upper end is clamped to an arm N, which projects at right angles to the mirror spindle.

This mirror spindle is pivoted at its ends in vertical spring checks JJ.

The spring K is made sufficiently rigid to transmit the piston's motion to the mirror arm without buckling, and at the same time allows the angular movement of the arm to take place; it will thus be evident that the mirror is given a rotation proportional to the movement of the indicator spring.

The reciprocating motion, proportional to the actual displacement of the main piston, is obtained by rotating the whole of the upper body of the indicator about the axis of its piston.

The main body of the indicator carrying the piston and mirror mechanism rotates about the axis upon a ring of balls S, under the control of a helical spring situated just above; this is shown in the diagram and also in the photographic reproduction, Fig. 112.

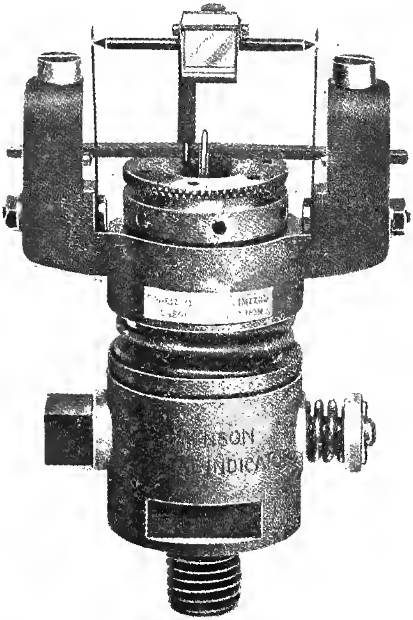

FIG. 112.—THE HOPKINSON INDICATOR.

The axial rotation of the mirror (which is, of course, perpendicular to its pressure-derived motion) can be controlled by a spring as in the ordinary paper drums of pencil indicators, or can be given a backwards and forwards or reciprocating motion positively.

This latter is a distinct advantage in high speed forms of indicator.

The method of rocking the body of indicator is illustrated diagrammatically in Fig. 113, and the reducing mechanism shown is self-explanatory.

The advantages of this type of indicator lie in the easy calibration of the pressure scale, for the instrument has only to be inverted and known weights placed upon the top of a rod inserted in the hole underneath the piston (communicating ordinarily through the tap with the engine cylinder).

Another advantage of this instrument is that the spring is out of contact with the hot gases, and its elasticity and deformations are therefore not affected thereby.

The disadvantages of this indicator can be enumerated thus :—

(1) Leakage of pressure (and gases) is apt to occur at the piston, which is provided with oil grooves, and also at the rotating ball ring.

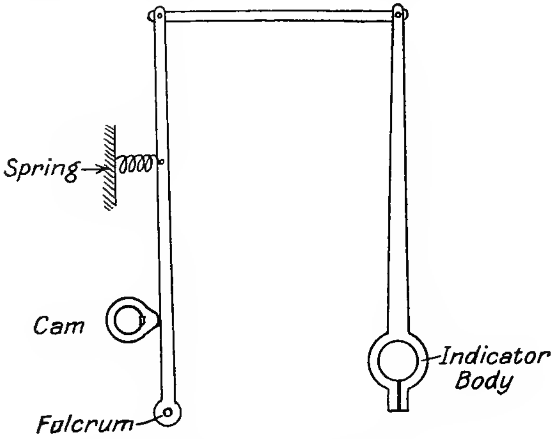

FIG. 113.—ROCKING MECHANISM.

(2) Friction of the piston against the walls of the cylinder, which acts in such a way as to reduce the area of the indicator diagram, similar to that of the pencil and drum of an ordinary steam engine type of indicator.

Friction also occurs at the swivels of the mirror arm.

(3) As a long piston is generally employed in order to reduce the leakage, it has an appreciable inertia effect—this is a common fault of piston forms of indicator.

The indicator gives very good results for speeds below about 700 revolutions per minute, above which the inertia effects become more and more serious.

An improved form of the Hopkinson indicator employs a steel diaphragm in place of the sliding piston, and the disadvantages due to the latter are thus overcome ; higher speeds can be indicated with this type.

The optical arrangements of this indicator are diagrammatically represented in Fig. 114.

A beam of light from the filament of a four-volt lamp at P is reflected from the plane mirror of the indicator M, and after reflection falls upon a convex lens R of about 18 inches focus.

The beam comes to a focus in the plane of R_2, at which is placed the ground glass screen.

With a sufficiently powerful source of light the path of the spot of light on the screen can be seen with the naked eye, but with a weaker source of illumination a second convex lens R_3 is placed behind the screen and about 10 inches from it, and the eye is placed at the principal focus of the lens.

The screen is engraved with vertical and horizontal lines, and the indicator diagram can be studied more easily with this.

For obtaining photographic records of indicator diagrams a light form of camera is employed.

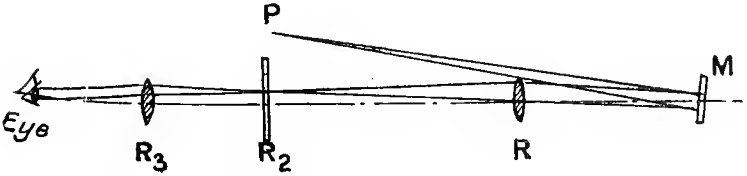

FIG. 114.—OPTICAL ARRANGEMENT OF HOPKINSON INDICATOR.

The pivoted mirror is made concave, instead of plane, in this case, and forms the image of the source of light upon the ground glass screen, which can be replaced when desired by a photographic plate in a suitable dark slide.

The source of light in this case is an electric filament lamp.

High Speed Indicators.

(*b*) It is proposed to describe the more successful types of high speed petrol engine indicator in turn, and then to consider in detail the essentials of, and the precautions necessary, in using these high speed indicators.

The problem of the high speed indicator has presented great difficulties in the past, and although there have been brought to public notice from time to time so-called successful indicators of this type, yet careful examination and experiment have shown them to be unreliable and defective.

It is only quite recently that a really reliable indicator, capable of working up to 2,500 revolutions per minute, has been evolved, although due credit must be given to the modified Hospitalier Charpentier indicator used by Prof. Callendar in his petrol engine experiments in 1904. In this case the indicator was very materially altered to conform to what has now been shown to be the correct principles; the modern high speed optical indicator is practically based upon the modified Hospitalier-Charpentier type of Prof. Callendar.

Most of the indicators designed for high speed work are characterized by the absence of the usual piston (on account of the inertia and friction effects), of the steam engine indicator, and by the employment of optical methods, in place of mechanical ones, for recording the diagrams.

The optical types of indicator will here be dealt with first.

The Perry Indicator.

One of the earliest forms of high speed indicator was that employed by Prof. Perry for showing large indicator diagrams of steam and gas engines to big audiences; a beam of light tracing out the diagram upon the walls or ceiling of the room.

The instrument consisted of a circular steel membrane clamped around its edges to a small cast-iron box provided with trunnions, one of the latter being hollow and communicating through a stuffing gland to the engine cylinder.

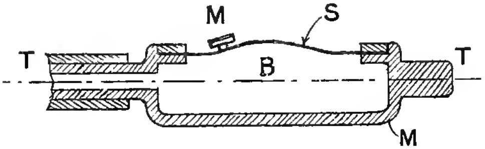

FIG. 115.—ARRANGEMENT OF PERRY INDICATOR.

The steel membrane S was shaped somewhat as shown in Fig. 115, and a small mirror M was fastened at a point of contrary flexure (or reversed curvature) to this membrane.

The piston pressure communicated with the space B and deflected the membrane, the deflections varying with the pressure, although not accurately proportional to same.

In order to obtain the piston displacement motion at right angles to the pressure motion the whole box M was partially rotated or rocked about the trunnions T, so that the beam of light's motion was proportional to the piston's displacement; thus a beam of light, incident upon mirror, was reflected to a suitable screen and traced out the indicator diagram upon same.

Although this arrangement is very simple, yet it is not suited to high speed requirements for one or two reasons. Firstly, the motion of the mirror is small, and cannot be magnified very much as the vibrations of the engine interfere with the motion of same.

Secondly, the scale of pressure is not uniform, being smaller at higher pressures.

The most convenient scale of pressures is a straight line through the origin of the pressure-deflection axes, in which case the area of the indicator diagram can be easily obtained by the usual area-measurement methods, and gives truly a measure of the work done.

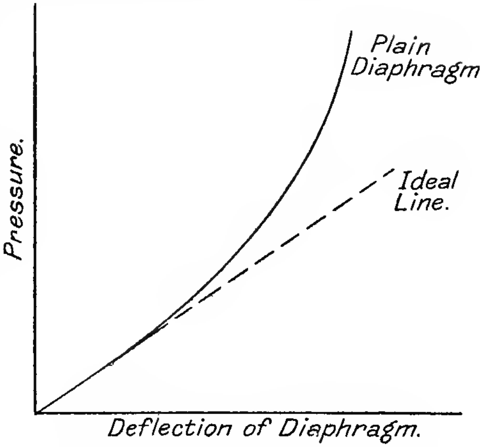

FIG. 116.—PRESSURE SCALE OF PLAIN DIAPHRAGM.

With the diaphragm employed upon this indicator the pressure-deflection line is curved, being approximately parabolic, as shown in Fig. 116.

The dotted line shows the correct line, or pressure scale, obtained with properly designed diaphragms and helical springs.

Thirdly it has been found that these diaphragms are unsuitable for internal combustion engines, as they will not withstand the rapid variations in pressure without fracturing.

The Hospitalier-Charpentier Indicator.

This instrument in principle consists of a spherical mirror which rests upon three points A, B and C, disposed in such a manner that the axes AB and AC are at right angles (Fig. 117), and the point A is fixed.

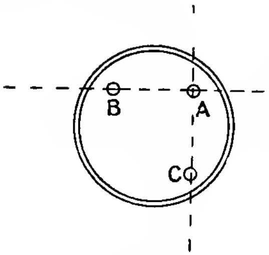

FIG. 117.

The point B is connected by means of a rod to the middle diaphragm, similar to that employed in the Perry indicator, and which has a motion nearly proportional to the pressure.

The point C is given a motion (that is, a partial rotation) proportional to the piston displacement, by means of the arrangement shown diagrammatically in Fig. 118, and consisting of a small lever with a fixed fulcrum at N, which is connected by a rod to a pinion wheel Q, eccentrically.

The wheel Q is driven by an equal pinion wheel P, which is in turn driven by suitable means off the engine shaft.

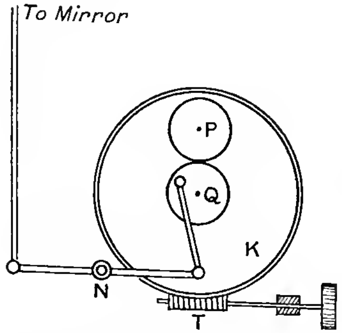

FIG. 118.—PHASE MECHANISM OF HOSPITALIER-CHARPENTIER MANOGRAPH.

The phase of the displacement motion of the mirror is adjusted by means of the tangent screw T, which acts upon the large worm wheel K, with its centre at the centre of Q.

This alters the position of P, and gives the necessary adjustment required to bring the spot of light from the mirror to the same position in its small displacement stroke that the piston is in its stroke

A flexible mechanism is employed to drive P, as its alignment changes with that of the driving mechanism; the usual driving gear supplied consists of long thin rods provided with Hookes's joints.

The optical arrangements of this indicator are shown diagrammatically in Fig. 119. The beam of light from the source R enters the light-tight box (or camera) through a tube O, and is reflected by the prism X on to the pivoted mirror M, previously described; from thence it is reflected to the ground glass screen S, which can be replaced by a photographic plate when required.

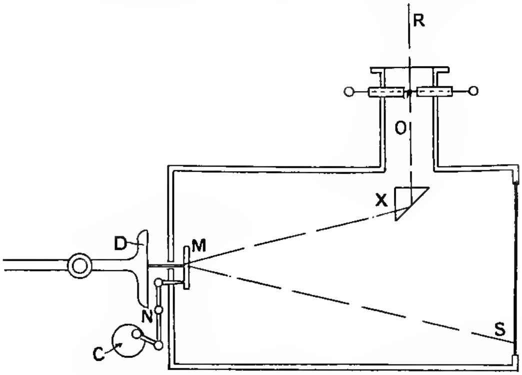

FIG. 119.—OPTICAL ARRANGEMENTS OF HOSPITALIER-CHARPENTIER MANOGRAPH.

The screen is placed at such a distance that the beam of light reflected from the mirror M is focused upon it.

The chief disadvantages of this indicator can be enumerated as follows :—

(1) The indicator is connected to the cylinder by means of a tube about 100 centimetres long, and with a bore of 3 millimetres, and these dimensions are found to give rise to what are known as "bore waves," which depreciate from the accuracy and value of the diagram.

(2) The flexible drive is quite unsuited to high speed work, owing to torsional deflections of the drive, which vary during the running, and so alter the phase of the indicator diagram.

(3) If the flexible drive be replaced by a solid shafting, a special phase-adjustment mechanism is required.

(4) With the single mirror receiving both pressure and displacement motions, it is found that the former motion affects the latter, whereas they should be quite unassociated as separate effects.

(5) The points A, B and C at which the mirror is pivoted consist of rounded rods in small recesses so that the points of contact, which should remain fixed in position, vary.

This can of course be obviated by employing pivoted points in larger conical recesses or holes.

(6) The pressure diaphragms do not give a uniform scale, and a further difficulty occurs, unless the discs are very tightly clamped, namely, that the pressure scale varies a good deal during a test or series of tests.

It has been found that it is possible to obtain different scales. as shown in Fig. 120, when the disc is very tightly, normally, and loosely clamped. The curves shown in the figure are approximate only, and are not intended to be to scale.

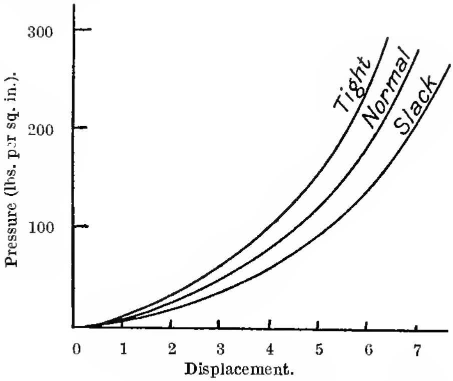

FIG. 120.—CALIBRATION CURVES FOR MANOGRAPH DIAPHRAGM.

Prof. Callendar employed a Hospitalier-Charpentier manograph in connexion with his researches upon the air-cooled petrol motor,* but modified his instrument in such a way as to obviate the disadvantages already mentioned.

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* "The Effect of Size upon the Thermal Efficiency of Motors," *Proc. Inst. Aut. Engineers,* 1906–7.

The long flexible copper tube for transmitting the pressure was found to cause a lag between the moment of pressure attainment in the engine and that recorded by the diaphragm, which at high speeds amounted to about 90 degrees of crank angle ; the amount of lag was found to vary with the speed of the engine.

In place of this long tube a steel tube of less than one-tenth in length was employed, and of more than three times its bore ; with this tube practically no lag was observed.

The long flexible shafting, which was found to give rise to varying piston phase errors, was replaced by a pair of short steel rods engaging through a bevel gear, so that the motion of the piston was accurately reproduced.

Other minor alterations were also made, in order to obtain satisfactory indicator diagrams at speeds up to 2,500 revolutions per minute.

The Schultz Indicator.

This instrument is in principle similar to the one previously described, and is shown diagrammatically in Figs. 121, 122 and 123.

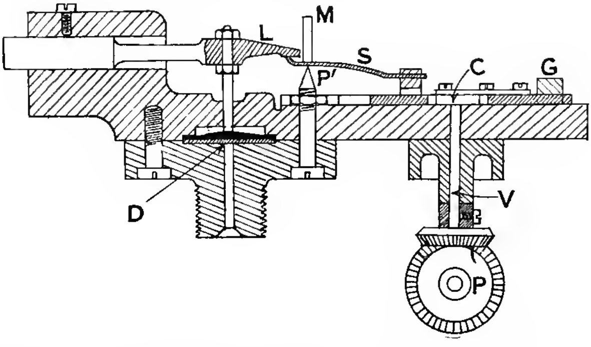


FIG. 121.—DIAGRAMMATIC ARRANGEMENT OF SCHULTZ INDICATOR.

The pressure recording arrangements consist of a compound diaphragm made up of a steel disk D above and a silver disk beneath it, on the side communicating with the engine ; this silver disc is to obviate the corrosive effects of the gases, the chief constituents of which are CO_2 and water vapour.

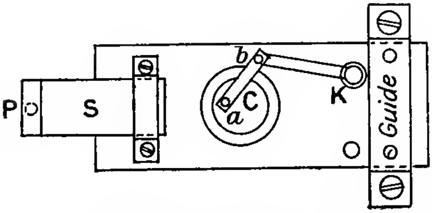


FIG. 122.—PISTON PHASE MECHANISM (SCHULTZ INDICATOR).

The pressure is taken partly upon the diaphragm D and partly by means of a cantilever spring L.

The free end of this cantilever is tapered down, and presses upon a knife edge attached at right angles to a concave mirror M, pivoted upon a point P. A broad steel band S presses upon the top surface of the base of the mirror, and serves to hold it in position.

The piston displacement motion is obtained through the bevel wheel P, attached to the engine shaft through suitable shafting and bevel gearing. This bevel P drives a similar bevel attached to the vertical shaft V, on the end of which is a disc C provided with an eccentric pin, which is connected by means of a small connecting rod *ab* (Fig. 122) to a lever, pivoted at K to a rectangular plate, which can be made to turn about the pivot P by means of the spring S.

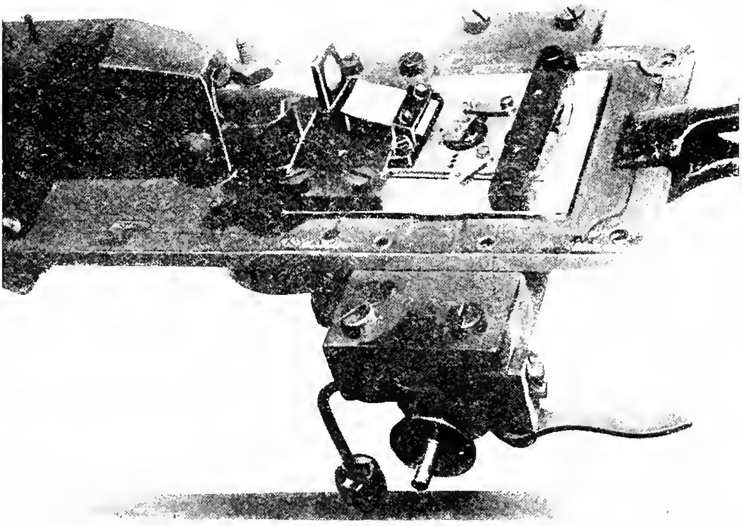


Fig. 123.—The Schultz Indicator.

Thus a rotation of the mirror M is obtained, in a direction at right angles to the pressure rotation, and which is proportional to the piston displacement; these two motions do not interfere appreciably with each other.

The whole instrument is encased in a light-tight box, and provided with suitable means of illumination, etc.

The means by which the "phase" of the displacement rotation is synchronized with that of the engine is interesting, and consists in altering the relative positions of one driving wheel with its driven wheel by an amount equivalent to half the pitch of the teeth at a time.

However, as there are only thirty-two teeth on the wheel, the minimum change of phrase which can be obtained is 6°, and this is equivalent to a fairly appreciable change in the area of the diagram.

The advantages of this form of indicator consist, firstly, in having a uniform pressure scale, due chiefly to the elasticity of the cantilever spring; secondly, in the non-interference of the pressure and displacement motions; and thirdly, in the compactness of the instrument.

The disadvantages of the indicator can be enumerated as follows:—

(1) There is no provision for cooling the diaphragm, which becomes very hot, although a water-jacketed communication tube is provided.

(2) The vibration of the disc is "damped" by allowing a slight leakage to occur through a hole under the disk, and also by providing frictional damping.

From indicator diagrams taken under different conditions of leakage, and non-leakage, it has been found that a considerable difference in area results by allowing leakage.

(3) The diaphragm is provided with a certain amount of clearance underneath, and this clearance, although in other respects an advantage, yet allows lubricating oil, etc., to accumulate, and this interferes with the pressure movements.

(4) The gearing for obtaining the piston phase motion consists of three sets of bevels (in the usual form), with long rods of small diameter.

The varying elasticity and torsional strains under the different torques, which occur at different parts of the working cycle, have the effect of varying the piston displacement's phase to an appreciable extent.

In addition, the varying velocity of the piston in its stroke causes the amount of backlash in the bevels to vary. This is in the nature of an inertia effect, and varies as the square of the velocity approximately.

The O.S. Manograph

An improved form of the Schultz indicator has been put upon the market recently under the above name.

The same form of mirror is retained, and as in the earlier instrument it is capable of rotation in two planes at right angles.

The apparatus is more compact, and the tube whereby the pressure diaphragm is connected to the engine is reduced in length, and further, is water-jacketed to prevent excessive temperature variations affecting the diaphragm.

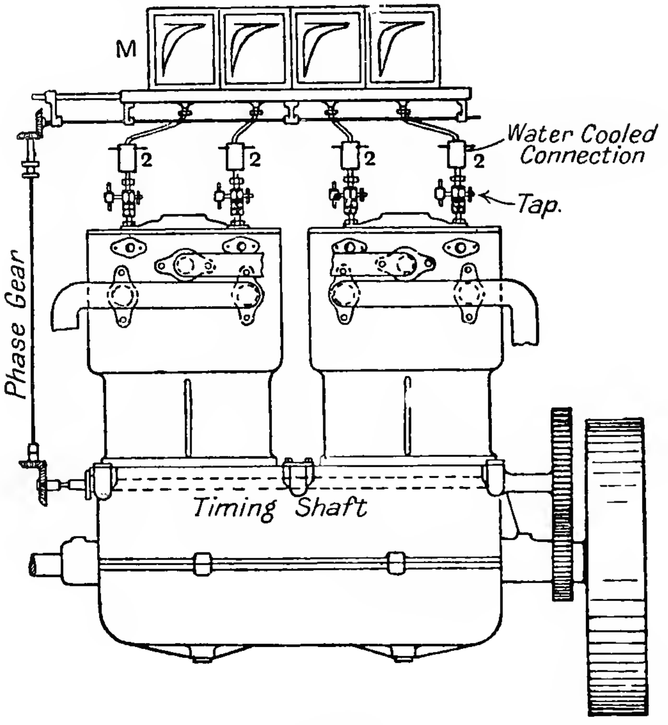


Fig. 124.—The O.S. Manograph fitted to a Four-Cylinder Petrol Engine.

As in the earlier form, the displacement motion is obtained through long shafting and bevel pinions. The source of illumination is a Nernst filament lamp.

With this manograph it is possible to obtain indicator diagrams from all four cylinders of a car engine at the same time, and to watch the diagrams of all cylinders at nearly the same moment, as shown in Fig. 124; this affords a useful means for observing the working of all the cylinders under different conditions of ignition, carburation and throttle opening.

The pressure scale for petrol engines is 3 mm. per kilogram per sq. centimetre, in this make of indicator.

The Watson Indicator.

This is probably the most successful form of indicator at present in use, and was expressly designed with a view to overcoming the inherent disadvantages in other forms of instrument. It has been employed in connexion with Dr. Watson's well-known researches upon petrol engines, and quite recently has been designed in a commercial form by Prof. Dalby.

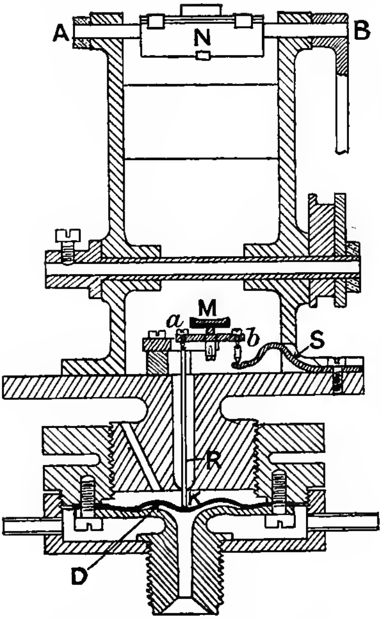


Fig. 125.—Sectional View of Watson Indicator.

The principle of the indicator is illustrated diagrammatically in Fig. 125, and photographically in Fig. 126.

The pressure is received upon a corrugated disc D, which is gilded upon its under side to minimize corrosion, and oxidized or cosletized upon its upper surface. This diaphragm is usually made of an alloy steel, such as silicon steel, in order to withstand the varying pressure effects, which tend to cause metal "fatigue," and is made corrugated in section, in order to obtain a uniform pressure scale; the diaphragm is clamped around its edges with a copper or asbestos washer to make the gas-tight joint. Resting upon the central trough of this diaphragm is a small rod R, passing through guides at K.

An enlarged view of the end of this rod is shown in Fig. 127. The upper end of the rod R is hollowed and engages a pivot *a* attached to one side of a light framing carrying the concave mirror M. The other pivot *b* rests upon a small rod, which in its turn rests upon a cantilever spring S.

The mirror frame *ab* is virtually a beam pivoted at its centre *c* (the pivots being underneath).

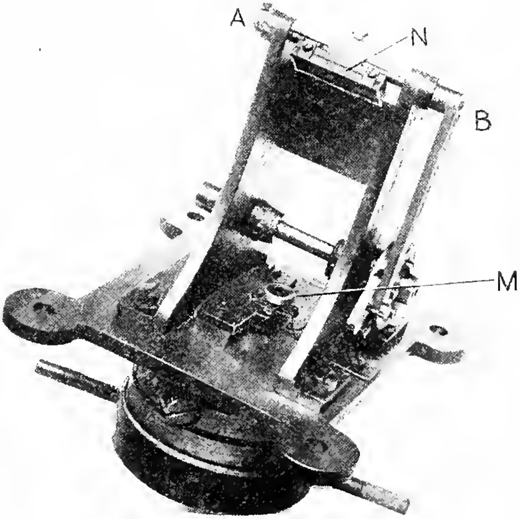


FIG. 126.—THE WATSON INDICATOR.

This arrangement allows the up-and-down motion of the rod R to be converted into an angular rotation of the mirror M, with very little constraint.

The piston phase is obtained by an entirely separate plane mirror N, receiving its motion from an eccentric, driven at engine speed.

The mirror N can rock about the axis AB in proportion to the piston displacement.

In order to vary the " phase " of the piston displacement mirror N, the eccentric is driven through an ingenious differential gear, Fig. 128, consisting of a pair of bevel wheels, E, F, with their respective axes in line, and connected by an intermediate bevel D.

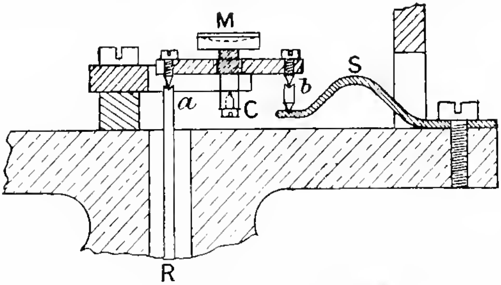


FIG. 127.—THE PRESSURE MOTION MECHANISM (WATSON INDICATOR).

The axis of D is carried upon a disc P, which can be rotated relative to a fixed disc Q about the axes of E and F, thus carrying the bevel D with it.

It will be seen that by rotating the bevel D about the common axis of the bevels E and F the phase of E (which is chain-driven off the engine shaft) can be altered relatively to that of F, which drives the eccentric, from which the piston displacement mirror derives its motion.

For large phase alterations, such as when changing over the three-way tap from one cylinder to the next, a series of holes are provided, and a spring-pin to engage in corresponding holes.

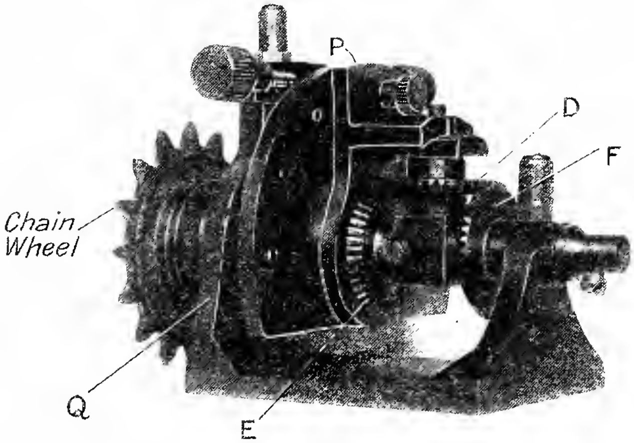


Fig. 128.—Phase Gear of Watson Indicator.

Thus, to alter the phase by 180°, the axis of D is rotated through an angle of 90° on the fixed disc Q.

For fine adjustments of phase, a pair of tangent screws are provided, which can be locked in position, and an exceedingly fine adjustment is rendered possible owing to the fineness of the pitch of these screws.

The optical arrangements are shown diagrammatically in Fig. 129. A beam of light from the crater of an arc lamp at A is focused by means of the lens B upon a plate C, provided with a hole (the size of which is variable), whence it diverges to the concave mirror M, and is from thence reflected on to mirror N, and finally reflected as a convergent beam to the screen placed at T.

The focus of the beam reflected from the mirrors M and N is upon the screen, so that an image of the hole at C appears as a bright spot of light upon the screen. The indicator diagram can be very readily seen upon the screen even in strong daylight, the motion of the image being so rapid that the diagram appears continuous.

The effects of variation of the control levers, etc., can be observed with ease in this type of indicator.

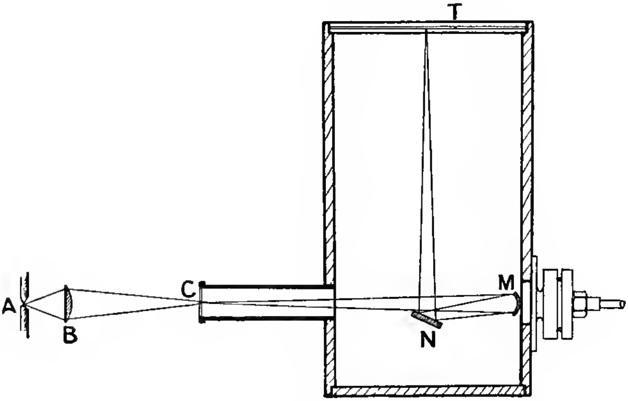


Fig. 129.—Optical Arrangements of the Watson Indicator.

The advantages of this type of indicator are as follows :—

(1) That the piston and pressure motions are quite independent of each other.
(2) The pressure scale is uniform.
(3) An exceedingly fine adjustment of the phase is provided.
(4) The clearance volume below the disc is small.
(5) The disc is water-cooled.
(6) The connecting tube to the three-way tap is short and free from "bore waves."

The Micro-Indicator.

This type of indicator is of the mechanical form, but is designed to obviate the difficulties previously experienced in applying the improved steam engine type of indicator to high speed petrol engines.

The principle of the instrument depends upon reducing the natural vibration period of the indicator to small dimensions, by means of a stiff helical spring, the period of vibration being about one-third that of the ordinary indicator.

The effect of having such a stiff spring is to reduce the pressure scale considerably.

In the instrument designed by Dr. Mader, shown illustrated in Fig. 130, the size of each of the diagrams taken is included in a rectangle of sides 2 millimetres, and about twenty-four diagrams can be traced out upon a smoked glass plate $1\frac{7}{8}$ inches long by $1\frac{1}{8}$ inches wide (or 48 by 28 millimetres).

A piston of one square centimetre area is employed for producing the pressure motion, the movement being controlled by the stiff external spring seen in the diagram.

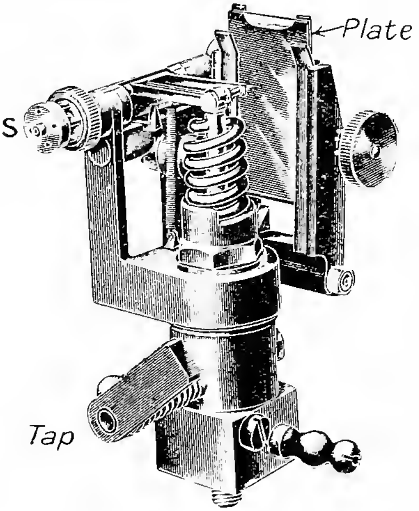


FIG. 130.—THE MICRO-INDICATOR.

Attached to the end of the piston rod is a fine pointer, which traces the microscopic indicator diagram upon a part of the smoked glass plate seen in the figure; after one diagram has been taken, the diagram slide is displaced, in the direction of piston's motion, by 5 millimetres, by means of a pawl, and is then ready for a second diagram.

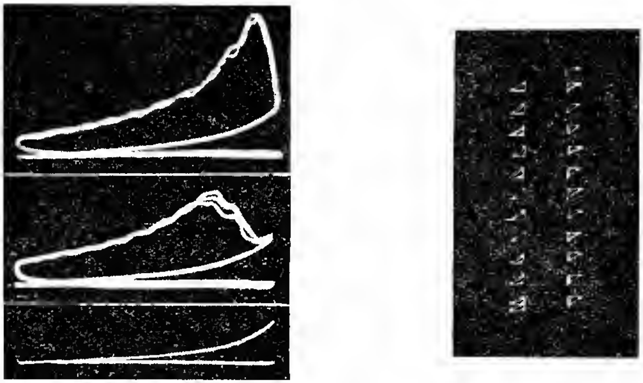
When twelve such diagrams have been taken, the glass slide is turned round, and is ready for another twelve. An example of the minute diagrams, and also the magnified diagram, is shown in Fig. 131.

The whole of the diagrams taken are fixed by immersion in Canada balsam, and when finished can be examined in a microscope, and copied in an optical tracing attachment, or enlarged photographically.

As regards the pressure recording arrangement, the spring, hollow piston rod and the piston are connected together in such a way that they can be changed together.

The instrument is usually screwed directly into the compression cock hole in the cylinder.

The piston displacement motion is obtained by means of a rocking lever attached to the shaft marked S in the illustration, and which is worked directly off a suitable crank or eccentric attached to a convenient portion of the engine shaft, the whole driving arrangement being fairly rigid.



Magnified. Actual Size.

FIG. 131.—EXAMPLES OF DIAGRAM TAKEN WITH MICRO INDICATOR.

The advantages of this indicator are :—

(1) Its compactness and convenience in manipulation.

(2) The short natural period of vibration.

(3) The lightness of the reciprocating parts.

The disadvantages of this type of indicator are :—

(1) That in the magnified diagrams the thickness of the pointer tracings is also magnified, and any fine pressure variations are masked. It is also difficult to obtain accurate area measurements as the lines are relatively thick.

(2) Friction of the piston, which is common to the piston form of indicator.

(3) The diagrams cannot be directly observed whilst the engine is running, and the effect of different adjustments accurately noted, as in the optical forms in which the actual diagram can be watched at the time of occurrence.

Explosion Recorders.

One of the chief functions of the ordinary indicator is to give the necessary information from which the work done during a few consecutive explosions, and therefore the horse power being exerted at the time can be deduced.

In calculating the horse power from indicator diagrams taken at intervals during a test, it is usually assumed that the cylinders are being fired regularly during the whole period of the test.

In the case of an engine running at, say, 2,000 revolutions per minute, two or three consecutive superposed indicator diagrams, as is usually obtained with high speed indicators, correspond with a period of running of about 0·2 second.

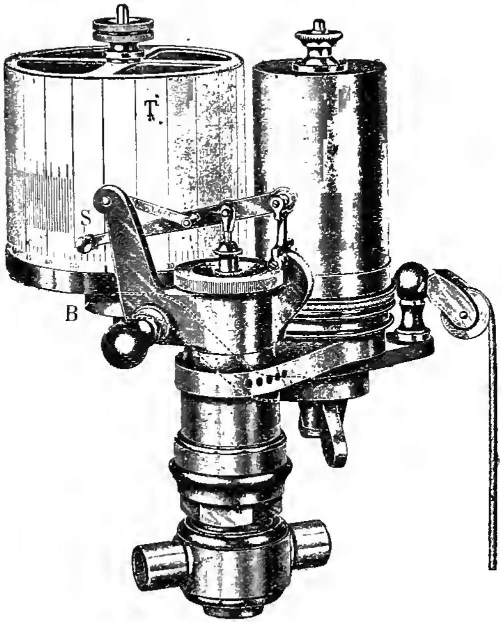


Fig. 132.—The Thompson Explosion Recorder, with Time Drum.

This period is usually very small compared with the actual time of the test, and it is therefore necessary to know whether during the non-indicated period the engine has been firing regularly.

In the case of multi-cylinder engines it is difficult to detect misfirings audibly, although such misfirings, if not too frequent, will not appreciably affect the horse power calculated from the indicator diagram.

In the case of engines controlled by a governor, and more especially a governor of the hit-and-miss type, it is necessary to know the actual number of working strokes.

Indicators are now made which give the total number of explosions and compressions during any given period of running.

The Mathot and Thompson indicators are designed for this purpose, the latter being illustrated in Fig. 132.

This instrument consists of an ordinary indicator piston, and straight line mechanism actuating a vertically moved pencil, which records a line, proportional to the pressure of the explosion, or when not firing, to the compression.

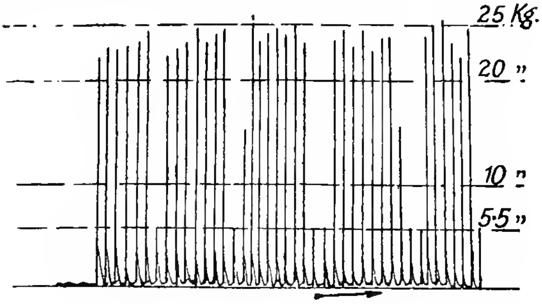


Fig. 133.—Example of Explosion Recorder Diagram.

The drum

remains practically stationary during the period of the pencil movement, and is rotated slowly by clockwork, so that a series of vertical lines are obtained as shown in Fig. 133.

The drum of the instrument illustrated rotates once in two minutes, the diameter of the drum being 4 inches.

It is possible to arrange for a continuous sheet of paper for prolonged tests, in place of a single sheet.

From the record of the pencil movements the number of misfires can at once be seen (for then compression only occurs) corresponding with the shorter lines.

The maximum pressures and their variation can also be noticed, and by running the drum faster a more detailed outline of the suction and compression can be obtained.

It must be remembered, in connexion with this type of explosion recorder, that the pressure readings are subject to the same errors as in the ordinary mechanical indicators.

The Okill Pressure Indicator.

This instrument is intended for the purpose of measuring the compression and the explosion pressures under running conditions.

It consists of a cylinder with an accurately fitting piston B (Figs. 134 and 135), the downward movement of which is prevented by a flange abutting against the upper end of the instrument's cylinder walls.

A calibrated helical spring of rectangular section opposes the upward motion of the piston, and it is the compression of this spring which is used to measure the compression or explosion pressure.

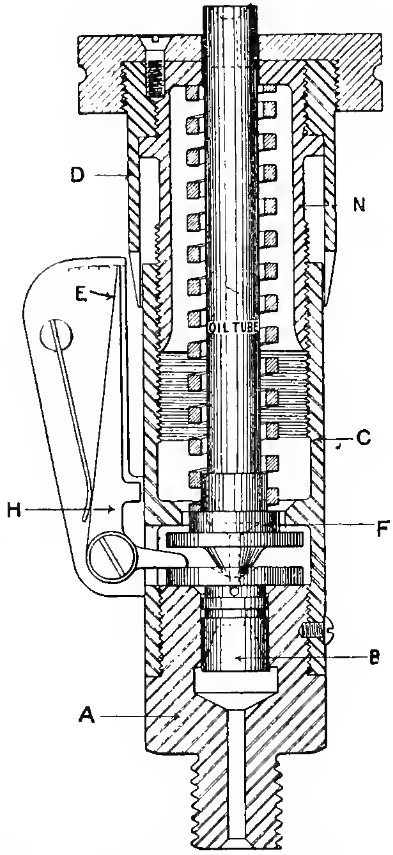


FIG. 134.—SECTIONAL VIEW OF OKILL PRESSURE INDICATOR.

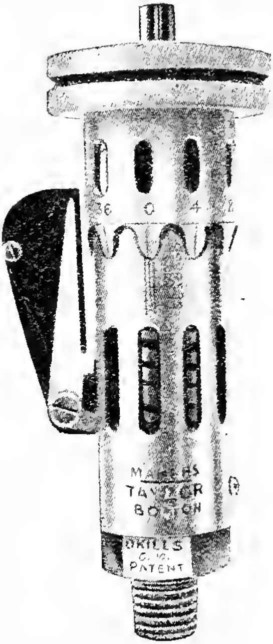


Fig. 135.—The Okill Pressure Indicator.

Knowing the exact load per unit axial compression of the spring, and the area of the small piston, the equivalent pressure per sq. inch can be estimated or obtained.

The upward pressure on the piston is balanced by the compression of the spring shown, the adjustment being made by rotating the milled nut with its threaded barrel N, and the spring-held pointer E serves as an indicator for this purpose.

The exterior of the main barrel is graduated with a scale reading, each division corresponding to 20 lbs. per sq. inch, and fractions of a division are obtained off the periphery of the milled nut barrel N.

The pressures can thus be read off to within 2 lbs. per sq. inch, the maximum reading obtainable with this instrument in the case of petrol engines being 400 lbs. per sq. inch.

In order to measure the explosion pressure in the cylinder the plug A is screwed into a convenient part of the combustion head, and the milled cap screwed down until the spring exerts a pressure in excess of the explosion pressure on the piston. The needle indicator E gradually ceases to vibrate at this stage. The cap is then gradually screwed back until the needle just commences to vibrate again, the mean position of non-vibration being the correct one, at which the reading is taken.

The compression pressure is measured in any engine cylinder by running the engine with this cylinder not firing and with open throttle (since the maximum compression pressure is only thus obtained).

This instrument is useful, not only for indicating the approximate compression and explosion pressures, but also for ascertaining whether all cylinders have the same compressions.

It can also be applied to gas, oil and steam engines for single pressure measurements.

For reliable readings the temperature of the spring should be kept low, as its elasticity diminishes rapidly at higher temperatures.

The Calibration of Indicators.

It is necessary to know with some degree of accuracy the scale of the pressure ordinates in all types of indicator.

In some forms, such as the Hopkinson type, this can be done by inverting the instrument and loading the piston directly with dead weights.

A very convenient method used in connexion with optical indicators is to employ a Crosby pressure gauge tester, similar to that shown illustrated in Fig. 136.

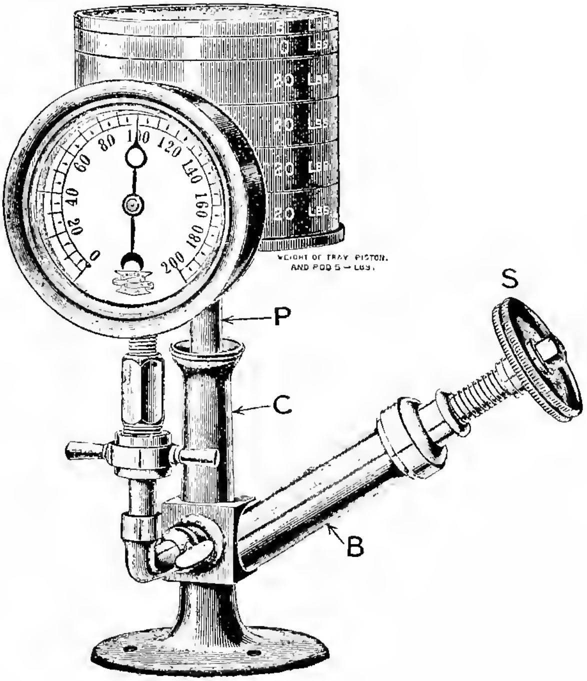


FIG. 136.—CROSBY GAUGE TESTER FOR CALIBRATING INDICATORS.

The piston or diaphragm space of the indicator is connected by means of copper tubing to the space usually occupied by the pressure gauge.

The tester consists of a cylinder C in which can slide a piston P of known area, ground and finished to be an accurate sliding fit in the cylinder.

The lower end of the cylinder communicates with the pressure gauge (or in our case with the indicator diaphragm) and with a barrel B containing a screw plunger S for varying the volume of the mineral oil in the tester itself, and for the purposes of adjustment.

The piston is loaded with known balanced weights, and the pressure per sq. inch is obtainable therefrom. It is essential in using this instrument to eliminate air bubbles, and means are provided for filling the whole of the interior with oil alone ; further, when making a calibration, the piston (with its balanced weights) should be rotated in order to avoid leakage and frictional or capilliary effects.

If the indicator is provided with a ground glass screen, a piece of tracing paper can be affixed thereto, and the position of the spot of light marked with a finely pointed pencil for each calibrated pressure.

The piston or diaphragm should be calibrated with ascending pressures by small steps to the maximum pressure, and then by descending steps to the lowest or atmospheric pressure, in order to obtain a check as to any hysteresis or viscosity pressure lag.

It is usual to take the mean of the ascending and descending readings.

All indicators of the diaphragm form should be calibrated before and directly after an engine test, as these diaphragms often fail by cracking, and the cracks are very difficult to discern visually.

The calibration of the diaphragm will reveal the presence of a crack, either by the erratic or inconsistent scale obtained, or by the presence of oil which has leaked through the crack at the higher pressures of calibration.

These methods of calibration are applied to the indicator piston or diaphragm when cold, and not under exactly the same conditions as when working ; the effect of temperature upon the elasticity and properties of the indicator diaphragm in this type of instrument are dealt with hereafter, but it may be here stated that unless the diaphragm be water-cooled efficiently the cold calibration will not always give reliable results.

The Requirements of High Speed Indicators.

In order that an indicator shall truly record the changes occurring within the cylinder of the engine, there are several conditions which it must fulfil, and also many precautions to be observed in its usage.

In the first place, it is absolutely essential that the ordinates of the indicator diagram obtained should accurately represent the pressure existing within the cylinder, and that the abscissae should truly represent the piston's phase.

In connexion with the former requirement, some reference has already been made concerning the methods employed for obtaining this result.

It has also been shown how the inertia, vibrating and friction effects of the steam engine form of indicator render the latter quite unsuitable for high speed work.

It is now almost universal to employ thin steel diaphragms in place of moving pistons ; this diaphragm should give a uniform pressure scale, and should be kept fairly cool by means of a water jacket. The reason for this is that the effect of temperature increase causes the elastic strength of the metal to alter and to thus yield different pressure scales at different temperatures, the pressure scale being more open with increased temperature, due to the reduction of elasticity of the metal.

Another factor connected with temperature is the cracking of the diaphragm, probably as a result of internal temperature stresses, for the diaphragm is generally tightly clamped around its edge.

It is further necessary to allow the air above the diaphragm space free ingress and egress.

One reliable and satisfactory form of diaphragm is that employed in Dr. Watson's indicator, which is corrugated in form, the corrugations being concentric with the centre of the disc, and stamped out of sheet metal in a press.

The material of the disc is an alloy steel, silicon steel being found satisfactory ; in some cases the discs are hardened and tempered.

It is necessary also to guard against the strongly corrosive effects of the exhaust gases upon the exposed side of the discs.

Gilding by electrical deposition has been found to be practically the only solution to the difficulty.

The Piston Motion.

In petrol engines the length of the connecting rod is fairly small, and the ratio of the length of this rod to that of the crank radius is also small, so that the piston's motion is not a simple harmonic one.

It is therefore necessary that the mechanism which produces the piston displacement motion of the indicator should allow for the connecting rod to crank ratio of the actual engine upon which it is used ; otherwise if the indicator motion is simple harmonic, the effect of the obliquity of the connecting rod will be to increase the area of the diagram.

In all indicators means should be provided for altering the connecting rod to crank ratio of the indicator to suit the engine being indicated.

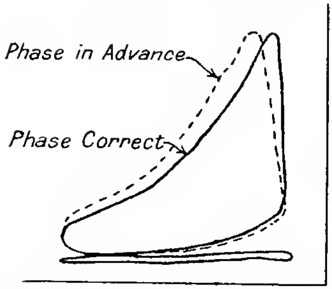


FIG. 137.—EFFECT UPON DIAGRAM OF PHASE ERRORS.

In the Watson indicator this can be done by altering the radius at which the connecting rod (actuating the rocking plane mirror) is attached to the eccentric.

It is also very important that the actual phase of the indicator's motion should be exactly the same as that of the engine; many indicator diagrams are rendered quite worthless on account of wrong phase adjustment.

The phase of the indicator is very apt to change with different speeds, and so it should be set for the speed at which the tests are to be run.

It has been shown that the effect of altering the phase of an optical indicator, initially correct, by an amount corresponding with 6 degrees crank angle, was to alter the mean effective pressure from 89 to 100 lbs. per sq. inch.

If the phase of the indicator is in advance of that of the engine, the area of the diagram obtained will be greater than the true area.

This is shown in Fig. 137, in which the indicator diagram for the correct phase is shown in full, and for an advance in the phase by the dotted lines; it is therefore essential that the spot of light

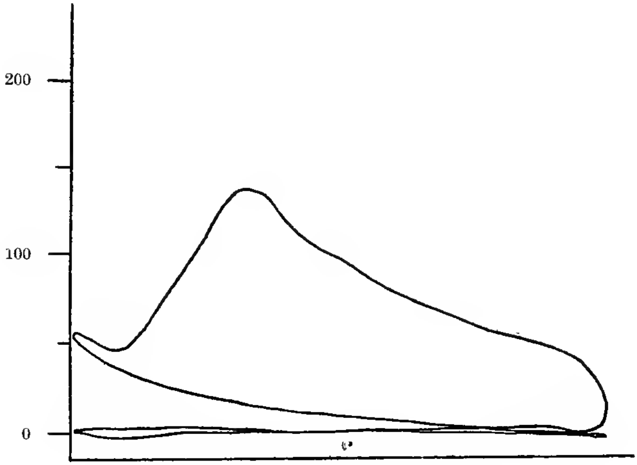


FIG. 138.—PHASE ADVANCED.

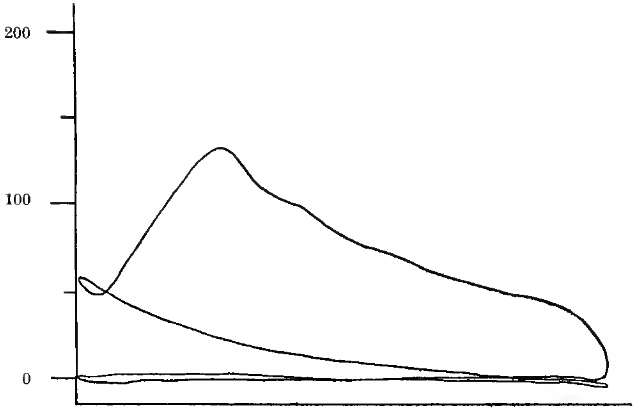


FIG. 139.—PHASE LAGGING.

in an optical indicator should be at the same part of its travel that the engine piston is in its stroke.

In the case of slow speed engines it is usually sufficiently accurate to run the engine and to switch off the ignition in the cylinder under consideration, and then to adjust the phase of the indicator until the expansion line returns upon the compression line.

In the case of engines running above speeds of 700 revolutions per minute, the best method is to run the engine at the desired speed with the spark retarded, and to adjust the phase until the end of the compression line is continuous.

The reason for this will be obvious from the diagrams given in Figs. 138 and 139.

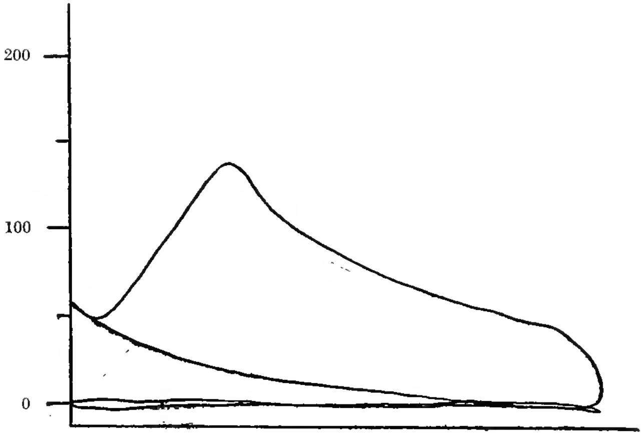


FIG. 140.—PHASE CORRECT.

In Fig. 138 the phase of the indicator is in advance of the piston, whereas in Fig. 139 it lags behind, thus forming a closed loop at the end of the compression stroke.

In Fig. 140 the phase is correct and the compression line returns upon itself, until the retarded ignition occurs.

In conclusion it may be repeated that it is very important to set the phase correctly before each test, and for the given speed required during the test.

Influence of the Length of Connexion Tube.

The dimensions of the connecting tube between the piston or diaphragm and the engine cylinder have a marked influence upon the accuracy and the shape of the indicator diagram in the case of petrol and high speed explosion engines.

As an example of the effect of the length of the connecting tube, a concrete example will be given for the case of an engine running at 1,200 revolutions per minute, the period of the stroke being ·025 seconds.

If the length of the connecting tube between the indicator and engine be $3\frac{1}{2}$ inches, a sound wave, which travels at 1,100 feet per second, will take ·0025 second to traverse this distance, that is, it will occupy a period of time corresponding with one-hundredth of the stroke of the engine.

Now the maximum rate at which pressure can be transmitted through a gas is the velocity of sound, so that the effect of a connecting tube of $3\frac{1}{2}$ inches in length is to cause a lag of $\frac{1}{100}$ of the stroke.

This retardation will vary with the engine speed and with the length of the connexion tube.

The effect of a long tube upon the indicator diagram is illustrated in Figs. 141 and 142, the former representing a normal diagram taken under light load with a short tube between the engine and indicator; in the second case a long tube was employed.

In both cases the spot of light on the screen was initially set by the dead centre for the correct phase.

The effect of the longer tube was to set up a "bore wave," due

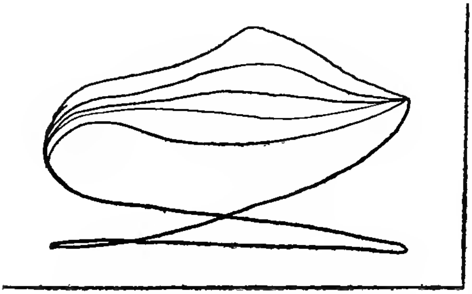


FIG. 141.—SHORT TUBE DIAGRAM.

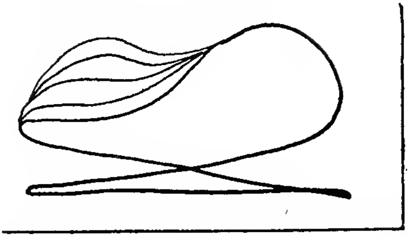


FIG. 142.—LONG TUBE DIAGRAM SHOWING "BORE WAVE" EFFECT.

to the pressure wave travelling up the tube and causing a heaping up of pressure at the end of the stroke.

This effect can be corrected, for any given speed, by altering the phase of indicator by an equal amount to the pressure retardation, and a normal type of diagram obtained.

The explosion often sets up resonance waves in the indicator, resulting in a wavy expansion line; this effect can be remedied by inserting wire gauze in the connexion tube to damp out the vibrations.

Dr. Watson has employed open spirals of copper wire inserted in the connecting tube to damp out this resonance effect in some tests upon a sleeve valve engine, in which the shortest tube employable was 4 inches long. The violent expansion curve waves set up by resonance effects were effectively damped out by the copper spiral of the correct length, without any throttling of the gases in the tube.

Fig. 143 represents an indicator diagram taken from the above-

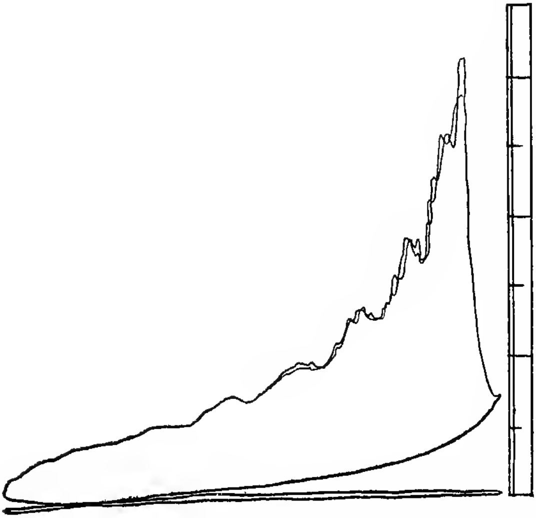


FIG. 143.—RESONANCE EFFECTS SHOWN BY INDICATOR DIAGRAM.

mentioned engine before the insertion of the spiral, the engine speed being 1,000 revolutions per minute.

The resonance waves during the explosion stroke are marked, and a series of waves of smaller amplitude and greater frequency are also apparent upon the expansion curve ; these may be due to vibrations of the steel diaphragm itself.

Effect of Bore of Connexion Tube.

If the bore of the connexion tube be too small, the gases therein will be throttled and the pressure considerably damped down.

The indicator diagram taken with too small a bore of tube will exhibit a much smaller pressure rise than the actual rise, and consequently a smaller area of diagram will result.

On the other hand, too large a bore will also yield unsatisfactory diagrams, on account of frictional and viscosity effects due to the increased motion of the gases in the tube ; further, a large volume of hot gases sweeping in and out of the indicator is very liable to damage the indicator.

The capacity of the indicator itself must not be appreciable compared with that of the combustion chamber space, or the true compression pressure of the engine will be altered.

The correct diameter of the bore of connexion tube in the case of optical diaphragm indicators is from 4 to 4·5 millimetres.

In order to examine for the presence of too large a bore the indicator diagrams should be taken—

(*a*) With the indicator cock full open, and
(*b*) With the cock partially closed in one or two different positions.

If the amplitude of the waves on the expansion line is diminished in the case (*b*), this is an indication of viscosity effects in case (*a*).

Cylinder Waves.

The effect of pressure waves, resonance, viscosity, and diaphragm vibrations have now been considered ; there is, however, another source of vibration yielding a wavy expansion line, which is due to vibratory effects occurring within the cylinder itself.

This is more particularly the case when the charge is fired from some extreme point in the combustion head, and the indicator is placed near this point.

In the case of a T-headed engine investigated by Dr. Watson, it was found that if the charge was ignited over each of the two valves in turn, cylinder waves were set up, but that if the charge was simultaneously ignited over the two valves the effects were neutralized and no cylinder waves occurred.

An example of cylinder wave effects, as shown by the indicator diagram, is given in Fig. 144, the waves or ripples being quite marked; the diagram was obtained when the charge was ignited over the exhaust in a T-headed engine running at 1,100 revolutions per minute.

The effect of simultaneous ignitions from two suitably disposed sparking plugs in this case was found to result in an increase of indicated horse power at 1,100 revolutions of 13 per cent., and at 1,600 revolutions of 12·9 per cent.

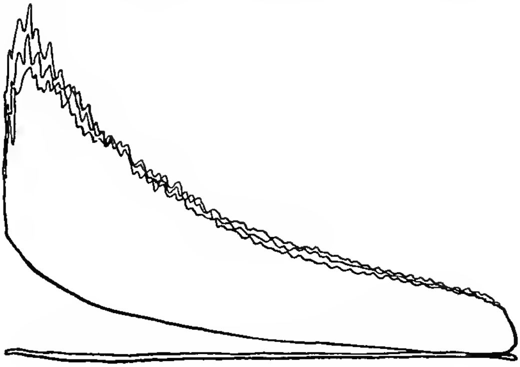


FIG. 144.—EFFECT OF CYLINDER WAVES.

Uses of the Indicator Diagram.

From the preceding remarks the value of the indicator diagram in revealing the pressure changes occurring within the engine cylinder will have been noticed by the observant reader; in order, however, to more fully appreciate the importance and value attached to a reliable indicator, it is proposed to here consider briefly some of the uses to which it can be put.

Dealing with the mechanical side first, one is able to examine the working of a particular engine, and from the indicator diagram records obtained to know definitely in which direction improvement may be made.

Thus by noting the values of the exhaust and inlet pressures as obtained by use of the indicator, the effect of different valve timing can be fairly accurately predicted.

This effect can be studied with precision by means of suction and exhaust diagrams taken with weak diaphragms, so as to obtain a magnified scale of pressure.

In the case of two-stroke engines using crankcase compression, the magnified pressure scale diagrams taken from the crankcase afford useful means for studying the action of this type of engine, and to assist in improvements of charge efficiency, timing operations, scavenging, etc.

An example of the ordinary and of the crankcase diagram is given in Fig. 145, the positive portion of the crankcase diagram corresponding to the expansion stroke of the piston. The fall in pressure in the crankcase when the inlet port is opened at A, and the maximum charge pressure just before A, will be noticed.

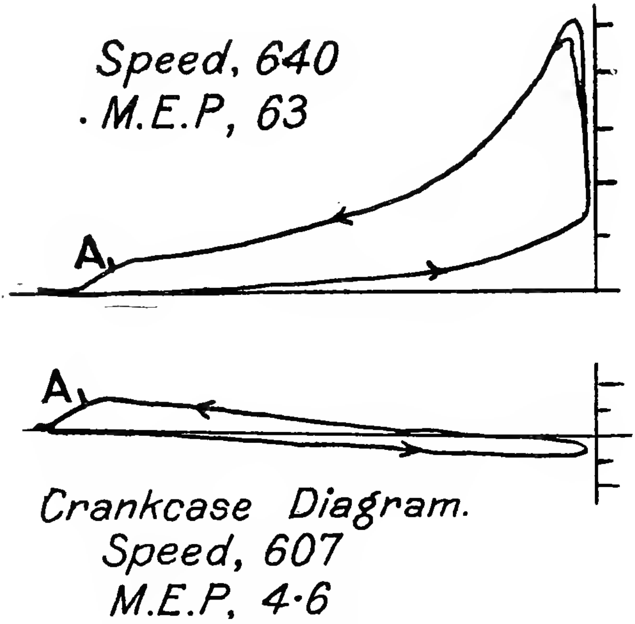


Fig. 145.—Two-Cycle Engine Diagrams.

The subjects of carburettor and silencer design can be studied in a rational manner by means of indicator diagrams taken from the inlet and exhaust manifolds; examples of this use of the indicator diagram have already been given.

It is evidently a much more commendable practice to endeavour to observe exactly what is happening within the cylinder by means of the indicator than by blindly groping about by trial and error methods, which are not only expensive but occupy much valuable time in most cases.

Many of the leading motor manufacturers are already adopting the indicator as a more accurate alternative to road and bench tests for experimental work in connexion with engine design and tuning, etc.

In the optical types of indicator, the diagrams can be watched upon the ground glass screen; it is fascinating and instructive to watch the effect upon the shape of the diagram of changes in mixture strength, throttle opening, and ignition timing, etc.

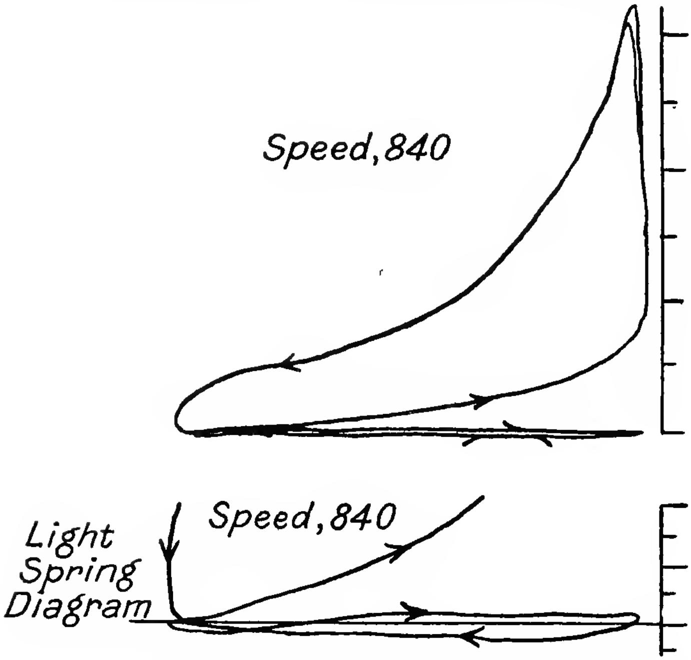


FIG. 146.—NORMAL AND LIGHT SPRING DIAGRAMS FOR PETROL ENGINES.

In considering the design of the valves and valve ports, the most suitable carburettor, the timing operations and cam contour, etc., the light spring diagrams of the suction and exhaust strokes have proved very useful.

An example of an indicator diagram, and a light spring diagram of the suction and exhaust strokes is reproduced in Fig. 146, from one of Dr. Watson's tests.

For the purpose of obtaining these light spring diagrams, a thinner diaphragm than usual is employed, above which is provided a stop to prevent the effects of the explosion pressure permanently distorting the diaphragm. It is generally, however, not possible to subject the disc to more than a limited number of explosions without fracturing it.

Use of the Indicator for Studying Ignition Phenomena.

Nearly all of the hitherto obscure ignition phenomena have now been successfully explained, with the help of the indicator diagram. Thus it was shown several years ago by Dr. Watson * that the nature of the spark itself has no influence at all upon the pressure developed during the subsequent explosion—that is to say, it does not matter whether the spark is a "fat" or weak one, provided that it occurs at the right moment.

The magneto was considered superior to the old make-and-break or trembler system, as it was thought that the spark obtained was hotter.

Experiments were made, employing the two methods of ignition, and indicator diagrams taken conclusively proved the fallacy of the existing belief.

The superiority of the magneto over the trembler coil was also shown by means of the indicator diagram, although it was also practically evident that the former was the better ignition means.

It was found that the loss of power when using the trembler coil was due to faulty timing, for if the spark advance was correctly set for one cylinder it would occur either too early or too late in the other cylinders.

The commutator was the errant factor, and by employing a special design of commutator to give correct timing for all the cylinders the engine yielded just as much power with the coil as with the magneto.

This is more particularly the case with engines having the sparking plug over the inlet valve, and with as pure a mixture as possible at the moment of firing.

Where the plug is situated elsewhere than in the inlet valve vicinity it was found that the hotter magneto spark was a distinct advantage, and more especially so when weaker mixtures were employed.

The use of displaced or purposely set "out-of-phase" indicator diagrams has been found of great assistance in studying the rate of pressure rise in the cylinder.

Thus if the spot of light (in the indicator) is set so that it is moving at its maximum speed during the time of burning of the charge

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* Vide *The Autocar*, July 25, 1908, p. 124.

the pressure curve will be exaggerated, and rendered better for the purpose of examination.

The pressure rise, on ignition, with correctly adjusted phase always occurs during the period in which the piston and the horizontal phase motion of the spot of light is travelling most slowly, so that the pressure rise curve is practically a vertical line.

By altering the phase until the spot of light is about in the centre of its path when the piston is at its top, the pressure rise curve is exaggerated.

Fig. 147 illustrates the diagram obtained by purposely altering the phase of the spot of light by about 60° of crank angle, in the case of a sleeve valve engine. The true diagram corresponding with and taken at the same time as this displaced diagram is given in Fig. 143.

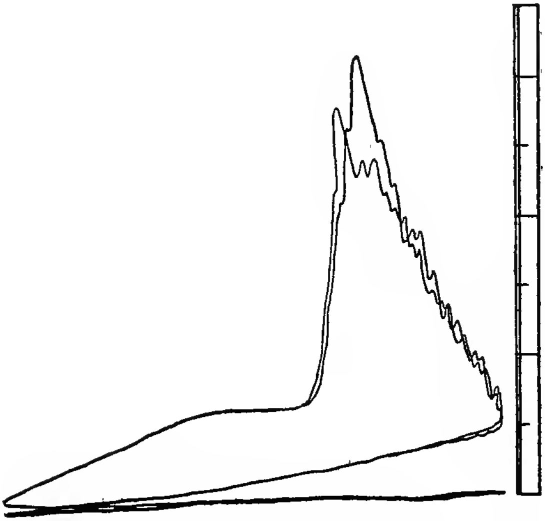

FIG. 147.—INDICATOR DIAGRAM WITH DISPLACED PHASE.

By means of these displaced diagrams the duration of the spark and the pressure rise were studied in the cases of the trembler coil and the magneto respectively. It was found that the period of the magneto spark was very much longer than that of the trembler coil; this is due to the period of vibration of the trembler blade being small compared with that of the engine revolutions.

The average period of vibration of the trembler coil employed on car engines is about 200 (breaks per second).

If the sector which makes contact on the commutator is 20°, that is, equivalent to 40° crank angle, then in the case of an engine revolving at 1,200 revolutions per minute the time of contact on the commutator is ·0056 second. As the period of the trembler coil blade is also ·005 second, it is evident that at and above this speed only one break will occur, and therefore only one spark (instead of the usual succession at lower speeds); at higher engine speeds the trembler will therefore be unreliable.

For starting up and slow running purposes the trembler is therefore most suited.

One other instance of the utility of the indicator diagram will be here given in connexion with the influence of duplex sparking upon power developed in the case of normal compression engines.

Dr. Watson has shown that in the case of a T-headed engine, in which the valves are in pockets upon opposite sides of the cylinder, that by arranging to ignite the charge simultaneously by means of plugs placed over each pocket, that a marked increase in power occurs.

Thus at 1,100 revolutions the I.H.P., with a single sparking plug over the inlet valve was 18·4, while with two plugs placed as above described the I.H.P. was 20·8, an increase in power of 13 per cent. At 1,400 revolutions the I.H.P., with two sparks, was 29·2, as compared with 26·0 obtained with the single spark.

The beneficial effect is due no doubt to the more rapid ignition obtained, for the time of attainment of maximum pressure from the moment of the spark occurring is a function of the length of the maximum path of travel of the explosion wave.

By employing two plugs, suitably placed, the maximum path of travel of the explosion or combustion wave is shortened.

It has been pointed out by more than one experimenter upon high speed engines that very little advantage accrues from the use of two plugs in such engines.

This is, no doubt, due to the high compressions employed in such engines, for the time of combustion is also a function of the compression pressure, the higher the latter is the more rapid does the ignition period become, and the hotter the charge before ignition.

With very high compression pressures the explosions approach the state of detonations. There is no doubt, however, that with compression pressures below about 95 lbs. per sq. inch duplex sparking is a marked advantage, this latter becoming more enhanced at higher speeds.

CHAPTER VI

THE MECHANICS OF THE HIGH SPEED INTERNAL COMBUSTION ENGINE

General Considerations.

IN the previous chapters the theoretical, physical, and experimental aspects of the subject of the development of power from the explosive mixture have been considered; thus the manner in which the chemical energy of the fuel is utilized, the peculiar changes accompanying its transformation into mechanical energy, the thermal and pressure changes, and their determination have been dealt with at some length.

It is proposed in the present chapter to study the factors involved in the practical utilization of the heat energy of the fuel, and to consider in some detail the mechanics of the working parts of the internal combustion engine.

The function of the mechanism of any engine is to provide a means whereby the heat energy of the fuel can be converted into useful mechanical work, and in a manner convenient for the employment of this work.

The investigations of the present Chapter will be confined to the subject of the reciprocating engine (that is, to the standard petrol engine mechanism), as the internal combustion turbine has not yet attained and does not appear likely, for some time, to gain sufficient foothold in the commercial world to warrant present discussion.

Various types of engine have from time to time been invented for which some advantage has been claimed over the ordinary connecting rod-and-crank system, but it is not proposed to deal with these, although their consideration will become greatly simplified if the principles of the ordinary reciprocating engine are fully understood.

The chief object of the mechanism of the petrol engine is to convert the relative reciprocating motion of the piston with the cylinder into a rotary one, this being the most convenient motion for adapting to various purposes.

Piston Position and Crank Angle.

It is of importance to the designer to know the exact piston position corresponding with any given crank angle.

This can be obtained either mathematically or graphically.

Dealing with the former method first, let P represent the position of the piston (which replaces the crosshead and piston rod of steam engine practice) along its stroke AB ($= 2r$), Fig. 148.

Let r = radius of crank pin circle
and l = connecting rod length
θ = angle of crank with line of stroke
x = displacement AP of piston.

We then have $x = AP = A^1E$,
where E is the point in which an arc of radius $PC = l$, from P as centre, cuts the line of stroke.

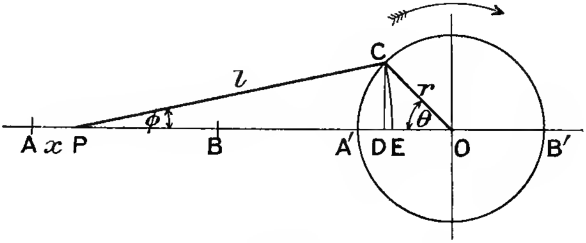

FIG. 148.

$$\begin{aligned} \text{And } A^1E &= A^1D + DE \\ &= (A^1O - DO) + DE \\ &= (r - r\cos\theta) + l - \sqrt{(l^2 - r^2\sin^2\theta)} \\ &= r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}\right\} \end{aligned}$$

If n = the ratio $\dfrac{\text{connecting rod}}{\text{crank}} = \dfrac{l}{r}$

$$\text{Then } x = r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{\sin^2\theta}{n^2}}\right\}$$

This may be written—

$$x = r(1 - \cos\theta) + l\left\{1 - \left(1 - \tfrac{1}{2}\frac{\sin^2\theta}{n^2}\right)\right\} \text{approximately}$$

$$\underline{= r(1 - \cos\theta) + \frac{r}{4n}(1 - \cos 2\theta)}$$

The above expression is very nearly true, since the other terms of the series on the right-hand side become very small.*

If the motion of the piston were a simple harmonic one [S.H.M.], that is, if the connecting rod were of infinite length, then the piston displacement would be given by

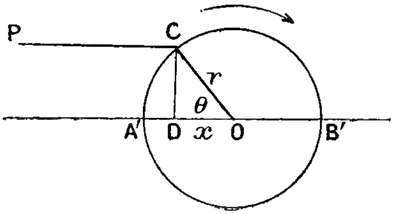

FIG. 149.

$$x = r(1 - \cos\theta), \text{ Fig. } 149,$$

so that the effect of a finite length of connecting rod is to introduce the second term, and the resulting piston displacement may be regarded as due to an initial S.H.M. due to the revolution of a point of radius r and a superimposed S.H.M. due to the revolution of a point in a circle of radius $\frac{r}{4n}$, and with an angular velocity double that of the original one.

Further, the "error" in the piston's position introduced, due to the obliquity of the connecting rod, is represented by the term

$$y = \frac{r}{4n}(1 - \cos 2\theta).$$

This is a maximum when $\frac{dy}{d\theta} = 0$, that is when $\theta = 90°$ or $270°$, and its value then becomes $\frac{2r}{4n}$, that is $\frac{\text{the piston stroke}}{4n}$

It is a minimum when $\theta = 0$ or $180°$, and is therefore zero at the two ends of the piston stroke.

Piston Displacement for Offset Cylinder.

In several examples of petrol engines the line of stroke of the piston is displaced at right angles to its normal position, for certain reasons which will be dealt with later; it is important in dealing with questions of balance, etc., to be able to determine the piston position at any crank angle.

In the diagram, Fig. 150, let b = the amount of offset, and for convenience, the distance PA $= x$ the piston position, and the distance OP $= a$.

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* For an exact expression for the piston position, see page 316.

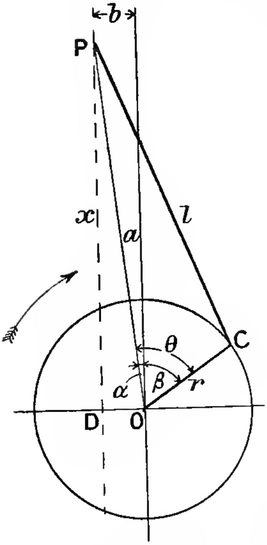

FIG. 150.—THE OFFSET CYLINDER.

With the notation given in the diagram, and considering the triangle POC, we have

$$l^2 = a^2 + r^2 - 2\ ar \cos \theta \quad . \quad . \quad (1)$$

$$\text{Also } \theta = \alpha + \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{and } \cos \alpha = \frac{x}{a} \text{ and } \sin \alpha = \frac{b}{a} \quad . \quad . \quad . \quad (3)$$

Further from (2) $\cos\ \theta = \cos\ (\alpha + \beta)$

$$= \cos\ \alpha\ \cos\ \beta - \sin\ \alpha \sin\ \beta$$

$$= \frac{x}{a} \cos\ \beta - \frac{b}{a} \sin\ \beta$$

Hence (1) becomes

$$l^2 = a^2 + r^2 - 2\ ar\left\{\frac{x}{a} \cos\ \beta - \frac{b}{a} \sin\ \beta\right\}$$

Also since $a^2 = x^2 + b^2$, this expression simplifies down to $x^2 - 2r \cos \beta . x - (l^2 + b^2 + r^2 + 2br \sin \beta) = 0$, a quadratic in x.

The solution of this equation by the usual algebraic rules is :—

$$x = r \cos \beta \pm \sqrt{r^2 \cos^2 \beta + l^2 - b^2 - r^2 - 2rb \sin \beta}$$

It will be evident from the figure that the negative root is inadmissible, and so, in terms of the cosine only, we get, since $\sin^2 \beta = 1 - \cos^2 \beta$

$$x = r \cos \beta + \sqrt{l^2 - b^2 - r^2 + r^2 \cos^2 \beta - 2br(\sqrt{1 - \cos^2 \beta})}$$

The second expression under the root sign can be expressed as a series for

$$(1 - \cos^2 \beta)^{\frac{1}{2}} = 1 - \frac{\cos^2 \beta}{2} + \frac{\cos^4 \beta}{4} - \frac{\cos^6 \beta}{8} + \text{etc.}$$

and we have

$$x = r \cos \beta + \sqrt{(l^2 - b^2 - 2br - r^2) + (r^2 + br) \cos^2 \beta + \frac{br}{4} \cos^4 \beta}$$

$$+ \text{ etc.}$$

This expression can be further simplified by neglecting higher powers of $\cos \beta$, and expanding out as a series the expression under the surd sign, and we have,
after substituting $\mathrm{A} = l^2 - b^2 - 2br - r^2$
and $\mathrm{B} = r(b + r)$

$$x = r \cos \beta + \mathrm{A}^{\frac{1}{2}} + \frac{\mathrm{A}^{\frac{5}{2}}}{2\mathrm{B}^2} \cos^2 \beta + \frac{\mathrm{A}^{\frac{9}{2}}}{2\mathrm{B}^4} \cos^4 \beta + \text{etc.}$$

This expression represents the position of the piston for any given crank angle, β very approximately, and can be employed for differentiation purposes, in connexion with the velocity and acceleration.

Graphical Method for Piston Displacement.

If the connecting rod be regarded as of infinite length, it is a simple matter to obtain the position of the piston corresponding to any crank angle.

Thus if the crank be in the position OC (see Fig. 151), and a perpendicular be dropped from C to the line of stroke at D, then A^1D represents the distance of piston from the outer end of its stroke.

For $A^1D = r(1 - \cos\theta)$.

From this it is evident that the piston displacement plotted against crank angle will give a curve of sines.

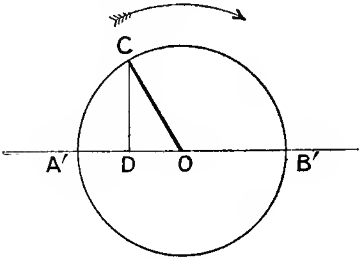

FIG. 151.

Next, consider the connecting rod of finite length PC (Fig. 148).

The position P of the piston for any crank angle θ is obtained by sweeping an arc of radius $CP = l$ from C as centre, the corresponding piston position being where this arc cuts the line of stroke.

If it is required to find the piston position corresponding with each position of the crank, this operation is repeated for each crank position.

Müller's Circles.

A somewhat more convenient method of obtaining in an easy manner the piston displacement for any crank position, due to Müller, is as follows:—

Let ABE be the line of stroke (Fig. 152) and Oc the crank radius, cP being the connecting rod.

With centre O, and radius $OI(= r)$ describe the circle IcI
,, ,, O ,, $OA(= l + r)$,, ,, X
,, ,, O ,, $OB(= l - r)$,, ,, Y

Now with radius l and centre I describe the circle Z.

The piston displacement for any crank angle Oa is given by the intercept ab on the crank arm, between the circles X and Z.

In order to prove this, join bI, bP and Ic respectively. Then

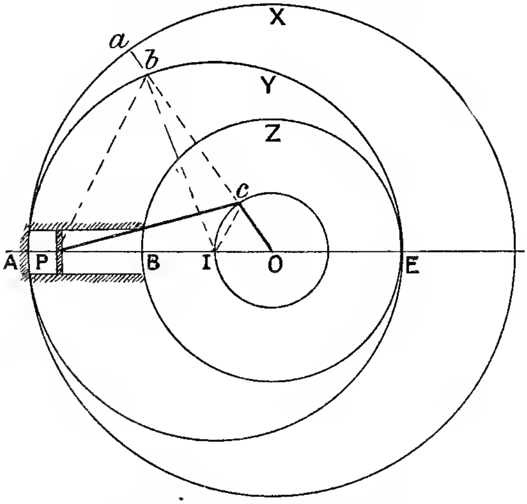

FIG. 152.—MÜLLER'S CIRCLES FOR PISTON DISPLACEMENT.

in the triangles Icb and IcP we have the sides bI and cP both equal to l, and cI is common to both, and

$$\angle bcI = \angle PIc.$$

So that the triangles are equal in all respects

and $cb = PI$

Also $ca = IA = l$.

Hence $ab = AP$ the piston displacement.

Curves of Piston Displacement.

It is often convenient, in the case of any given engine, to be able to quickly find the position of the piston for a given crank angle, and vice-versa.

The polar curve is very suitable for this purpose.

The principle of the use of this curve is that the intercept of the crank arm on the curve represents the distance of the piston from its mid-position.

The curves are constructed graphically by first finding the corresponding point E, to the piston's position, and then intercepting the distance OE upon the crank arm, thus obtaining a point R.

The locus of R for all crank angles gives the polar curve.

An example of a polar displacement curve is shown in Fig. 153, and it will be evident that the intercept of the crank OR represents the corresponding piston position from the mid-point of its stroke, or CR, its distance from the extreme point of its stroke.

For an infinite connecting rod this polar curve becomes a pair of circles of diameter equal to the crank radius, and with their centres on the line of stroke, and the greater the obliquity of the connecting rod, the more distorted do these initially circular curves become.

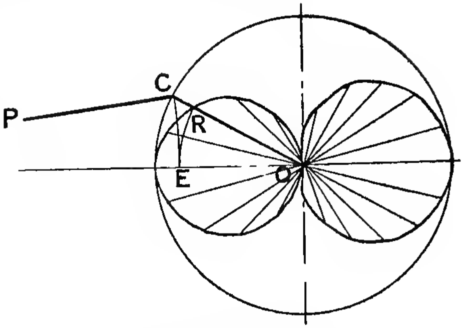

FIG. 153.—POLAR CURVES OF PISTON DISPLACEMENT.

Another method of representing conveniently the piston displacement for different crank positions is to plot upon a crank angle base the distances of the piston from the centre of its stroke, as ordinates.

Piston Velocity.

It is required to obtain an expression for the velocity of the piston at any moment in terms of the crank angle stroke and connecting rod length.

Dealing with the case where the connecting rod is very long, so that the motion of the piston follows a simple harmonic law, we have, with the notation in the diagram, Fig. 149—

The piston displacement from the centre $x = r \cos \theta$.

The velocity $V_p = \frac{dx}{dt} = r \sin \theta \frac{d\theta}{dt}$

But $\frac{d\theta}{dt}$ is the rate of change of the angle θ, that is, its angular velocity, and the linear velocity of the crank pin $C = r\frac{d\theta}{dt} = V_c$

$$\text{Hence } V_p = V_c \sin \theta$$

So that the velocity of the point P varies according to the law of sines, and is a maximum when $\theta = 90°$; it is then equal to the velocity of the piston, and the velocity is a minimum at the two extreme ends of its path of travel, and is then zero.

Effect of Connecting Rod Obliquity.

Consider next, the case for a finite length of connecting rod l. Referring again to Fig. 148, we have already obtained an expression for the position of the piston in terms of the crank angle, and in order to obtain the velocity it is necessary to differentiate this expression in regard to the time.

Thus we have given $x = r(1 - \cos\theta) + l\left\{1 - \frac{1}{l}\sqrt{l^2 - r^2\sin^2\theta}\right)$

Then $\mathrm{V}_p = \frac{dx}{dt} = r\sin\theta\frac{d\theta}{dt} + \frac{r^2.2\sin\theta.\cos\theta}{2\sqrt{l^2 - r^2\sin^2\theta}}.\frac{d\theta}{dt}$

Writing $\mathrm{V}_c = r\frac{d\theta}{dt}$ and simplifying we have for the velocity of the piston:

$$\mathrm{V}_p = \mathrm{V}_c\sin\theta\left\{1 + \frac{r\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right\}$$

It will be seen that the effect of the obliquity of the connecting rod is to introduce the second term

$$\frac{r}{2}\mathrm{V}_c\frac{\sin 2\theta}{\sqrt{l^2 - r^2\sin^2\theta}}$$

The velocity is a maximum at a position which can be determined from the above expression by substituting actual numerical values, or by differentiating the expression.

Thus by differentiating the expression for V_p we obtain

$$\frac{d\mathrm{V}_p}{dt} = \frac{\mathrm{V}_c}{r}\left\{\cos\theta + \frac{rl^2\cos 2\theta + r^3\sin^4\theta}{(l^2 - r^2\sin^2\theta^{\frac{3}{2}}}\right\}$$

and by equating to zero, and simplifying, the expression for the values of the crank angle at which the velocity is a maximum is obtained, and we then have

$$\sin^6\theta - \frac{l^2}{r^2}\sin^4\theta - \frac{l^4}{r^4}\sin^2\theta + \frac{l^4}{r^4} = 0$$

The solution of this cubic equation gives the values of the crank angle for maximum velocity.

As an example of the application of this expression to practical purposes, the following values for connecting rod to crank ratios occurring in practice have been calculated.

Ratio $\frac{l}{r}$	Crank Angle (in degrees) from top dead centre at which the velocity of the piston is a maximum.
3	73·18
4	76·71
5	79·11
6	80·60

From these results it will be evident that the greater the obliquity of the connecting rod, the earlier in the stroke is the velocity of the piston a maximum; and the longer the connecting rod in relation to the crank, the nearer to its mid-position does the maximum velocity of the piston occur—that is, the motion of the piston approaches a simple harmonic one.

In an exactly similar manner the velocity at any point of the stroke of the piston having an offset or " desaxe " cylinder can be obtained.

It is only necessary to differentiate the expression for the piston displacement in order to arrive at an expression for the piston's velocity at any crank angle.

The Graphical Method for Piston Velocity.

With the notation employed in Fig. 154, let V_p represent the piston's velocity at any point P, and V_c the uniform crank pin linear velocity.

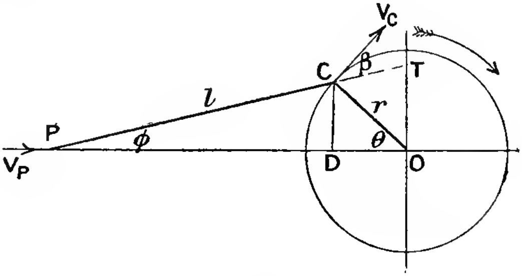

FIG. 154.

It will be evident that at the crank pin C its direction of motion will be tangential to the crank pin circle.

Further, the resolute of the piston's velocity along the connecting rod must be equal to the resolute of the crank pin's velocity along this rod, since it is a rigid mass.

We thus have $V_p \cos \varphi = V_c \cos \beta$

$$\text{or } \frac{V_p}{V_c} = \frac{\cos \beta}{\cos \varphi} = \frac{\sin \text{OCT}}{\sin \text{OTC}} = \frac{\text{OT}}{\text{OC}} = \frac{\sin(\varphi + \theta)}{\cos \varphi}$$

Hence the intercept OT of the connecting rod upon the vertical diameter of the crank pin circle at once gives a measure of the piston velocity for this position of the crank (the radius of the crank pin circle being proportional to the uniform crank pin velocity).

It will be evident from this that the piston velocity is a maximum when the angle OCT is 90° on either side of the line of stroke, and is equal in value to $\frac{V_c}{\sin OTQ}$

This will be seen to occur before the crank is perpendicular to the line of stroke, as was shown previously mathematically.

Curves of Velocity.

If the method of intercepts be employed to obtain the piston velocity, relative to the crank pin, and if further the intercept OT be marked off upon the crank arm, for all positions of the crank, then the locus of all such points on the crank arm will give a polar curve of piston velocity.

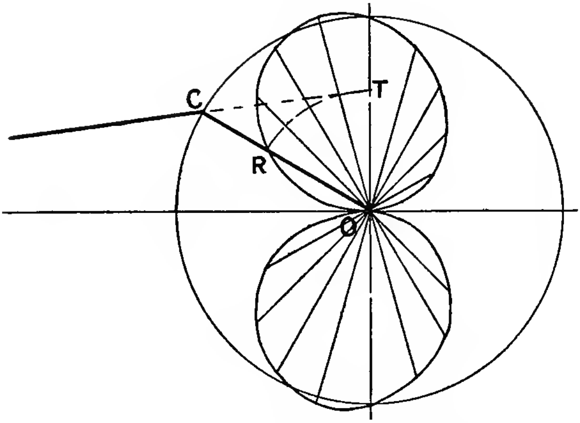

Fig. 155.—Polar Curves of Piston Velocity.

The method of construction is indicated and an example of the curves for a connecting rod crank ratio of 4 given in Fig. 155.

To obtain the velocity of the piston for any crank angle from these curves it is only necessary to measure the length of the crank pin intercept OR, then, if OA represents the crank pin velocity to scale, OR will to a similar scale represent the piston's velocity.

If the connecting rod is very long the two polar curves will approach true circles having diameters equal to the crank pin radius, and their centres upon a diameter of the crank pin circle perpendicular to the line of stroke.

A general method of construction of the velocity curve upon a piston stroke base, due to Unwin, is given in Fig. 156, and further the polar velocity curve has been described from this, and is also given in the same figure.

In this figure O*d* represents the crank length *r*, and *dc* the connecting rod *l*.

Produce O*d* to *f*, making *df* equal to the crank pin velocity V_c. Draw *fg* parallel to the connecting rod to cut the perpendicular from *c* in *g*.

Then *cg* represents the piston velocity when the piston is at *c*.

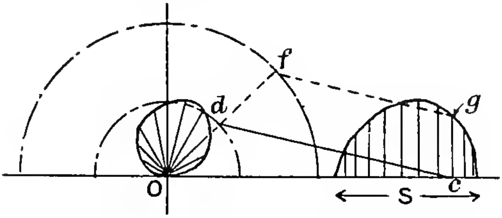

FIG. 156.—CURVES OF PISTON VELOCITY.

For all positions of the crank points such as *g* can be determined, and a curve of piston velocity upon the piston stroke base is obtained.

When the connecting rod is very long, this curve approaches an ellipse.

The Angular Velocity of the Connecting Rod.

It is required to obtain an expression for the angular motion of the connecting rod about the gudgeon pin, as this at once gives a measure of the rubbing velocity at the bearing surfaces of the latter.

Using the diagram of Fig. 154, we have

$$l \sin \varphi = r \sin \theta = \mathrm{CD}$$

$$\text{or } l \cos \varphi . d\varphi = r \cos \theta . d\theta$$

$$\text{So that } \frac{d\varphi}{d\theta} = \frac{r \cos \theta}{l \cos \varphi} = \frac{\mathrm{DO}}{\mathrm{DP}} = \frac{\mathrm{CT}}{\mathrm{CP}}$$

Hence the ratio of the connecting rod to crank pin angular velocities is given by the segments into which the crank pin C divides the line PT.

It follows from this that the angular velocity of the connecting rod will be zero when CT = O, that is, when the crank is perpendicular to the line of centres.

The angular motion of the rod will be a maximum when $CT = r$, that is, when the crank is on the dead centres, and its value will then be

$$W_R = W_c.\frac{r}{l} = \frac{W_c}{n}$$

where W_R and W_c represent the angular velocities of the connecting rod and crank respectively.

Rubbing Velocity of Gudgeon and Crank Pins.

The actual maximum rubbing velocity of the small end upon the gudgeon pin is then given by

$$V_G = W_R.a = a.\frac{W_c}{n}$$

where V_G is the rubbing velocity, a the radius of the gudgeon pin, and W_c the angular velocity of the crank.

Expressed in a more convenient form, in terms of the revolutions per minute N of the crank pin, we have $V_G = \frac{a}{n}.2\pi N$

Similarly the crank pin rubbing velocity at any moment is V_N = angular velocity of OCP × b, where b = radius of crank pin.

We thus have: Angular velocity of AP relative to PC given by

$$\frac{d(\varphi + \theta)}{d\theta} = \frac{PC + CT}{CT} = \frac{PT}{CT}$$

Then rubbing velocity of crank pin $= b.\frac{PT}{CT}W_c$

This is a maximum when $CT = r$, that is, when the crank is in the dead centre position.

Expressed in terms of the revolutions per minute, the maximum rubbing velocity of crank pin $= b.\frac{l + r}{r}.2\pi N$.

If the lengths be expressed in feet, the rubbing velocity will be in feet per minute.

Mean Piston Velocity.

If the crank pin velocity be V_c and the crank pin radius r, then the crank pin moves through a distance $2\pi r$, whilst the piston moves through $4r$ (twice the stroke).

Hence the mean piston velocity $V_{p1} = \frac{4r}{2\pi r}.V_c = \frac{2}{\pi}V_c$

On the polar curves this mean velocity can be represented by a circle of radius $\frac{2}{\pi}V_c$ drawn with O as centre, and in the case of linear curves by a straight line of height $\frac{2}{\pi}V_c$

Piston Acceleration.

As in the method adopted for the treatment of piston velocity, the mathematical side will be first considered for the case of the acceleration, and the graphical methods afterwards.

Referring again to the diagram shown in Fig. 148, the piston displacement is given by—

$$x = r(1 - \cos\theta) + l\left\{1 - \frac{\sqrt{l^2 - r^2\sin^2\theta}}{l^2}\right\}$$

By differentiating this expression with respect to the time, we have piston velocity at $\mathrm{P} = V_p = \frac{dx}{dt}$

$$= V_c \sin\theta\left\{1 + \frac{r\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right\} \text{as before.}$$

Differentiating again, we obtain the piston acceleration at the position P, that is

$$A_p = \frac{dV_p}{dt} = \frac{d^2x}{dt^2} = \frac{V_c{}^2}{r}\left\{\cos\theta + \frac{rl^2\cos^2\theta + r^3\sin^4\theta}{(l^2 - r^2\sin^2\theta)^{\frac{3}{2}}}\right\}$$

From this expression it will be evident that the piston acceleration when the crank is on the in-centre (and the piston at the outer end of its stroke) will be given by substituting $\theta = 0$.

And we have acceleration at in-centre $= \frac{V^2}{r}\left(1 + \frac{r}{l}\right)$

and for the out-centre since $\theta = 180°$

we have acceleration at out-centre $= \frac{V^2}{r}\left(1 - \frac{r}{l}\right)$

that is, a retardation or negative acceleration.

Or if the revolutions per minute of the crank shaft N be given, we have

$$\text{Acceleration at in-centre} = \frac{4\pi^2 N^2 r}{3600}\left(1 + \frac{r}{l}\right)\text{feet per sec.}^2$$

$$\text{and acceleration at out-centre} = \frac{4\pi^2 N^2 r}{3600}\left(1 - \frac{r}{l}\right)\text{feet per sec.}^2$$

The acceleration at any other position of the piston can be determined from the expression given, but usually, for design and similar purposes, it is only required to know the values at the two ends, and the value of the crank angle for which the acceleration is zero.

This occurs when the velocity is a maximum, and it has already been shown that by equating the expression for the acceleration to zero a cubic equation $\sin^6\theta - \frac{l^2}{r^2}\sin^4\theta - \frac{l^4}{r^4}\sin^2\theta + \frac{l^4}{r^4} = 0$ is obtained, and the solution of this equation gives the values of θ for zero acceleration.

It can also be shown that the distance of the piston from the outer end of its stroke, when no acceleration occurs, is given by the expression

$$d = \frac{s}{2}\left(1 + n - \sqrt{1 + n^2}\right)$$

Also, when the crank is at right angles to the line of centres, by substituting $\theta = 90°$ in the acceleration expression we get

$$\text{Acceleration } \theta_{90°} = \frac{v^2}{\sqrt{l^2 - r^2}}$$

These expressions are convenient for constructing the acceleration diagram.

Corresponding values for θ, when the crank is at right angles to the connecting rod, that is to say, values of the crank angle when the velocity of the piston is a maximum, are given upon page 268.

If the motion of the piston were simple harmonic (that is, if the connecting rod were infinite), then the acceleration at any position of the crank would be given by

$$A_p = \frac{d^2x}{dt^2} = \frac{V_c^2}{r} \cos\theta$$

so that in the expression for the acceleration of a finite length connecting rod the second term represents the "error" introduced by the obliquity of the connecting rod.

The Graphical Method for Piston Acceleration.

If the velocity of the piston be known at different positions of the stroke, the corresponding values for the acceleration can be deduced, since acceleration is the time rate of change of velocity.

The general method of obtaining the acceleration will first be considered.

In the case of the polar curve of piston velocity, consider a number of radii vector drawn at equal time intervals, that is, imagine the crank pin circle to be divided up into a number of equal parts (since the crank pin velocity is uniform).

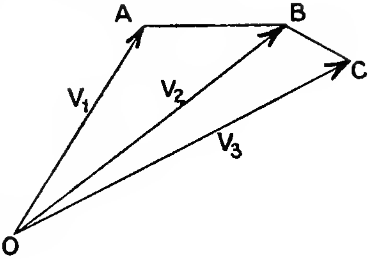

FIG. 157.

Then consider three consecutive radii vector drawn in this manner. Let OA, OB, and OC represent the velocities for these positions of the crank (that is the intercepts upon the polar velocity curve).

If now V_1, V_2 and V_3 represent the magnitudes of these respective velocities, we have—

$$\left.\begin{array}{l}\text{Average acceleration between}\\ \text{crank positions OA and OB}\end{array}\right\} = \frac{V_2 - V_1}{\delta t} \qquad \delta t = \text{the time interval}$$

$$\text{and between OB and OC} = \frac{V_3 - V_2}{\delta t}$$

Now in the limit when the time interval is exceedingly small the points A, B and C will be very close together. The acceleration at the point B will be represented by the tangent to the curve at that point.

Similarly, in the case of the linear velocity curve upon a crank angle base, the ordinates at equal intervals will represent equal time intervals, and the difference in height of consecutive ordinates will represent the average acceleration over the intermediate time interval.

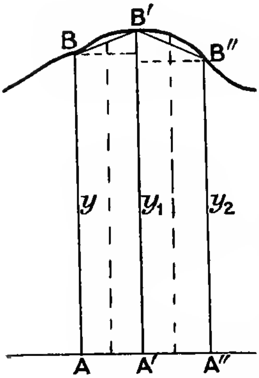

FIG. 158.

Thus, referring to Fig. 158, if the time intervals be δt, then the average acceleration between A and A^1 will be represented by $\left(\frac{y_1 - y}{\delta t}\right)$, and by erecting an ordinate midway between A and A_1, proportional $y_1 - y$, a curve of acceleration upon a crank angle base is obtained. Alternatively, if a series of tangents are drawn to the velocity curve at the points B, B′, B″, etc., the values of the tangents of the angles made with the base line AA″ are proportional to the accelerations at these points, If the base line represents piston displacement, it should be divided into equal time interval positions, the divisions, of course, being unequal in this case.

The above general method for obtaining the acceleration graphically can be applied to the velocity curves of any mechanism, but it is only approximate in practice, since it is difficult to draw the tangents at a point accurately, and the time intervals, necessarily, have to be finite.

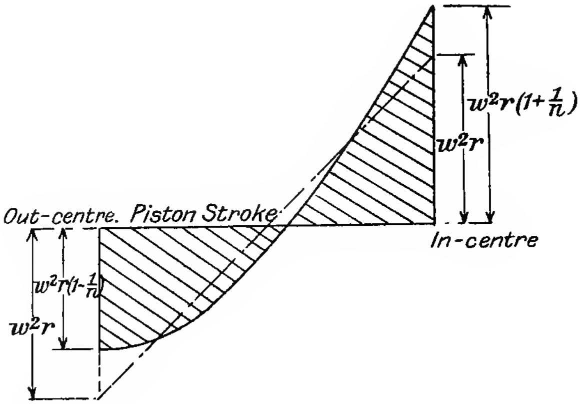

Fig. 159.—Curve of Piston Acceleration on Stroke Base.

The usual method adopted, in the case of the engine mechanism, is to plot the acceleration of the piston upon a piston-stroke base.

The values of the acceleration can be calculated at the two extreme ends of the stroke, and the position of no acceleration obtained by methods already considered.

By calculating values of the acceleration from the general expression for one or two other intermediate positions, sufficient points are obtained to enable the curve of acceleration to be drawn fairly accurately.

An example of a piston acceleration curve is given in Fig. 159, and it will be observed that the acceleration is a maximum at the beginning of the downward stroke of the piston (that is, towards the crank shaft), and gradually diminishes to zero before half of the stroke is accomplished, after which a negative acceleration (or retardation) occurs of increasing magnitude, finally attaining a maximum value at the inner end of the stroke, The dotted line shows the corresponding acceleration curve for an infinite connecting rod.

The greater the obliquity of the connecting rod, the greater will be the value of the initial maximum acceleration, and the difference between the maximum positive and negative accelerations.

It should be mentioned that it is the initial maximum value of the piston acceleration which determines the magnitude of the inertia stresses due to the piston's weight, etc., to which the moving parts are subjected.

Klein's Piston Acceleration Method.

A graphic method for finding the acceleration of the piston for any position is shown in Fig. 160.

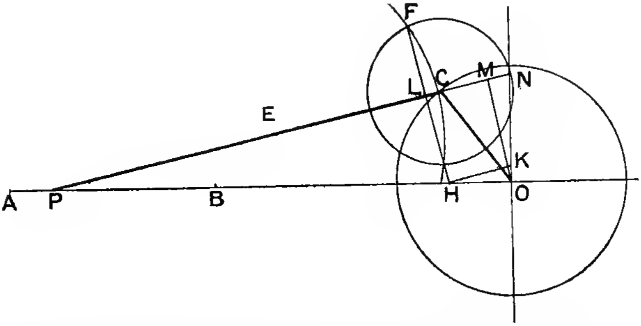

Fig. 160.—Klein's Construction for Piston Acceleration.

The connecting rod PC cuts the vertical diameter in N. With C as centre and CN as radius, a circle is described.

Bisect the connecting rod in E and sweep out the arc FG to cut the smaller circle at these points, the radius of the arc being EC.

Join FG, and if necessary, produce it to cut the line of stroke in H.

$$\left.\begin{array}{c}\text{Then the acceleration of the piston}\\ \text{at P}\end{array}\right\} = \text{HO} \times \left\{\begin{array}{c}\text{angular velo-}\\ \text{city of crank}\end{array}\right\}^2$$

The proof of this method is arrived at by dropping a perpendicular OM on CN and drawing HK parallel to LM.

It has previously been shown that if V_c be the crank pin velocity, and ω_R and ω_c the angular velocities of the connecting rod and crank, respectively,

$$\text{Then } \omega_R = \omega_c . \frac{CN}{PC}$$

The acceleration of C along CO is $\omega_c{}^2CO$, and the resolute of this acceleration along the connecting rod is $\omega_c{}^2CM$.

The acceleration of C along the connecting rod, due to the component of the connecting rod's angular acceleration, is $\omega_R{}^2CP$.

$$\text{Now } \omega_R{}^2CP = \frac{\omega_R{}^2.CN^2}{PC} = \frac{\omega_R{}^2CF^2}{PC} = \omega_R{}^2LC$$

The total acceleration of the piston along the connecting rod is made up of the resolute of the crank's acceleration in its direction, and of its own acceleration in the direction of its length.

Hence acceleration of piston along PC $= \omega_R{}^2 (LC + CM)$

$= \omega_R{}^2.LM = \omega_R{}^2HK$

Resolving this acceleration along the line of stroke PO we finally obtain the acceleration of the piston in its direction of motion as

$$\omega_R{}^2HK.\frac{HO}{HK} = \omega_R{}^2HO.$$

Inertia of Reciprocating Parts.

It has already been shown that the piston experiences an acceleration during the earlier period of its inward movement, and a retardation towards the latter end of the stroke.

In order to accelerate the mass of the piston, a certain force is required.

Thus if M be the mass of the piston in pounds and f the acceleration in feet per second, then the force F required to accelerate it is given by

$$F = \frac{Mf}{g},$$ where g is the acceleration due to gravity [$= 32{\cdot}12$ feet per sec.]

Now it is possible to calculate f for any position of the piston, and therefore the value of F corresponding can be ascertained.

Evidently since M and g are constant, F will vary directly as f, and the diagram of piston acceleration is also, to a suitable scale, a diagram of piston accelerating force F.

Hence the inertia force at the beginning of the down-stroke (that is, for the in-centre crank position) is given by

$$F_i = \frac{M\omega^2r}{g}\left(1 + \frac{1}{n}\right), \text{ where } n = \frac{l}{r}$$

and for the out-centre—

$$F_o = \frac{M\omega^2 r}{g}\left(1 - \frac{1}{n}\right)$$

This may be expressed in terms of the engine revolutions per minute by writing—

$${}_iF_o = \frac{M}{g} \cdot \frac{4\pi^2 N^2}{3600} . r\left(1 \pm \frac{1}{n}\right)$$

By reference to Fig. 159, which may now be regarded as a diagram of accelerating force, it will be seen that whereas the initial acceleration is equivalent to a reduction in the effective pressure upon the piston during the earlier part of its stroke, yet during the latter portion this pressure is enhanced by the retardation occurring.

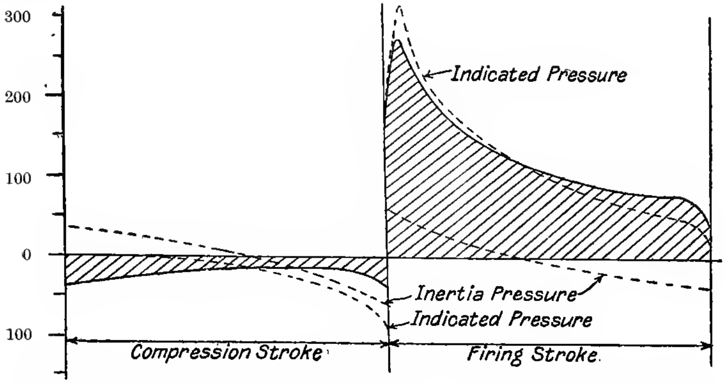

FIG. 161.—PISTON PRESSURES CORRECTED FOR INERTIA [1000 REVS. PER MIN.].

This effect during the working (or firing) stroke of the piston can be more clearly understood by reference to the diagram of piston pressures upon a piston stroke base, as plotted from an indicator diagram, and shown in Fig. 161.

The shaded curve shows the resultant effective pressure per sq. inch upon the piston when corrected for the piston inertia effect.

In order to obtain the real correction for the inertia effect, the accelerating force F at any point must be divided by the area of the piston.

Thus the equivalent inertia pressure $= \frac{4F}{\pi d^2} = \frac{4Mf}{\pi d^2 g}$

where d = diameter of piston.

Further, in the case of vertical engines the weight of the piston itself has to be added to the force causing acceleration, during the downward motion, and subtracted during the upward motion.

This is equivalent to adding or subtracting a small constant pressure $\frac{4W}{\pi d^2}$ to the effective explosion pressure. Since, however, in most car and aeronautical engines the dead weight of the piston is only a matter of a pound or two, and the area of the piston several square inches, the dead weight equivalent pressure will only be a fraction of a pound per square inch as compared with explosion and inertia pressures (at high speeds) of a hundred or two pounds per square inch.

It will be observed from Fig. 161, which illustrates an actual example, to scale, that one effect of piston inertia is to tend to render the effective explosion pressure more uniform during the firing stroke; also during the compression stroke the effect of the retardation of the piston towards the end of the stroke is to diminish the actual effort due to the energy stored in the flywheel upon the compressed gas.

This retardation force, called into play, actually diminishes the pressure between the small end of the connecting rod and the gudgeon pin.

In order to reduce these inertia pressures, for a given maximum speed, the reciprocating parts have, necessarily, to be made as light as possible, since at any given speed the inertia force due to the acceleration or retardation of the piston varies as the mass of the accelerated or retarded parts.

It must also be remembered that whatever energy is absorbed or used up in accelerating the reciprocating parts is not lost, since an equal amount of energy is given back during the ensuing retardation.

This will be evident from the curve of piston inertia given, for the areas for the acceleration and retardation periods are equal.

Lightness of reciprocating parts is an invaluable aid in reducing the bearing pressures between the working surfaces, for with heavy pistons and rods these pressures may become excessive, and since friction is an irreversible process, the loss of power is continuous for both acceleration and retardation. At high speeds, with heavy parts, the lubrication is generally ineffective, since the lubricant becomes squeezed out of the bearings.

Further, it will be shown later that the degree of vibration due to unbalanced parts varies directly as the mass of the reciprocating parts, and is a source of inefficiency, wear and tear, and discomfort.

In high speed engines the inertia pressures often become so great that they sometimes exceed the explosion pressures and cause a marked change in the distribution of resultant pressure and torque.

At speeds of 2,500 revolutions per minute and over in the case of car and cycle engines, the inertia pressures may begin to equal or exceed the explosion pressures, and the actual wearing effect of the explosion pressures becomes insignificant in comparison with inertia wearing effects; in fact, practical experience in the case of high speed engines shows that the side of the crank pin remote from the piston during the firing stroke (i.e., nearest to the crankshaft centre) wears the most.

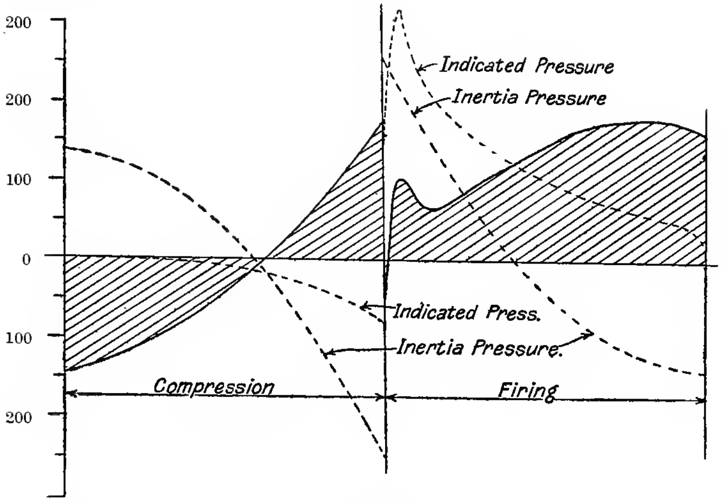

Fig. 162.—Piston Pressures corrected for Inertia [2,500 revs. per min.].

Fig. 162 represents to scale the piston pressure corrected for inertia effects for a speed of 2,500 revolutions per minute, the reciprocating mass being 3 lbs. per cylinder as before.

It will be observed that the inertia pressures are very high, and that they completely alter the character and shape of indicator diagram (shown on a stroke base).

In the diagram, areas above the base line represent work done by the piston, whilst areas below this line represent work done upon the piston either to accelerate the reciprocating mass or to compress the gas.

It will be noticed that a considerable amount of inertia energy is given back towards the end of the compression stroke, greatly in excess of that needed for compression purposes.

This diagram may also be taken to represent the force on the gudgeon pin (to a suitable scale, that is, by multiplying by the piston area).

The areas above represent positive pressures by the piston upon the gudgeon pin, whilst those below represent pressures exerted by the connecting rod upon the gudgeon pin (in the direction of the line of stroke).

The sudden reversal of this total pressure on the pin at the end of the compression stroke is very marked, and tends in practice to give rise to a knock when any play exists between the small end bearing and gudgeon pin.

Crank Effort or Torque.

In the diagram shown in Fig. 163 let P be the total pressure upon the piston when in the position *p* and let Q be the resultant connecting rod thrust and R the normal thrust between the piston and cylinder walls.

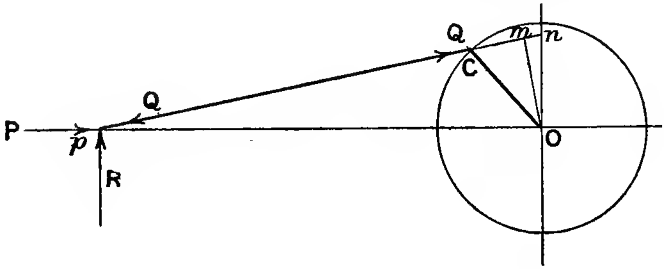

FIG. 163.

Then we have from the diagram, since the triangles *pn*O and O*nm* are similar,

$$P = Q \cos \varphi = Q. \frac{op}{pn} = Q. \frac{Om}{On}$$

Hence the moment of the crank effort $Q.Om = P.On = Pr \sin \theta \left(1 + \frac{r \cos \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}\right)$

So that, knowing the value of P (the total pressure upon the piston), the value of the torque, or crank-effort, can be calculated for all values of θ.

Alternatively, the torque can be graphically obtained by measuring the length O*n* for different crank positions, and multiplying by the total piston pressure.

The torque is therefore a function of the piston pressure and the crank angle.

If the value of P be obtained from the indicator diagram (corrected for inertia and dead weight, but neglecting friction effects), then the torque at different crank positions is obtainable.

If now the values of the torque thus obtained be plotted against values of crank angle, a diagram of crank-effort will result.

The area of this diagram will represent the amount of work done per revolution. This follows since area of torque curve $= \int_0^{360} T.d\theta$, which is also an expression for the work done per revolution.

It is next proposed to briefly consider the torque curves of some of the types of petrol engine employed in practice, so that their relative merits, from this point of view, may be compared; later on the question of the balance of these examples will also be considered, so that a more complete comparison of different types may be made.

From the designer's point of view the mean and maximum values of the torque for any given engine are very important, for it is the value of the former which determines the power output at any given speed, whilst the maximum torque governs the strength of the crankshaft; further, the amount of variation of the actual torque has an important bearing upon the permissible stresses in the crankshaft and transmission, etc.

Torsional Oscillations.

Another important factor that is governed by the variation of the actual torque is that of the torsional vibrations set up, for while the engine is working there is a reaction torque, equal and opposite to the torque upon the crankshaft. It is the variation of this torque which tends to set up vibrations in the supports, more especially when these are not adequately stiff, and when the natural period of vibration approaches that of the torque variation periods.

It may be of interest to consider this subject a little further, and to examine the case of elastic oscillations under the influence of a periodic torque.

Imagine a body capable of rotation about a fixed centre.

Let the moment of inertia of the whole body about this centre be denoted by I, then it can be shown that the time of vibration is given by

$$t = 2\pi\sqrt{\frac{I}{R}}, \text{ where } R = \frac{\text{torque applied}}{\text{angular displacement caused}}$$

In the case of a circular shaft of diameter d and modulus of rigidity C, if T be the applied torque, then the angular displacement due to this torque is given by

$$\theta = \frac{32\ l.T}{\pi d^4 C}, \text{ where } l = \text{the length of shaft}$$

$$\text{and } \theta = \text{angular displacement in radius.}$$

$$\text{Hence substituting for } R = \frac{T}{\theta} = \frac{\pi d^4 C}{32 l}$$

$$\text{we get time of oscillation } t = 2\pi\sqrt{\frac{32.l.I}{\pi d^4 C g}}$$

$$= \sqrt{\frac{128\pi l.I}{d^4.C.g}}$$

This principle is true for all torsional oscillations, and can be applied to the case of the crankshaft, transmission, etc., and also to the case of the reactive torque upon the engine cylinders and framing, although its application would require a knowledge of the moment of inertia, and the deflection per unit couple applied, in order to determine the value of R in the expression.

Critical Speeds and Torque Vibrations.

What is more important, in the above consideration, is the fact that the whole framing, and crankshaft, etc., have definite periods of vibration of their own, and that when the frequency of variation of the torque due to the engine approaches that of these parts resonance effects occur, and the vibrations may then assume unpleasant dimensions.

It is a well-known fact that nearly every high speed internal combustion engine has its "critical speeds" at which these vibrations are marked and it should be one of the objects of the designer to guard against these effects as far as possible.

The engine may be balanced to a nicety, in so far as the inertia pressures and centrifugal forces are concerned, and yet be subjected to "critical" vibrations.

Thus it is a feature of six-cylinder vertical engines that, although their mechanical balance is as perfect as possible—only the sixth harmonics being unbalanced—yet at definite engine speeds "critical" vibrations, manifested as a tremor through the car (or engine framing), occur.

As the engine speed is increased or decreased above or below this critical speed, the vibrations die away and smooth running is resumed.

There is usually two or three definite speeds in the ratio 1 : 2 : 3 within the running speed range of four- and six-cylinder engines at which these vibrations are felt, these speeds corresponding to multiples or harmonics of the natural frequency of the crankshaft under the influence of torsional vibrations.

The six-cylinder engine is more liable to these vibrations than the ordinary four-cylinder engine, on account of its greater length of crankshaft.

The effects of these "critical" vibrations also tends to upset the balance of the engine due to the torsional deflections affecting the angles of the cranks, and this effect, although the angular deflections are small, may become quite appreciable.

Designers of better classes of car engines have successfully attempted to obviate these torsional effects by introducing "vibration dampers" to the crankshaft, the function of which is to damp out these critical speed vibrations by mechanical or fluid frictional damping devices.

Some manufacturers have employed stiffer crankshafts in known cases of critical speed effects, whilst others have sought to meet the difficulty by adopting different stroke-bore ratios, for the longer stroke engines are the greater offenders in respect to torsional vibrations.

In gas engine practice, the addition of an extra flywheel to the front end of the crankshaft has been successfully employed.

In the Daimler and Lanchester four- and six-cylinder engines a "vibration damper" working upon the fluid friction principle is employed; this is shown illustrated in Fig. 164.

It will be seen to consist of a multi-disc clutch (which is further utilized to drive the fan, as a pulley for the fan-belt), running in a rather viscous oil bath, one set of clutch discs, or plates, being keyed to the crankshaft extension, and the other set to the fan pulley flywheel proper.

In the normal way, the whole damper rotates as a rigid mass, but at the critical speeds the fluid friction of the oil between the clutch plates is very effective in damping out quickly the crankshaft torsional vibrations.

The chief advantage of the torsional vibration damper is that

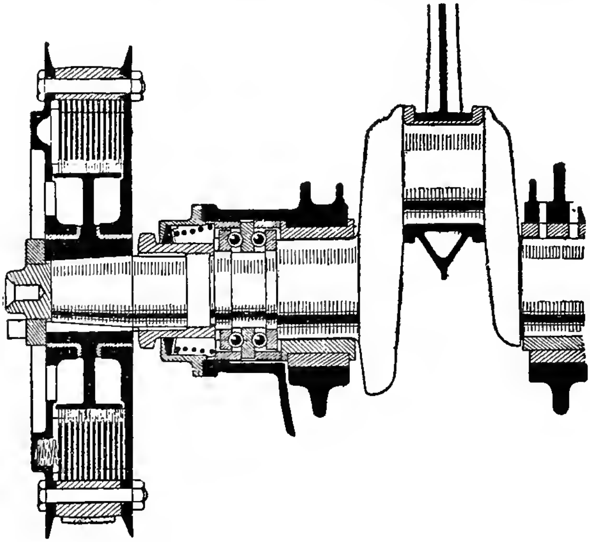

FIG. 164.—THE LANCHESTER VIBRATION DAMPER.

it prevents an accumulation of amplitude effect under the influence of periodic torques, which would otherwise tend to cause synchronous deflection effects of marked magnitude.

Torque Diagrams for Different Types of Engine.

It is now proposed to consider a few of the more important cases of the application of crank-effort or torque curves to examples occurring in practice.

For the purposes of comparison, it will be more convenient to assume that all the types of engine have the same sized cylinder, the engine speed being constant in each case.

In the examples considered, the bore and stroke have been taken as 3 and 4 inches respectively, and the engines to be running at 1,000 revolutions per minute.

Further, a connecting rod crank ratio of 4 ($= n$) has been assumed, and the weight of the reciprocating parts to be 4 lbs.

A standard example of indicator diagram has been taken, plotted out on a crank angle base, and corrected for inertia effects in the manner previously explained.

The equivalent inertia pressures in lbs. per square inch have been estimated to be 59 and 36 for the in- and out-centres respectively.

In each case the mean value of the torque for one complete cycle of operations of a single cylinder or two crank revolutions has been obtained from the mean height of the torque diagram, and further, the ratio of the maximum to the mean torque is given in each case.

Case I.—The single-cylinder type, as illustrated diagrammatically in Figs. 165 and 165A, gives a torque diagram as illustrated in the upper part of the same figure.

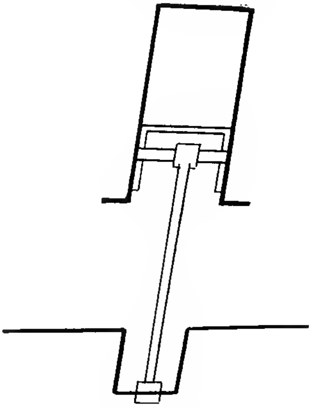

FIG. 165.

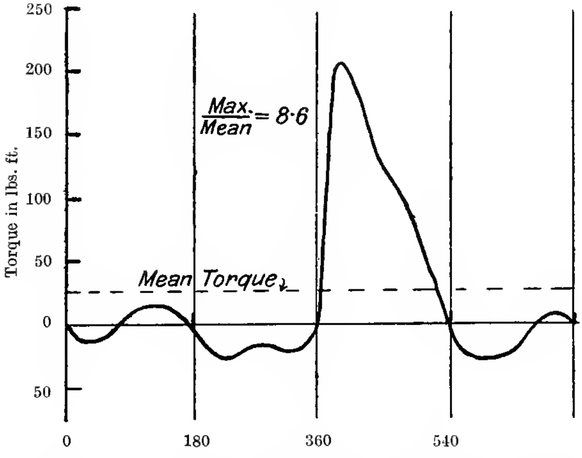

FIG. 165A.—THE SINGLE CYLINDER ENGINE TORQUE CURVE.

It will be noticed that the variation in the crank-effort during the firing stroke is very marked in this case; the ratio of maximum to mean torque being 8·6.

It will be seen later that such a high value of this ratio necessitates relatively larger sizes of the torque transmission parts, and heavier flywheel, than in the succeeding cases.

Case II.—The diagrams given in Fig. 166 illustrate the torque curves for a twin-cylinder side-by-side type of engine with cranks at 180°, and firing alternately.

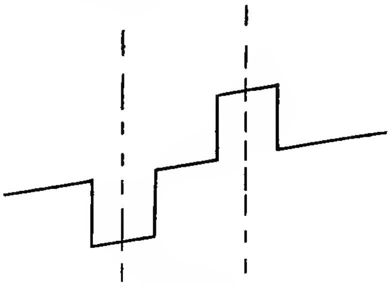

FIG. 166.

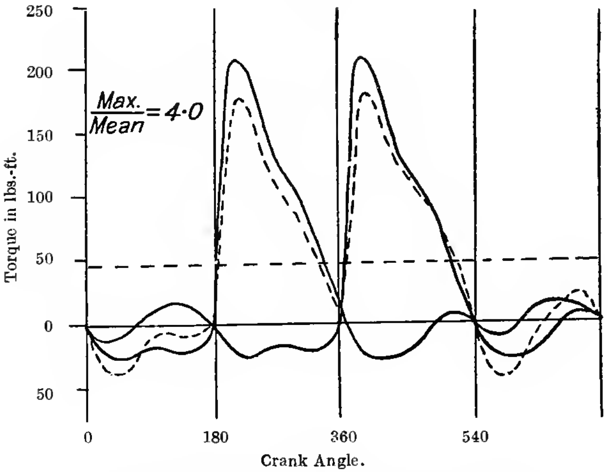

FIG. 166.—THE TWO-CYLINDER VERTICAL ENGINE TORQUE DIAGRAM.

The dotted curve in the diagram represents the resultant torque due to the combination of the two separate torques of the respective cylinders.

In this case the maximum torque is just four times the mean, which is represented by the dotted horizontal straight line.

Further, it will be seen that two explosions follow each other consecutively, and then there is an idle period of one revolution, and two more firing strokes during the next revolution, and so on.

Case III.—Two opposed cylinders, with cranks at 180°. The crank effort curves for this type of engine, which is illustrated in Figs. 198 and 199 , are shown in Fig. 167.

It is noticeable that the frequency of occurrence of the firing strokes is quite regular, so that the periods of occurrence of maximum torque will also be regular.

The ratio of maximum to mean torque during two revolutions in this case is 3·9.

Case IV.—Next in consideration is the **V** type of engine, commonly employed in aeroplane and motor-cycle practice.

The diagrams in Fig. 168 are for cylinders set at 90° and firing alternately.

The interval between consecutive explosions is not quite regular, the intervals being alternately $\frac{3}{4}$ and $\frac{1}{4}$ of a revolution, or expressed in strokes, three strokes and one stroke respectively of idle working.

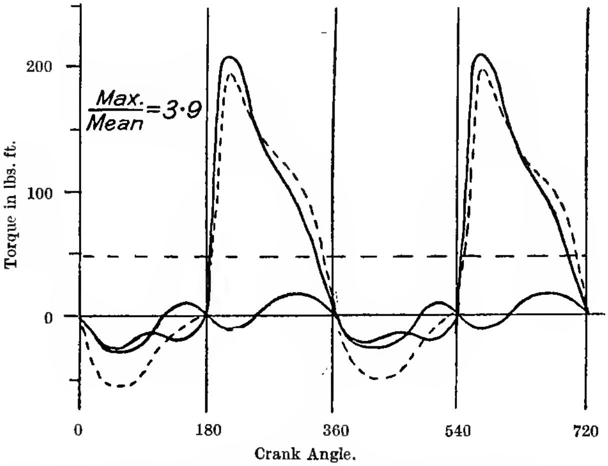

FIG. 167.—THE TWO-CYLINDER OPPOSED ENGINE TORQUE DIAGRAM.

The ratio of maximum to mean torque is 4·0.

It will be apparent that the smaller the angle between the cylinders, the nearer does the total or combined torque curve approach that of a two-cylinder engine with one crank only, and the more regular do the firing periods become, that is, the more uniform the periods of maximum crank effort.

Case V.—Four-cylinder with cranks in pairs at 180°, as shown in Figs. 169 and 169A, the order of firing being 1, 2, 4, 3—this being the ordinary motor-car engine arrangement.

The crank effort diagram indicates that the firing periods are quite regular and follow on consecutively with no interval between.

There are four periods of firing or useful crank effort during every two revolutions, and consequently four peaks to the combined torque curve.

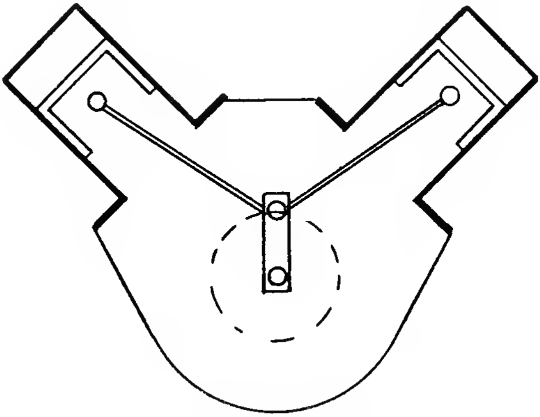

FIG. 168.—THE 90° TWIN ENGINE.

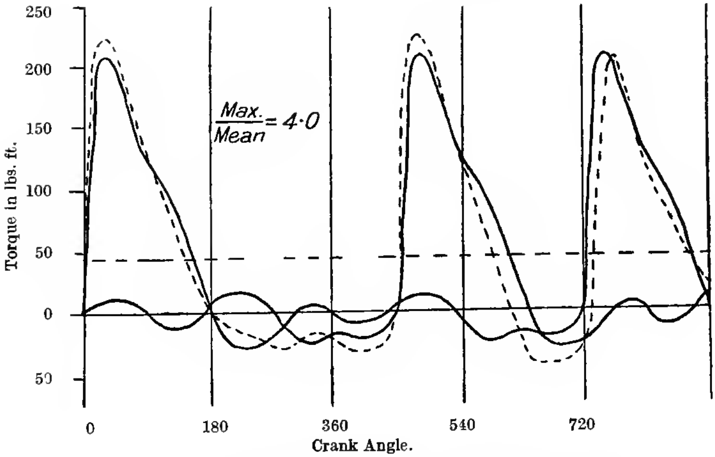

FIG. 168A.—THE 90° TWIN ENGINE TORQUE DIAGRAM.

The ratio of the maximum to the mean torque in this case is 2·0 approximately.

Case VI.—The six-cylinder type with cranks at 120°, the firing order being 1, 4, 2, 6, 3, 5, and representing the ordinary type of six-cylinder car engine.

There are now six firing periods during every two revolutions, and consequently six maximum torque peaks, as shown in Figs. 170 and 170A.

The ratio of maximum to the mean torque in this case is 1·4, so that one effect of cylinder duplication is evidently to raise the height of the mean torque line, and to cause it to more nearly approach the maximum torque height.

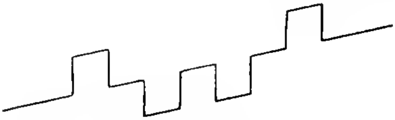

Fig. 169.—The Four-Cylinder Engine.

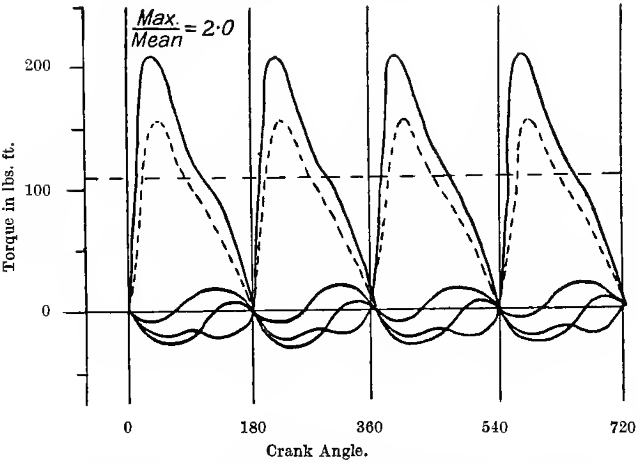

Fig. 169A.—Four-Cylinder Engine Torque Diagram.

From the examples considered, it should be a simple matter to determine the resultant crank effort curve for any other arrangement of cylinders, as it is of importance in internal combustion engine design to be able to analyse the effective efforts upon the crankshaft, and later on it will be possible to compare the relative merits of different engine types both from the point of view of crank effort and of engine balance, flywheel proportion, etc.

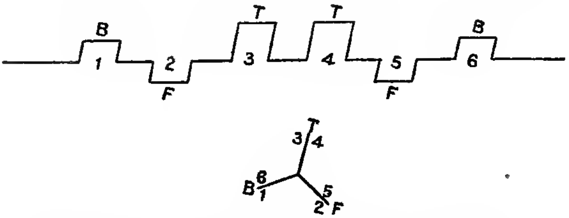

FIG. 170.—THE SIX-CYLINDER VERTICAL ENGINE.

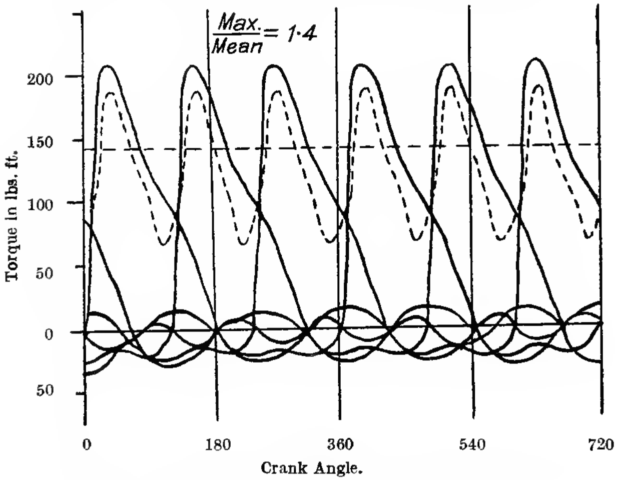

FIG. 170A.—THE SIX-CYLINDER TORQUE DIAGRAM.

Thrust on Cylinder Walls.

Case I.—The obliquity of the connecting rod produces a side thrust on the cylinder walls. The value of this thrust increases as $\frac{l}{r} = n$ decreases, assuming a definite value for the motor couple or torque.

Let T be the side thrust

also F the force at the gudgeon pin,

and R the force exerted along the connecting rod.

By taking moments about O, Fig. 171

$$T.OB = R.OC.$$

$$= \text{Torque or couple of the motor.}$$

The investigation may be proceeded with analytically. Considering the equilibrium of the three forces at B

$$\frac{T}{F} = \tan \varphi$$

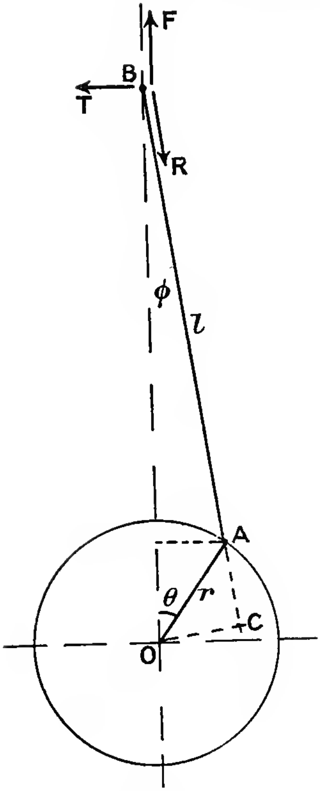

FIG. 171.

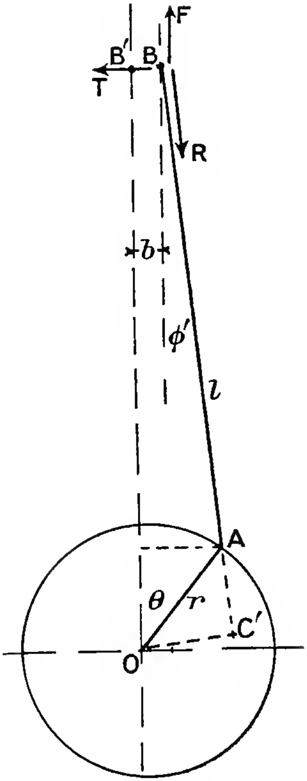

FIG. 172.

Putting φ in terms of l, r, and θ; since $r \sin \theta = l \sin \varphi$

$$\tan \varphi = \frac{\sin \varphi}{\sqrt{1 - \sin^2 \varphi}} = \frac{\frac{r}{l} \sin \theta}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}} = \frac{r \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}$$

$$\text{That is } \frac{T}{F} = \frac{r \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}$$

Case II.—When the engine is offset by an amount b, Fig. 172, the balance of the moments about O is

$$T.OB^1 + F = R.OC^1$$

As before, the alternative consideration of the forces at B gives

$$\frac{T}{F} = \tan \varphi^1$$

Determining φ^1 in terms of $\theta.l.r.b.$ from $l \sin \varphi^1 = r \sin \theta - b$, we get

$$\tan \varphi^1 = \frac{r \sin \theta - b}{\sqrt{l^2 - (r \sin \theta - b)^2}}$$

Now the numerator is less and the denominator in Case II as compared with Case I, so that the value of $\frac{T}{F}$ is less. Hence for a given value of F, the thrust T is smaller, so long as sin θ has a positive value, that is, during down strokes. When sin θ is negative —corresponding with up strokes—the value $\frac{T}{F}$ is greater ; but as the real value of F for compression or exhaust strokes is small the increase in T is not of great moment. The diminution of T on the firing stroke when F is large is, however, quite appreciable. It is to this factor that the adoption of offsetting may be put down.

The Flywheel.

The function of the flywheel is to minimize the fluctuation of engine speed caused by the variation of the crank effort and by the variation in load upon the engine.

The actual crank effort at any moment, in the case of an internal combustion engine, depends upon the piston position, connecting rod to crank ratio, number of cylinders, etc., and is always varying ; sometimes the energy imparted to the flywheel and crankshaft is in excess of the mean resistance or load upon the engine, and sometimes below it, so that the speed of the flywheel will tend to undergo corresponding fluctuations.

Alternatively the actual load upon the engine may vary periodically, but this condition does not often occur in the case of petrol engines, unless they be employed for driving pumps or compressing air.

Thus in the former mode of variation of output energy during a revolution, if the line MN (Fig. 173) represent the mean height of the torque diagram this line may be regarded as the mean torque equivalent to the steady load, or resistance, imposed upon the engine.

Then the shaded area above the line MN will represent the excess of energy obtained during the explosion stroke and imparted to the flywheel, thereby causing an increase in its speed.

Similarly the shaded area below the line MN shows the energy obtained at the expense of the kinetic energy of rotation of the flywheel and required for the idle strokes (exhaust, suction, and compression) of the engine.

This absorption of energy from the flywheel will cause a reduction in the speed of the flywheel.

Further, at the points A and B the rates at which work is being done on and by the crankshaft is the same, and hence the flywheel's speed will be neither increasing nor decreasing at these points.

At A the flywheel will just have ceased to supply energy to the crankshaft, and therefore its speed will be a minimum at this point, as indicated by the diagram in Fig. 173, which represents the speed fluctuation curve for a working cycle.

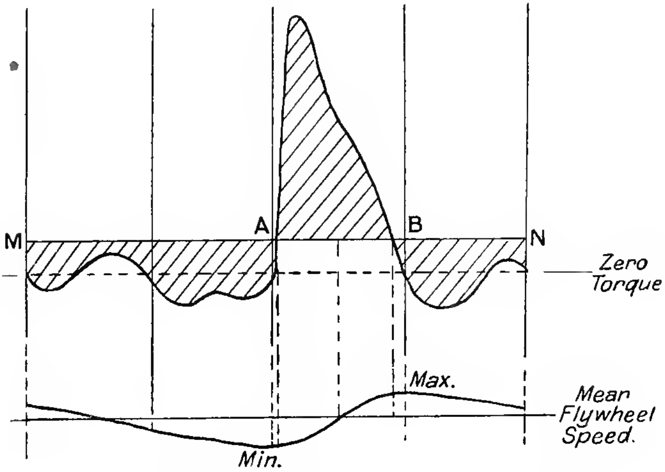

Fig. 173.—Constant Load Flywheel Effect.

Similarly at B the flywheel will have just finished receiving surplus energy from the crankshaft, and so its speed will be a maximum here, as shown in the lower diagram.

Obviously the excesses and deficiencies of energy are equal for a cycle of operations, or in other words, the algebraic sum of the positive and negative areas for a complete cycle is zero, and are obtained at the expense of the energy of rotation of the flywheel.

If the area of the torque curve for one complete cycle of operations (for any type of engine) be denoted by E, and the greatest of the separate areas above or below the mean torque line be denoted by δE, then the ratio $\frac{\delta E}{E}$ is termed the Coefficient of Fluctuation of Energy.

The following table gives some actual values for the energy fluctuation coefficient in different types of petrol engines.

TABLE XXXV

Type of Engine.	Energy Coefficient.
Single-cylinder	97 per cent.
Two-cylinder, vertical cranks at 180°	60 ,,
Two-cylinder, 90° twin	74 ,,
Four-cylinder vertical	5 ,,
Six-cylinder ,,	2·4 ,,
Seven-cylinder Gnome	2·6 ,,

We have assumed the load upon the engine, until now, to be constant during a revolution, but more generally if this load be regarded as variable, such as when the engine drives a pump (hydraulic or pneumatic) or drives a lay shaft which is bent and periodically binds in its bearings, if either the clutch or tyres slips during a cycle of operations, etc., then the curve of load torque would be no longer represented by a line such as MN in Fig. 173, but by a curve such as M^1N^1 as shown in Fig. 174; then the fluctuation of energy will be represented by the shaded areas, and the points A and B will still represent the places of minimum and maximum speeds of the flywheel, but their actual maximum values will not be the same.

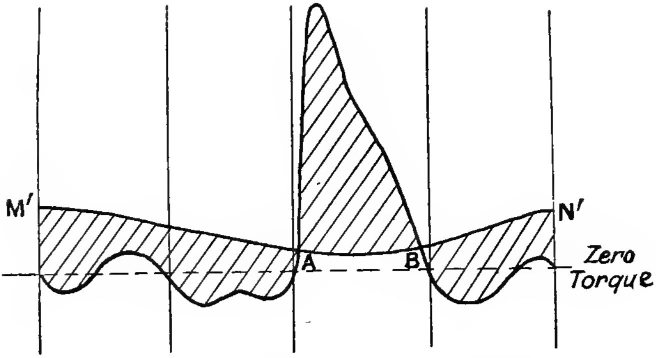

FIG. 174.—VARIABLE LOAD FLYWHEEL EFFECT.

For a single cylinder, if the separate areas above and below the load torque line be equal, then the speed variation of the flywheel will be the same above and below the mean speed, that is to say, the minimum speed will be as much below as the maximum speed is in excess of the mean.

The ratio of the extreme range of speed to the mean speed of the flywheel, resulting from the variation in the engine effort and load "effort," is termed the Coefficient of Speed Fluctuation, and it is the function of the flywheel to keep this coefficient down to an assigned limit.

Since the energy which can be stored in a flywheel is proportional to its moment of inertia and to the square of its angular velocity, there is considerable latitude in which to design an appropriate flywheel to fulfil its functions, for either the mass, diameter, and disposition of the material can be varied, generally, at will, so as to obtain an appropriate moment of inertia.

As an illustration of the values of the speed fluctuation coefficient (expressed as a percentage) employed in practice the following list is given.

TABLE XXXVI

Type of Engine.	Coefficient of Speed Fluctuation.
Single-cylinder petrol engine . . .	5 to 10
Four-cylinder ,, ,, ...	1 to 2
Six-cylinder ,, ,, . . .	0·5 to 1·0
Seven-cylinder Gnome ,, ...	0·1 to 0·4
Steam engine driving machine tools	2·8
,, ,, ,, pump	5
,, ,, ,, electrical machinery . .	0·6

Determination of Flywheel Proportions

It is required to find the size of flywheel which will give a certain limiting speed variation under working conditions.

Let the mean speed of the flywheel, expressed as an angular velocity, be ω_o, and q the coefficient of speed fluctuation given.

If ω_1 and ω_2 be the maximum and minimum speeds, then $q = \frac{\omega_1 - \omega_2}{\omega_o}$

If I be the moment of inertia of the flywheel and E_o the energy of rotation at its mean speed of the flywheel

$$\text{Then } E_o = \frac{1}{2} I\omega_o^2$$

and the fluctuation of energy δE will be given by

$$\delta E = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} I(\omega_1 - \omega_2)(\omega_1 + \omega_2)$$

$$= I(\omega_1 - \omega_2)\omega_o = 2E_o\left(\frac{\omega_1 - \omega_2}{\omega_o}\right)$$

$$= 2E_o q$$

Whence $E_o = \dfrac{\delta E}{2q}$

From this result the energy E_o of the flywheel necessary to keep the speed variation within the assigned limits is known, since the fluctuation δE is known from the torque diagram.

If the mean speed ω_o be known, we have

$$I = \frac{2E_o}{\omega_o^2} = \frac{\delta E}{q\omega_o^2}$$

and since $I = Mk^2$, where M is the mass and k the radius of gyration of the flywheel, either of these quantities may be variable in choosing an appropriate size of flywheel.

Example.—If the flywheel be made up of an annular rim connected to the central boss by arms, then if W_R and W_S be their respective weights, and k_r and k_s their respective radii of gyration, we have $I = W_R k_r^2 + W_S k_s^2$.

If the rim of flywheel be of rectangular section, and of internal and external radii R_1 and R_2, then we have $k_r^2 = \dfrac{R_1^2 + R_2^2}{2} = R^2$ approximately in practice.

Usually $k_s^2 = \dfrac{R^2}{3}$

so that $I = \left(W_R + \dfrac{1}{3}W_S\right)R^2 = \dfrac{\delta E}{q\omega_o^2}$

Hence it will be evident that in order to make q, the speed variation, a minimum for a given energy fluctuation, the weight and radius of the flywheel rim should be as large as conveniently possible.

The radius of the flywheel is, however, limited in practice by the centrifugal tension or stresses produced in the material of the flywheel itself by the centrifugal force due to rotation.

Thus it can easily be shown that the centrifugal tension in the rim of a flywheel is $\dfrac{\omega v^2}{g}$ lbs. per sq. inch, where v is the peripheral velocity in feet per second and ω the weight per foot of a bar 1 sq. inch in section.

Now for cast-iron, such as is used in flywheels, $\omega = 3{\cdot}37$ lbs. per foot per sq. inch section, and the ultimate tensile strength is about 8 tons to the square inch, or 18,000 lbs. per sq. inch.

The bursting speed is then given by

$$v^2 = \frac{32{\cdot}2 \times 18{,}000}{3{\cdot}37}$$

from which $v = 415$ feet per second

The safe working peripheral speed is usually obtained with a factor of safety allowance of about 18, and will be seen to be equivalent to a peripheral speed, in cast-iron, of 97 feet per second.

Much, however, depends upon the actual design of the flywheel, and upon the frequency and magnitude of the energy fluctuations, etc., and it has been found, in practice, inadvisable to exceed a peripheral speed, in cast-iron, of 80 feet per second.

Numerous cases of " burst flywheels " have resulted in employing peripheral speeds exceeding this figure.

Another factor concerned in flywheel strength is that due to initial casting stresses, which may very appreciably weaken the rims and their supports.

Further, with cast-iron, the metal is not usually homogeneous, air-bubbles being often interspersed with the metal, and the balance may be thereby impaired, thus causing additional stresses on the flywheel rim.

The adoption of forged steel and cast-steel, for flywheels, is becoming the practice for high speed engines and machinery, and in certain cases steel rims have been shrunk on to the rims of cast-iron flywheels, and even cast-steel wire rope has been employed with cast-iron flywheels, enabling the working speed with the latter material to be trebled.

With good cast-steel flywheels the working peripheral speed should not exceed 180 feet per second, and for forged or stamped mild steel or iron wheels 140 feet per second.

For any other material, the safe working rim speed will vary as the square root of its ultimate tensile strength, and inversely as the square root of its density or weight per linear foot.

$$\text{Thus } V_s = k.\sqrt{\frac{f_t}{\omega}}$$

where k is a constant, usually between 10 and 14, and f_t and ω are the ultimate tensile strength in pounds per square inch, and weight per foot run per square inch section of the material.

This formula is directly deduced from the considerations already given.

Further data in connexion with the design of flywheels for internal combustion engines will be found in a subsequent companion volume upon design.

Inertia of the Connecting Rod.

Hitherto the inertia effect of the piston and those parts reciprocating in a linear direction only have been considered, but it will be evident that the connecting rod itself will be subjected to inertia effects, but of a different nature, since the connecting rod experiences at one end a reciprocating motion identical with that of the piston, and at the other a rotation exactly the same as that of the crank pin.

Evidently its resultant motion will be a combination of a translatory and rotary motion.

For approximate purposes in inertia and balancing problems it is usual to assume the connecting rod to be divided into two parts, inversely as the distances of the small and big ends of the rod from the C.G. Then the whole mass of the rod is imagined to be divided up in this proportion and concentrated at the two ends, so that the portion at the small end may be treated as an additional mass to the reciprocating mass, and the big end portion treated as a rotating mass concentrated at the crank pin, and appropriately balanced.

This method, although rather rough, yields quite satisfactory results for most practical purposes.

For an exact solution of the motion of the rod it is necessary to consider the whole of the separate motions to which it is subjected.

The motion of the rod may be analysed as consisting of—

(*a*) A translation of its C.G. with the velocity and acceleration of the piston at any instant $= f$.

(*b*) An angular acceleration about P, the gudgeon pin (see Fig. 175) of $\frac{d^2\varphi}{dt^2}$, where φ is the angle CPO.

And (*c*) an angular velocity $\left(\frac{d\varphi}{dt}, \text{ at any instant}\right)$ about P.

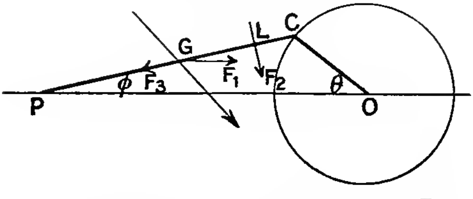

Fig. 175.—Inertia of the Connecting Rod.

It will be apparent that the rod will be under the influence of three separate forces acting as shown in Fig. 175, and consisting of—

(a^1) A force F_1 acting at the C.G. of rod, causing it to accelerate similar to the piston, the magnitude of this force being given by

$$F_1 = \frac{M_c f}{g}, \text{ where } M_c = \text{mass of rod.}$$

(b^1) A force F_2 which acts through the centre of percussion, and which may be arrived at as follows: The angular acceleration of the rod about P may be considered as being due to a torque T such that

$$T = \frac{I}{g}\frac{d^2\varphi}{dt^2}, \text{ where } I = \text{moment of inertia of rod about P}$$

$$= \frac{M_c}{g}k^2\frac{d^2\varphi}{dt^2}, \text{ where } k \text{ is the radius of gyration of rod about P.}$$

The distance of the centre of percussion from P is $\frac{k^2}{PG}$ (the same as the length of the simple equivalent pendulum). Hence the force F_2 acting at L, the centre of percussion is given by

$$F_2 \times PL = T = \frac{M_c}{g}k^2.\frac{d^2\varphi}{dt^2}$$

$$\text{whence } F_2 = \frac{M_c}{g}.PG.\frac{d^2\varphi}{dt^2}$$

(c^1) The other remaining force acting upon the rod consists of the centrifugal force towards P, and which is given by

$$F_3 = \frac{M_c}{g}.PG.\left(\frac{d\varphi}{dt}\right)^2$$

The methods for obtaining the values of angular velocities and accelerations of the connecting rod in terms of the crank angle, are given in the earlier portion of this chapter (page 271).

Knowing now the magnitude, direction and points of application of these forces acting upon the rod, the resultant force R may be obtained by the ordinary graphical (or analytical) methods.

The resolute of the resultant force along the line of stroke will give the connecting rod inertia force at any instant in this direction.

Also by resolving R along the rod and perpendicular to it, the resultant thrust due to the inertia effect of the rod can be obtained, and the thrust due to the piston pressure must be corrected for this inertia effect, exactly as the piston pressure was corrected for piston inertia.

In obtaining the torque of the engine at any instant, the resultant of the piston thrust and inertia effect of the connecting rod resolved along itself is obtained, as indicated above, and multiplied by the perpendicular distance of the rod (or its prolongation) from the crankshaft centre.

The torque curves should therefore be corrected both for piston and connecting rod inertia, in order to obtain accurate results, for these two latter effects become of great importance at high engine speeds.

The weight of the rod itself should be taken into account by treating it as an additional force to F_1, F_2 and F_3 in finding the resultant force R, if greater accuracy is required, but relative to these three forces it is generally negligible.

An alternative analytical method of treating the inertia of the connecting rod, together with the forces acting at the big and small end pins, should be consulted in *The Steam Engine* (Prof. Perry), p. 544.

Inertia Forces producing Bending Action on Connecting Rod.

Let the connecting rod be situated as shown in Fig. 176, so that it is at right angles to the crank.

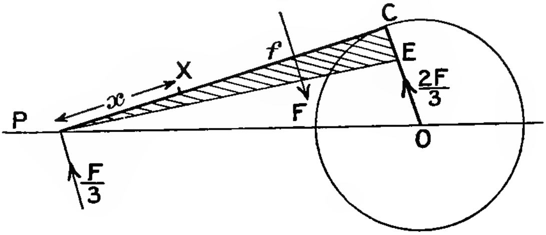

FIG. 176.

Then the resultant acceleration of the whole connecting rod in a direction parallel to OC will cause an outwardly-acting normal inertia force F upon the rod, acting at some point *f*, and causing a bending action to occur.

Let m^1 be the weight per unit volume of the rod, then assuming the section of the rod to be constant throughout its length, the weight per unit length is given by $m = \frac{m^1}{A}$, where A is the sectional area of the rod.

(1) Dealing with this simplest case of a uniform section rod first,

the outward acceleration at C is given by $\omega^2 r$, where ω is the crank pin angular velocity.

At any point X situated at a distance x from P the normal acceleration will be $\frac{x}{l}.\omega^2 r$, and hence the acceleration is proportional to the distance from P, and can therefore be represented by the triangle PCE, whose breadth at any part represents the normal acceleration at that place.

The resultant normal force to the rod is evidently the integral of all the component accelerations similar to the elementary length δx at x from P.

The force causing acceleration of such an element is

$$\delta \mathrm{F} = \frac{m}{g}.\omega^2 r.\delta x.\frac{x}{l}$$

Hence the resultant force $\mathrm{F} = \int_{x=0}^{x=l} \frac{m}{g}\frac{\omega^2 r}{l}.x dx$

$$= \frac{m}{2g}.\omega^2 l r$$

and acts at $\frac{2}{3}l$ from P.

The reactions at P and C are therefore $\frac{\mathrm{F}}{3}$ and $\frac{2}{3}\mathrm{F}$ respectively.

The rod may now be treated as a beam supported at the ends and loaded with a force or weight F.

The bending moment at any given point such as X is then given by $\mathrm{M} = \frac{\mathrm{F}}{3}.x - \frac{m}{g}\omega^2 r.\frac{x^3}{6l}$

$$\text{or } \mathrm{M} = \frac{m\omega^2 r}{6lg}(l^2 x - x^3) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

It will be seen from this that the bending moment varies not as the distance x from B, but as $x\ (l^2 - x^2)$, and that the curve of bending moment has the equation $y = kx(l^2 - x^2)$, where y is the B.M. and k a constant.

The maximum B.M. occurs where $\frac{dy}{dx} = 0$

$$\text{that is } l^2 - 3x^2 = 0$$

$$\text{or } x = \frac{l}{\sqrt{3}} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

Substituting this value of x in (1) we have

$$M_{max} = \frac{m\omega^2 r}{16g}.l^2$$

For the purposes of design it is necessary to be able to compute M_{max} in this way, in order to determine the maximum bending stress in the rod. Thus if Z be the "strength modulus" of the section of the rod, the maximum stress is given by

$$f = \frac{M_{max}}{Z}$$

For a round rod $f = \frac{M_{max}}{\frac{\pi d^3}{32}} = \frac{32M_{max}}{\pi d^3}$ d = diameter of rod at maximum B.M. place, and for a rectangular rod of breadth b and depth d

$$f = \frac{6M_{max}}{bd^2}$$

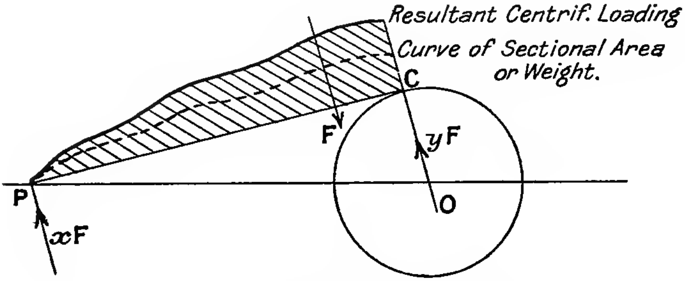
Resultant Centrif. Loading
Curve of Sectional Area or Weight.
C
F
yF
P
O
xF

Fig. 177.—Bending of Irregular Sectional Connecting Rod.

(2) If the section of the rod be not uniform, but varies all along the rod, as in Fig. 177, according to some law of sectional variation $K = \varphi(x)$, where x is the distance from P.

Then the centrifugal force upon an elementary length δx at X is given by

$$\delta F = \frac{m^1}{\varphi(x)} . \frac{\omega^2 r}{g} . \delta x . \frac{x}{l}$$

the first term $\frac{m^1}{\varphi(x)}$ now representing the mass of the element.

The resultant force F upon the rod will then be given by

$$F = \frac{m^1\omega^2 r}{gl}\int_{x=0}^{x=l} \frac{x}{\varphi(x)}.dx$$

the value of which will depend upon the law of variation of the section.

Thus if the section increase uniformly from P to L (neglecting the big and small ends)

then $K = a.x$, where a is a constant

$$\text{and } F = \frac{m^1\omega^2 r}{gl}\int_{x=O}^{x=l}\frac{x}{ax}\,dx$$

$$= \frac{m^1\omega^2 r}{g}.\frac{a}{l}$$

For more complex variations of section it is simpler to perform the integration graphically by dividing the rod up into a number of parts by parallel planes to the crank OC, and finding first the mass of each part from the average sectional area, and then the resultant force upon each mass due to its acceleration in the direction parallel to OC.

The resultant of all of these component forces can be then easily obtained.

The Connecting Rod as a Strut.

Besides being subjected to inertia bending forces, the connecting rod is also subject to axial thrust due to the piston and piston inertia pressures, which will create buckling stresses quite apart from the inertia stresses.

The rod may be treated as a strut hinged at each end and loaded with a load equal to the greatest thrust which can occur, due to the piston pressure corrected for inertia effects.

Thus if T be the maximum axial thrust in rod,

$$T = \frac{1}{\cos\varphi}\left[P - \frac{M\omega^2 r}{g}\left(1 \pm \frac{1}{n}\right)\right]$$

where P is the *total* piston load and M the reciprocating mass, φ being the angle CPO (Fig. 176).

The maximum value of this expression can easily be determined by foregoing methods.

Knowing T, then the stress to which the rod is subjected, as a strut, is given by

$$\frac{k.T}{A} = \frac{\pi^2 EI}{l^2}$$

where k = the factor of safety, usually lying between 6 and 10, under the above conditions of load application

A = the sectional area (assumed uniform at the centre)

E = modulus of elasticity of material of rod

I = its least moment of inertia of cross section at middle of rod, about an axis through its C.G.

The value of the working stress $f_c = \frac{kT}{A}$ can thus be calculated.

Resultant Connecting Rod Stress.

In order to determine the strength of the connecting rod under the combined influence of bending and end thrust it is necessary to calculate the stresses due to these two causes separately, and to add them together for the resultant stress.

$$\text{Thus resultant stress } f_R = f + f_C$$

and from this value f_R the dimensions of the rod are determined, knowing the properties of the material employed.

Thus if the rod be made of mild steel f_R should not exceed 7,000 lbs. per sq. inch, more especially on account of the alternating action of the loading, which is compressive for the firing, exhaust and compression strokes, and tensile for the suction and any misfiring or idle strokes.

For cast-steel rods, f_R should not be greater than about 12,000 lbs. per sq. inch, whilst for the alloy steels such as nickel steel, nickel chrome, chrome vanadium, etc., the value of the permissible working stresses will depend upon the ultimate breaking stress, elastic limit, and fatigue-resisting properties of these materials.

CHAPTER VII

ENGINE BALANCE

THE balance of the working parts of an engine is a very important factor in engine design. A perfectly balanced engine is one in which the relative motions of the component parts have no tendency to make the engine vibrate as a whole.

The elimination of vibration, especially in automobile and aeronautical practice, should be one of the chief objects of every designer of high speed internal combustion engines.

An engine, if perfectly balanced, would, if suspended perfectly freely in space, exhibit no vibratory or other movements. In connexion with this method of regarding engine balance, mention might here be made of an experimental apparatus of Prof. Perry's, consisting of four discs keyed to a shaft driven by means of a belt from an electric motor. The discs can be made to be out of balance to any extent; and since the whole frame carrying the motor and shaft bearings is suspended by wires from an external support, any out-of-balance effect of the discs when rotating causes the whole frame to vibrate or move.

If an engine be perfectly in balance, it will, theoretically, require no foundation bolts or means of holding it to the ground; but if unbalanced, the reactions of the movements of the unbalanced parts will be transmitted to the foundations or frame, through the bedplate or supporting arms.

Actual engines can only be said to approximate to the condition of perfect balance.

There are two distinct modes of vibration which can occur in engines. The vibrations which may happen are—

(*a*) Vibrations due to the elastic deformations of the material of the component parts of the engine, such, for instance, as the torsional oscillations of the crankshaft under the influence of the periodic piston effort.

(*b*) Vibrations of the engine as a whole, considered as a rigid system of parts, that is, vibrations of the engine and framing as a whole, in which no elastic yielding of the components occurs.

The question of engine balance is concerned chiefly with the latter kind of vibrations, and it is broadly true that these vibrations are a direct consequence of out of balance effects.

There are several ways in which the framing and foundations (or supports) of an engine may vibrate under the influence of unbalanced parts, and these may be briefly enumerated thus—

(1) Vertical linear, or translational vibrations due to unbalanced vertical forces.

(2) Horizontal linear or translational vibrations due to unbalanced horizontal forces.

(3) Rotary oscillations in vertical or horizontal, or indeed in any plane, due to unbalanced couples in that plane.

Before dealing with any specific cases for engine, a consideration of a few of the fundamental principles involved in questions of engine balance may be of advantage, as a proper grasp of these principles will render much easier the task of explanation and the understanding of the examples treated herewith.

Centrifugal Forces and Couples.

If a mass M be rotated about a fixed centre, with an angular velocity ω, and if R be the radius of the circular path described, then the body will experience a radial force F acting outwards given by $F = M.\omega^2.R$, or writing $\omega = 2\pi N$, where N = the number of revolutions per second,

$$F = M.4\pi^2N^2.R$$

If R be in feet and M in pounds, then the value of the force in pounds weight is given by

$$F^1 = \frac{M}{g}.4\pi^2N^2.R$$

As a general case, consider a body of mass m, whose centre of gravity is moving in any path whatever. Let r be the radius of curvature, Fig. 178 of the path of the C.G. at any instant, then the centrifugal force acting upon the body is

$$f = m\omega^2r$$

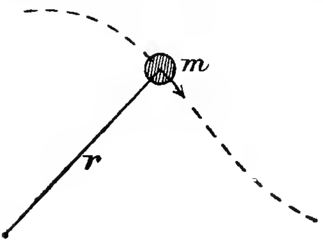

FIG. 178.

and f will be a function of both ω and r.

Proceeding from this general case, imagine a number of particles of masses m_1, m_2, m_3 . . . revolving about a common centre at different radii r_1, r_2, r_3 . . . and each moving with an angular velocity ω, as if attached to the same rotating disc. Then the axis of rotation of the disc will be subjected to a number of pulls of magnitude $m_1\omega^2r_1$, $m_2\omega^2r_2$, $m_3.\omega.^2r_3$ . . in the respective directions of the radii r_1, r_2, r_3 . . .

The resultant pull upon the axis will be represented by the resultant of all the component pulls, that is, by vector sum of $\omega^2.(m_1r_1 + m_2r_2 + m_3r_3 + \ldots)$.

The resultant, as given by the force diagram, will represent the single centrifugal force equivalent to the component forces, and an equal and opposite force to this will completely balance the system of component pulls.

Thus if the vector sum of the forces be a single force of magnitude X, the forces may be balanced by a mass M, at a distance R from the axis, and in a direction diametrically opposite to the single resultant, provided that M.R = X.

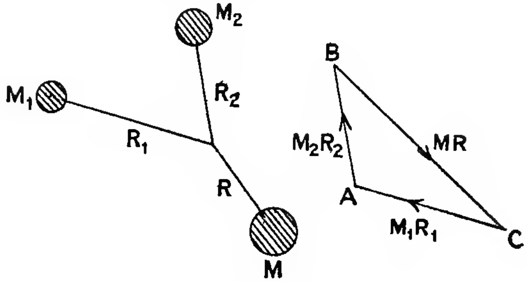

Fig. 179.

It is obvious that any mass M, or any radius R, can be chosen to suit other conditions provided that the product fulfils the above condition.

As an example, consider the case of two masses M_1 and M_2 at radii R_1 and R_2 respectively (Fig. 179).

It is required to balance these by a single mass placed at a radius R.

Draw AB parallel to the radius R_1 and proportional to M_1R_1, and AC parallel to the radius R_2 and proportional to M_2R_2. Then BC will represent in magnitude and direction the resultant single centrifugal force, say, F.

Hence the two masses M_1 and M_2 may be balanced by a mass $=\frac{F}{R}$ placed as shown at O_c.

This procedure may be followed in a simple application. The

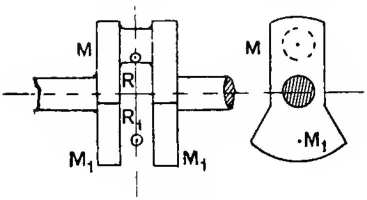

FIG. 180.

weight of the webs and crank pin of a single crank are assumed to be of value M, and it is required to balance this by two webs, such as is diagrammatically shown in Fig. 180, upon the other side of the crankshaft (i.e., extensions of the crank webs).

Let r = distance of C.G. of main crank from the axis of rotation and M_1 = mass of each web balance weight, the C.G. of which is at R_1 from centre.

$$\text{Then } M\omega^2 r = 2M_1\omega^2 R_1$$

$$\text{or } M_1 = \frac{Mr}{2R_1} \qquad \text{(see Fig. 180)}$$

This ensures the centrifugal force, and its moments being in balance.

Usually unit angular velocity is assumed in these cases.

Centrifugal Couples.

If a shaft AB of length a be provided with arms of equal length r at its ends, and equal weights M be attached to these arms (Fig. 181), then the centrifugal force upon each arm when the shaft is rotating will be $M.\omega^2.r$, and the shaft will experience a couple of moment $M.\omega^2.r.a$ tending to rotate or twist it in the manner indicated by the arrow, that is, about an axis perpendicular to the direction of rotation and of the centrifugal forces themselves. It is important to notice that this couple will be of constant magnitude but of varying direction—the axis of the couple rotating in a plane perpendicular to the axis of the shaft and at the same angular velocity.

Such a couple would occur in the case of a two-cylinder engine with cranks at 180°, and would remain unbalanced. In several other types of engine also unbalanced couples occur and are resisted by the reaction of the supports of the engine.

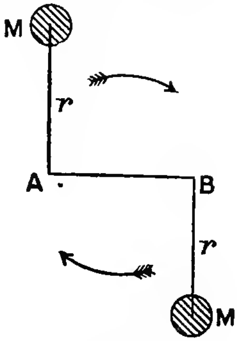

FIG. 181.

The Balancing of Crank Webs.

It has been shown that each crank pin and its webs which give rise to radially acting centrifugal forces can be balanced by means of a pair of balance weights upon the opposite side of the crankshaft. In practice, however, this is not always done—in fact, it is the exception rather than the rule. In multi-cylinder car engines the difficulty of obtaining uniformity in manufacture and the cost of providing such weights, which is considered out of proportion to the results obtained, are the main reasons given for doing away with balance weights.

The result of leaving these cranks unbalanced is to increase the bearing pressures upon the main bearings, and to cause severe bending actions to occur upon the crank case.

Taking the case of a four-cylinder crankshaft (Fig. 182) of the three-bearing type, with a cylinder bore and stroke of 4 inches and 6 inches respectively, at a speed of 1,500 revolutions per minute, there is a resultant centrifugal force of 1,700 lbs. at each crank pin. These unbalanced forces cause additional pressures upon the main bearings of 700 lbs. each for the end and 1,400 lbs. for the central bearings.

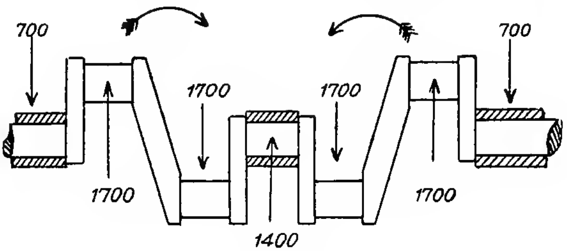

FIG. 182.

These forces rotate with the direction of the plane of the webs, that is, at crankshaft speed, and give rise to an alternating bending action upon the crank case which may cause appreciable distortion, the magnitude of this bending moment in the example given being 8,500 inch pounds, which is equivalent to half a ton acting at the centre of the crank case, if it is taken as supported at the ends.*

The next consideration is that of an example involving the balancing of both forces and couples due to a single weight of mass M. The possible solution by using two balance weights is

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* *Vide The Automobile Engineer*, Oct. 1, 1913, for "Crankshaft Balancing."

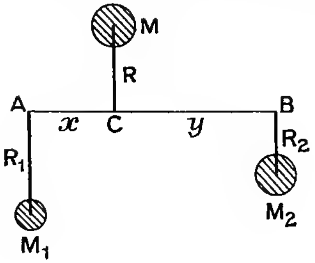


FIG. 182A.

represented in Fig. 182A, in which the conditions for balance are—

(1) That the resultant centrifugal force must be zero. For this $M.r = M_1r_1 + M_2r_2$.

(2) That the resultant centrifugal couple must be zero. For this $M_1r_1x = M_2r_2y$.

(3) The weights M, M_1 and M_2 must be in the same plane.

It will thus be evident that the rotating parts can be perfectly balanced by suitably disposed weights, and that the balance of the cranks of any type of engine is only a matter of the application of the foregoing methods.

Nevertheless, it is not always a simple and easy matter to make the necessary calculations. The application of these methods to the general case of the balancing of any number of cranks in any direction, and placed anywhere along the crankshaft, will be considered next, both graphical and analytical solutions being presented.

General Case of Engine Balance.

A number of rotating masses in different transverse planes are to be balanced by two masses in two given transverse planes, the magnitudes of these masses to be found.

(1) *Graphical Method.*—Consider the crankshaft OZ in metric projection as shown in Fig. 183. Let A and B be the given transverse planes of the balance weights whose masses are to be found. We are given the mass m, radius r, and angular direction a, and also the distance x, for each of the rotating masses, so that the position of any mass can be defined by ${}_xmr_a$.

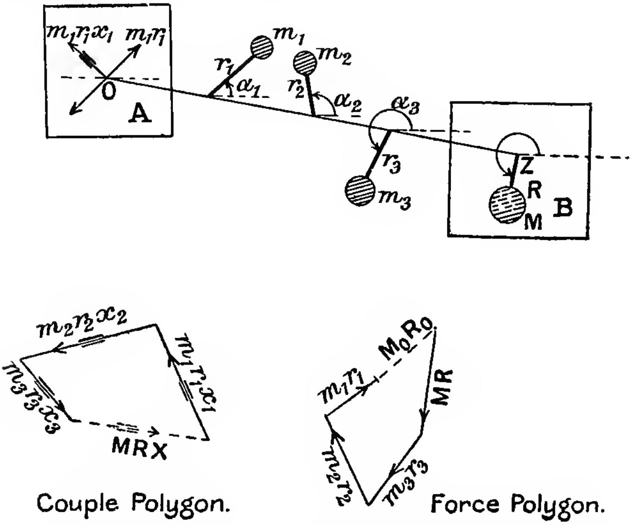
First consider the centrifugal forces acting, then every centrifugal force like $m.r$ (unit angular velocity of 1 radian per second being assumed throughout) can be replaced by a force mr in the reference plane A and a centrifugal couple of moment mrx perpendicular to the direction of the force in the plane A. The direction of the axis of such a couple is indicated in the figure by the broad lines parallel with the arrow. All of the centrifugal forces therefore can be replaced by equivalent forces in the reference plane A and couples.

We can now proceed to find the resultant of all the couples referred to the plane A.

A "couple" polygon is drawn, the sides of which, being drawn in the proper order, are proportional to the magnitudes of the couples, and in the same direction as the axes of couples. The closing side of this polygon gives the magnitude MRX and direction α_B of the resultant couple.

Since X is given, we know MR, and can choose either M or R for convenience, so long as the product is constant, and we also know the direction of the arm of the balance weight M, namely, $(\alpha_\beta + 180°)$. This gives the position of balance weight, and its value, in plane B.

The resultant of all the centrifugal forces (including MR) referred to the plane A is similarly obtained by a diagram of forces, and the closing side gives the magnitude M_0R_0 and direction α_A of the resultant force in plane A.



Couple Polygon. Force Polygon.

FIG. 183.

An equal force M_0R_0 in an opposite direction $(\alpha_A + 180°)$ gives the magnitude and direction of the equilibriant, and the position of the balance weight in plane A is now known.

This graphical method is equivalent to solving the vector equations—

$$(a)\ m_1r_1x_{1(\alpha_1+90)} + m_2r_2x_{2(\alpha_2+90)} + \text{etc.} + \text{M.R.X}_{(\alpha_\beta+90)} = 0$$

and $(b)\ m_1r_{1\alpha} + m_2r_{2\alpha} + \text{etc.} + M_0R_{0\alpha_A} + MR_{\alpha_B} = 0.$

(2) *The Analytical Method.*—In order to obtain an analytical solution, the couples and forces are referred to the plane A, as before. A pair of convenient axes OX, OY at right angles is taken, and the couples and forces resolved along them.

Since the resolute of any number of forces in a given direction is equal to the resolute of their resultant in the same direction, we get the following relations—

$$\mathrm{MRX}\sin(a_B + 90) = \Sigma m_1r_1x_1 \sin(a_1 + 90) + m_2r_2x_2 \sin(a_2 + 90) + \text{etc.}$$

$$\mathrm{MRX}\cos(a_B + 90) = \Sigma m_1r_1x_1 \cos(a_1 + 90) + m_2r_2x_2 \cos(a_2 + 90) + \text{etc.}$$

from which MR and a_B are obtainable.

Also for the resultant of the forces in plane A we have

$$\mathrm{M_0R_0}\sin a_A = \mathrm{MR}\sin a_B + \Sigma m_1r_1 \sin a_1 + m_2r_2 \sin a_2 + \text{etc.}$$

$$\mathrm{M_0R_0}\cos a_A = \mathrm{MR}\cos a_B + \Sigma m_1r_1 \cos a_1 + m_2r_2 \cos a_2 + \text{etc.}$$

from which $\mathrm{M_0R_0}$ and a_A are obtainable, so that a complete solution is thus effected, analytically.

The Reciprocating Parts.

It has been seen that the centrifugal forces in connexion with the rotating parts can be perfectly balanced, although in actual practice it is not always possible to effect such a balance, some of the reasons for which have already been referred to.

In the case of the reciprocating parts, of which the piston is a good example, the inertia forces to which the motion of these parts gives rise will, if unbalanced, occasion rocking and vibration in the engine framing and its support.

It can be shown that if in the case of the single-cylinder engine the obliquity of the connecting rod be neglected, then the periodical variation of velocity and direction of the moving parts gives rise to inertia forces which are equal in magnitude, but opposite in direction, at the two ends of the piston stroke. If, however, the obliquity of the connecting rod be allowed for, then the inertia forces at the two ends of the stroke are unequal, and this factor is of considerable importance in connexion with the balancing of the reciprocating parts.

It will be shown later that the motion of the piston can be represented by the displacement (or component along the line of stroke) of a number of simple harmonic motions, of different amplitudes and of different periods.

The harmonics of the same period as the piston can be allowed for and balanced, but the secondary, tertiary and higher orders of harmonics either have to be arranged to balance themselves in different types of engine or have to be left unbalanced. The secondary forces, due to the second harmonic of the piston's motion may, in some cases, be of appreciable importance.

Often the secondary forces in an engine are, by a suitable choice

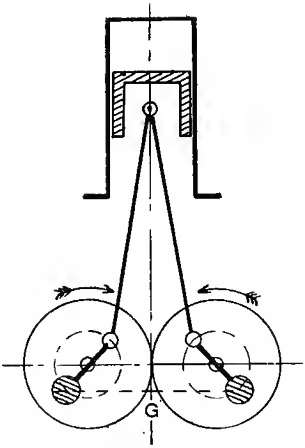


FIG. 184.—METHOD OF BALANCING SINGLE CYLINDER ENGINE.

of crank positions, made to balance themselves, and the higher harmonics reduced to negligible proportions.

The balance of a mass having a reciprocating motion can only be effected perfectly by the introduction of an equal and opposite reciprocating mass, in the proper phase and same line of motion as the original.

An interesting method of approximately balancing the reciprocating forces due to the mass of the piston, and part of the connecting rod, is illustrated in Fig. 184, and is due to Lanchester.* Taking the example illustrated of a single cylinder engine, two flywheels of equal moments of inertia were provided with balance weights placed opposite to the cranks on each flywheel, and were geared together so as to rotate in opposite directions, as indicated by the arrows.

It will be seen that the C.G. of the two balance weights has a simple harmonic motion along the centre line of stroke of the piston, but opposite in direction.

Thus the primary reciprocating forces can be perfectly balanced by this means.

A perfectly balanced engine is obtained by having the cylinders upon opposite sides of the crankshafts, as shown in Fig. 185,

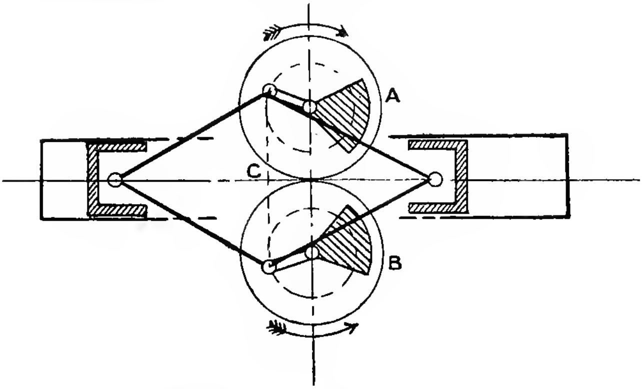


FIG. 185.—LANCHESTER'S EARLY TWO-CYLINDER ENGINE.

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* *Vide* "Engine Balancing" (F. W. Lanchester), *Proc. I.A.E.*, 1914.

which represents the arrangement of the old Lanchester type of engine of 1896 to 1903.

The two opposed cylinders were arranged symmetrically upon opposite sides of the crank axes, and the connecting rods formed a symmetrical parallelogram system in all positions. The balance weights A and B were made to balance the rotating portions of the cranks, and also the reciprocating parts of the engine. It will be noticed that the whole system of pistons and connecting rods is symmetrical about the point C along the line of strokes, and that the motion of this system is strictly simple harmonic, and can be balanced by oppositely rotating weights on the crankshaft at A and B.

In reality the higher harmonics balance themselves, provided that they all lie in the same plane, and therefore do not introduce rocking couples. In this particular type of engine this was actually the case, for, as Lanchester remarks, "the engine was not troubled with any rocking moment owing to the fact that the whole of its reciprocating parts had 'looking-glass symmetry' about the transverse vertical plane."

The General Case of Engine Balance.

It is proposed to consider the general case for the motion of the piston in the case of the single-cylinder engine, and for this purpose both analytical and graphical methods will be employed. The results of a detailed study of the above primary example will then be used in solving the proposition for different types of engine.

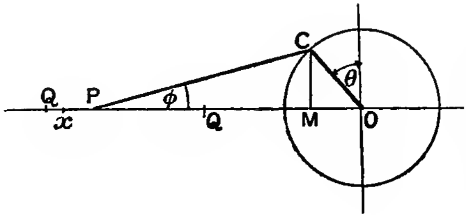


FIG. 186.

It will be necessary to consider the expressions for the displacement, velocity, and acceleration of the piston somewhat more fully than in the earlier portion of this chapter. Let OC be the crank arm of length r, PC the connecting rod of length l, and Q and Q^1 the inner and outer dead centre positions of the piston, Fig. 186.

Assume $\frac{r}{l} = m$

The piston displacement PQ is denoted by x.

$$\text{Now } x = PQ = OM + MQ - OP$$
$$= r \sin \theta + l \cos \varphi - l$$

But $\frac{\sin\varphi}{\cos\theta} = \frac{r}{l} = m$

and $\cos\varphi = \sqrt{1 - m^2\cos^2\theta}$

Hence $x = r\sin\theta + l\sqrt{1 - m^2\cos^2\theta} - l$

$$\text{or } x = r\sin\theta + l\left\{-\frac{1}{2}m^2\cos^2\theta - \frac{1}{8}m^4\cos^4\theta + \frac{1}{16}m^6\cos^6\theta + \text{etc.}\right\}$$

$$= r\left\{\sin\theta - \frac{1}{2}m\cos^2\theta - \frac{1}{8}m^3\cos^4\theta + \frac{1}{16}m^5\cos^6\theta - \frac{5}{128}m^7\cos^8\theta + \text{etc.}\right\}$$

$$\text{Now } 2^{n-1}\cos^n\theta = \cos n\theta + n\cos(n-2)\theta + \frac{n(n-1)}{1.2}\cos(n-4)\theta + \text{etc.}$$

And substituting in the general expression definite values for n

$$\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$$

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$$

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$$

$$\cos^8\theta = \text{etc.}$$

Hence we may express the displacement of the piston by a "Fourier Series," by substituting the multiple angle cosine terms for the powers of the cosine terms obtained.

The most convenient method is to write the displacement as $x = r\{\sin\theta - (k + \varrho_2\cos 2\theta + \varrho_4\cos 4\theta + p_6\cos 6\theta + \text{etc.})\}$

Then the values of the constants are obtained by substituting for the cosine power terms, and are—

$$k = \frac{1}{4}m + \frac{3}{64}m^3 + \frac{5}{256}m^5 + \frac{175}{(121)^2}m^7$$

$$\varrho_2 = \frac{1}{4}m + \frac{1}{16}m^3 + \frac{15}{512}m^5 + \frac{35}{2048}m^7$$

$$\varrho_4 = \frac{1}{64}m^3 + \frac{3}{256}m^5 + \frac{35}{642}m^7$$

$$\varrho_6 = \frac{1}{512}m^5 + \frac{25}{2048}m^7$$

$$\varrho_8 = \frac{5}{(128)^2}m^7$$

$$\varrho_{10} = \text{etc.}$$

In order to afford some idea as to the values of these constants for certain cases occurring in practice the following table has been prepared.

TABLE XXXVII

Table of Fourier Constants

$\frac{\text{Crank}}{\text{Connecting Rod}}=m$	$\frac{1}{4}$	$\frac{1}{4\frac{1}{2}}$	$\frac{1}{5}$
k	·0632	·0561	·0504
ρ_2	·0635	·0563	·0505
ρ_4	·000261	·000177	·000140
ρ_6 . .	·00000205	·00000112	·0000065
ρ_8 . .	·00000000	—	—

It will be found that the analysis of the piston displacement gives very accurate results if taken to five terms only, as the higher harmonics die away very rapidly after the fourth term.

Hence we can express the piston's displacement as

$$x = r\left\{k + \sin\theta + \varrho_2 \sin\left(2\theta - \frac{\pi}{2}\right) + \varrho_4 \sin\left(4\theta - \frac{\pi}{2}\right) + \text{etc.}\right\}$$

It will be noticed that only the *even harmonics* are present; this is accounted for by the fact that the piston displacements are the same for symmetrical crank positions, as shown in Fig. 187.

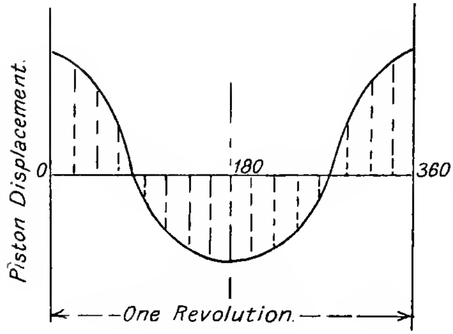


FIG. 187.

From the above expression it will be seen that the displacement of the piston may be regarded as being due to the component displacements upon the line of stroke of

(*a*) A constant displacement rk

(*b*) A crank of radius r, moving at crank velocity

(*c*) A crank of radius $r\varrho_2$, moving at twice crank velocity and out of phase by 90°

(*d*) A crank of radius $r\varrho_4$, moving at four times crank velocity and out of phase by 90°

and so on.

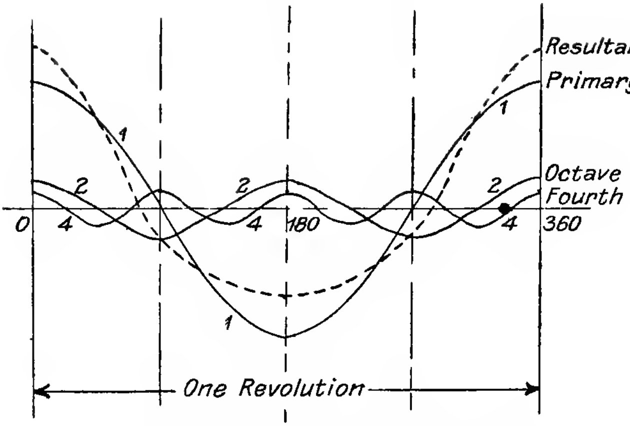


FIG. 188.—ANALYSIS OF PISTON'S MOTION.

The total displacement x may be expressed by means of a number of sine curves of different amplitudes and periods, somewhat as illustrated in Fig. 188.

Piston Acceleration.

By differentiating the expression for the piston displacement twice, with respect to the time we obtain the acceleration, thus we have—

$$\frac{d^2x}{dt^2} = \omega^2 r\left\{\sin(\theta+\pi) + 4\varrho_2 \sin\left(2\theta+\frac{\pi}{2}\right) + 16\varrho_4 \sin\left(4\theta+\frac{\pi}{2}\right) + 36\varrho_6 \sin\left(6\theta+\frac{\pi}{2}\right) + \ldots\right\}$$

The harmonics here become of greater importance as they are magnified, 4, 16, 36, etc., times respectively, and these harmonics become important in engine balancing problems, since the inertia forces are proportional to the above acceleration.

Corresponding to the table previously given (Table XXXVII), a similar one has been prepared for the values of the acceleration amplitudes.

TABLE XXXVIII

TABLE OF ACCELERATION AMPLITUDES

$\frac{\text{Connecting Rod}}{\text{Crank}}$ Ratio	4	$4\frac{1}{2}$	5
$4\rho_2$	·254	·225	·202
$16\rho_4$	·0041	·0028	·0021
$36\rho_6$	·000074	·000040	·000023

The results of the preceding investigation can now be applied for the purpose of obtaining the piston acceleration effect, in which the obliquity of the connecting rod is taken into account.

The acceleration may be considered to be due to the displacement of—

(*a*) A crank of radius $\omega^2 r$, rotating at crank velocity, and out of phase by $+\pi$

(*b*) A crank of radius $4\omega^2 r\varrho_2$, rotating at twice crank velocity, and out of phase by $+\frac{\pi}{2}$

(*c*) A crank of radius $16\omega^2 r\varrho_4$, rotating at four times crank velocity, and out of phase by $+\frac{\pi}{2}$

(*d*) A crank of radius $36\omega^2 r\varrho_6$, rotating at six times crank velocity, and out of phase by $+\frac{\pi}{2}$

etc.

This is represented graphically (Fig. 189) by a vector O1, proportional in magnitude to $\omega^2 r$ and 180° in advance of the main crank, and vectors O2, O4, O6, etc., proportional to $4\omega^2 r\varrho_2$, $16\omega^2 r\varrho_4$, $36\omega^2 r\varrho_6$, etc., respectively, and rotating at twice, four, six, etc., times the crank velocity.

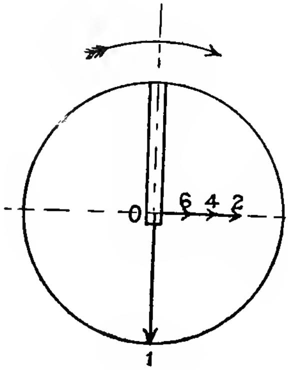


FIG. 189.

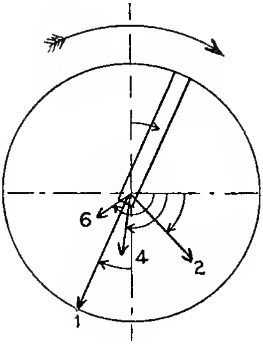


FIG. 190.

For any position θ of the main crank, the first, second, third, etc., vectors go through angles θ, 2θ, 4θ, etc., respectively (Fig. 190), and the resultant acceleration may be determined graphically by finding the displacement ON due to separate displacements, that is, by algebraically summing the respective ordinates or projections upon the base line (Fig. 191).

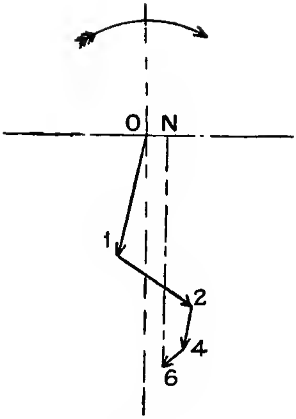


FIG. 191.

Applications to Balancing Problems.

The expression for the acceleration force in the case of a single cylinder engine is—

$$F = M\omega^2 r\left\{\sin(\theta+\pi) + 4\varrho_2\sin\left(2\theta+\frac{\pi}{2}\right) + 16\varrho_4\sin\left(4\theta+\frac{\pi}{2}\right) + 36\varrho_6\sin\left(6\theta+\frac{\pi}{2}\right) + \ldots\right\}$$

where M = mass of reciprocating parts.

If the $\dfrac{\text{connecting rod}}{\text{crank}}$ ratio be denoted by n, then by substituting

$$\varrho_2 = \frac{1}{4n},\ \varrho_4 = \frac{1}{64n^3},\ \varrho_6 = \frac{1}{512m^5},\ \text{etc.,}$$

we have

$$F = M\omega^2 r\left\{\sin(\theta+\pi) + 4.\frac{1}{4n}.\sin\left(2\theta+\frac{\pi}{2}\right) + 16.\frac{1}{64n^3}\sin\left(4\theta+\frac{\pi}{2}\right) + 36.\frac{1}{512m^5}\sin\left(6\theta+\frac{\pi}{2}\right) + \text{etc.}\right.$$

The first harmonic may be represented by a mass of $\dfrac{M}{4n}$ at the crank radius r rotating with velocity 2ω for the accelerating force $= \dfrac{M}{4n}(2\omega)^2 r = \dfrac{M\omega^2 r}{n}$ Similarly for the second harmonic we have an equivalent mass $\dfrac{M}{64n^3}$ at crank radius, and so on.

The Single-Cylinder Engine.

In the case of the single-cylinder, in so far as the crank and crank pin are concerned, these may be balanced by means of suitable counterbalance weights upon the opposite side of the shaft.

In the balancing of the connecting rod it is usual to consider the big end of the rod as a rotating part, and to balance it by an additional counterbalance weight; the small end of the rod being treated as a reciprocating part, and a corresponding addition is made to the weight of the piston and its component parts.

The method of allotting the weights of the two parts is usually taken as being inversely proportional to the distances between the end centres and the C.G. of the rod.

For a single cylinder, the balance of the reciprocating parts in respect to the initial simple harmonic component is usually effected by introducing an equivalent counterbalance weight to a portion of this mass, *not* to the whole of the reciprocating mass.

This is an important point, for if a rotating balance weight equal to that of the reciprocating parts be employed to effect a balance for the latter, then resultant unbalanced forces will be introduced having a reciprocation at right angles to the line of stroke, and equal in magnitude to the original unbalanced forces.

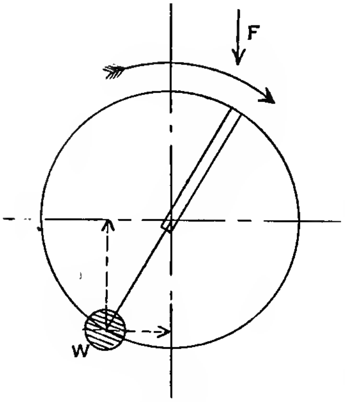


FIG. 192.

This will be evident from Fig. 192, from which it will be seen that whilst the resolute along the line of stroke balances the initial harmonic movement of the reciprocating parts, yet the perpendicular resolute is unbalanced.

In some cases the construction of the engine is such that it is better able to resist vibration forces in the direction perpendicular to the line of stroke, but generally speaking, a compromise has to be adopted between wholly balancing the reciprocating parts by an equal rotating mass on the opposite side of crank shaft and not balancing them at all.

The usual compromise is to balance half of the reciprocating weight by a rotating counterbalance weight, in which case the unbalanced force is equivalent to a mass equal to the added balance weight rotating in the direction opposite to the engine's motion.

A proof of this statement may be obtained by assuming the primary harmonic motion as equivalent to two masses of half the reciprocating weight revolving at crankshaft velocity, one in the direction and the other in a reverse direction to the main crank, as illustrated in Fig. 193. The mass moving in the direction of the main crank can be balanced by an equal and opposite counterbalance weight, and we are left with an unbalanced effect due to the mass rotating in a reverse direction.

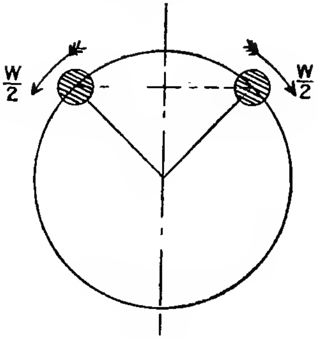


FIG. 193.

The degree of compromise in attempting to obtain the balance of the reciprocating masses is entirely a question of the conditions of mounting, usage, etc., of the engine.

An interesting arrangement for perfectly balancing the reciprocating forces in a single-cylinder engine is that of the Gobron-Brille, as employed in both car and aviation engines, Fig. 194. There are two pistons A and B working in the same cylinder, but in opposite directions, the compression and combustion space being between the two. The top piston is arranged to work by means of an overhead crosspiece and long side connecting rods, on to a pair of cranks symmetrically disposed in relation with and at 180° to the main crank.

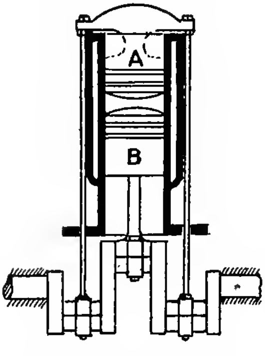


FIG. 194.—THE GOBRON-BRILLE ENGINE.

This is an example of balancing a reciprocating mass by an exactly similar one, but opposite in direction.

The engine, necessarily, occupies a greater vertical height than normal types, but otherwise it possesses advantages over other types, in the matter of balance.

The Two-Cylinder Engine, Cranks at 180°.

The line diagram shown in Fig. 195 illustrates the arrangement of this type.

Dealing with the fundamental motion effect first, there will be seen to be two forces of magnitude $M\omega^2r$ acting in the directions shown. These constitute a disturbing couple of magnitude $M\omega^2r.b$, with its axis at C, the direction of the latter being norma to the crankshaft axis.

There will therefore be no resultant vertical force, but a resultant couple tending to act about an axis perpendicular to the line of stroke and centre of crank axis, the magnitude of the couple at any instant about this axis being the resolved part of the main rotating couple in this direction.

The maximum value of the resolved couple $M\omega^2r.b$.

The axis of this couple, which is always perpendicular to the plane of cranks, revolves about the axis of crankshaft at engine speed.

Effect of the Harmonics.

The octave harmonic effect is illustrated in the same diagram in Fig. 195.

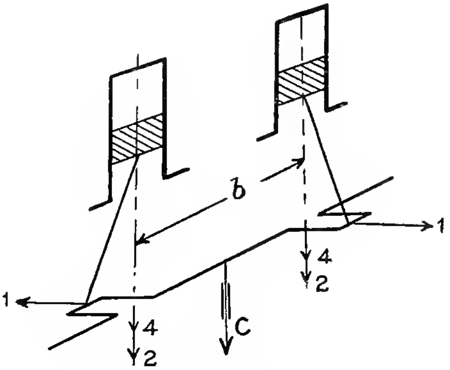


FIG. 195.—THE TWO-CYLINDER ENGINE (CRANKS AT 180°).

This harmonic is such that the mass $\frac{M}{4n}$ at equivalent crank radius for the two cranks will have moved through twice the crank angle (being of double the fundamental period) and will thus be as shown in the diagram in their relative positions.

These harmonics lag by $\frac{\pi}{2}$ behind the fundamental, which for convenience is taken in phase with the main cranks.

The net effect of the octave components is equivalent to two forces $\frac{M\omega^2r}{n}$ acting at right angles to the respective cranks in the same direction, and revolving at twice crankshaft speed, giving rise to a hammering action of twice the fundamental frequency.

Similarly the effect of the next, or 4θ component, can be studied, and will be seen to be equivalent to two forces at right angles to the respective cranks, in the same direction, and revolving at four times engine speed. These harmonics give rise to a hammering action of four times the fundamental frequency.

The Two-Cylinder Engine, Cranks at 90°.

In this type of engine, which is illustrated diagrammatically in Fig. 196, the fundamental component of the pistons' motion is represented by forces $M\omega^2 r$ acting in the directions shown.

If a point C midway between the centre lines of engine cylinders be chosen, the force at A may be replaced by an equal force at C, and a couple of transference $M\omega^2 r . \frac{b}{2}$

Similarly for the replacement of the force acting at B.

Then we have two equal couples, whose axes are represented in direction by the barred lines, of magnitudes $M\omega^2 r . \frac{b}{2}$ and two forces of magnitude $M\omega^2 r$ acting parallel to the original forces upon the cranks at A and B.

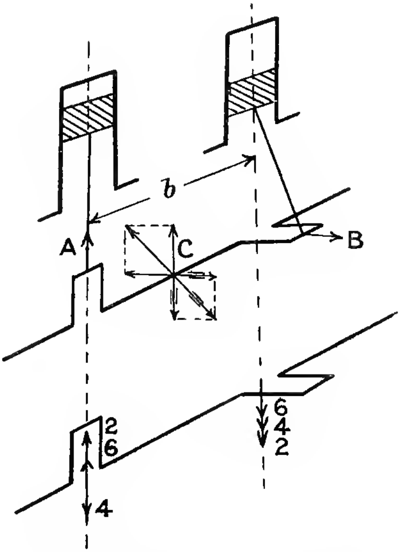


FIG. 196.—THE TWO-CYLINDER VERTICAL ENGINE (CRANKS AT 90°).

The two couples may be replaced by a couple of magnitude $\frac{M\omega^2 r . b\sqrt{2}}{2}$ acting at 45° with the crank arms, and whose axis is also perpendicular to the resultant force $M\omega^2 r\sqrt{2}$ of the two separate forces acting at C.

The resultant force due to the combined inertia effects of the two pistons, and their parts, will be the resolute of the above force in the directions of the cylinder axes, and will be a maximum when the cranks are symmetrical with the plane of the lines of stroke, and its value will then be $M\omega^2 r\sqrt{2}$. At any other moment its value is $M\omega^2 r\sqrt{2}\cos(\theta - 45°)$, where θ is the angle made by the leading crank with line of stroke.

Similarly the resolved part of the resultant couple in a plane perpendicular to the resultant force will give the resultant inertia couple effect due to the unbalanced reciprocating parts.

The unbalanced force gives rise to vertical vibrations of a frequency equal to the piston's frequency, whilst the unbalanced couple effect will tend to rock the engine about a horizontal axis, from side to side, the frequency of the angular oscillation being the same as that of the piston.

The maximum value of the unbalanced couple is $\frac{M\omega^2 rb\sqrt{\ }}{2}$, and occurs when the resolved part of the resultant force in the cylinder axes direction is a minimum, or when the leading crank has gone through 135° from its inner dead centre.

The Harmonics.

The position of the second, fourth and sixth harmonics is shown in the lower part of the figure, from which it will be seen that the octave or secondary harmonics are equivalent to a couple of magnitude $\frac{M\omega^2 rb}{n}$ rotating at twice engine speed, the resolved part of which in the plane perpendicular to cylinder axes will represent the resultant effect.

This couple sometimes enhances and sometimes detracts from the effect of the primary couple.

Thus these two couples may be compounded into a single resultant couple rotating at engine speed, the resolute of which in the perpendicular plane to cylinder axes will give the resultant couple effect.

A good exercise for the student would be to find the resultant couple due to the primary and octave harmonics, and to plot the magnitude of this resultant as a radius vector, the vectorial angle representing the direction of the axis of couple (not the crank angle).

The fourth harmonics give rise to a hammering action of four times the fundamental frequency, and of maximum magnitude $\frac{M\omega^2 r}{4n^3}$. The sixth harmonics will give rise to a couple rotating at six times crankshaft speed, and of maximum value $\frac{36M\omega^2 r}{512n^5}.b$, which will sometimes oppose and sometimes enhance the resultant of the primary and octave couples.

The octave couple may become very appreciable for small connecting rod crank ratios, a point that should not be, but occasionally is, overlooked. Thus for $n = 4$, its maximum value is $\frac{M\omega^2 r.b}{4}$, whilst that of the primary is $\frac{M\omega^2 rb}{1{\cdot}414}$, so that the octave effect is about half that of the fundamental couple.

The Two-Cylinder V Engine, Axes at 90°.

In this arrangement the two pistons drive one crank, as shown in Fig. 197.

The relation of the inertia forces in this type can best be studied by the principle of reverse cranks, in which a simple harmonic reciprocating motion can be represented by two equal rotating masses, of half the reciprocating weight, rotating in opposite directions as indicated in Fig. 197. The C.G. of these weights oscillates with a S.H.M. along the line of piston stroke and in the same phase as the piston, and exactly represents the piston's motion and inertia forces, for the fundamental harmonic.

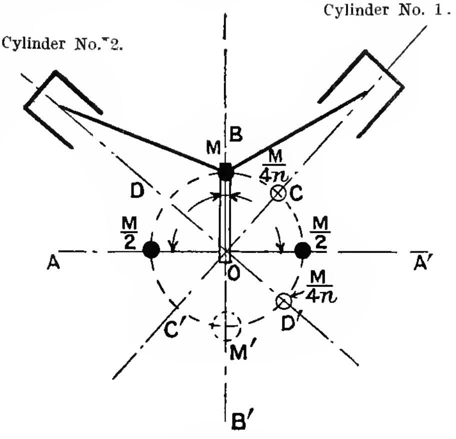


Fig. 197.—The Two-Cylinder 90° Engine.

For the secondary or octave components, the equivalent masses at crank radii will rotate (in proper phase relative to the fundamental harmonic) at twice crankshaft speed, and so on for the higher harmonics.

Referring to Fig. 197, it will be evident that the primary reciprocating motion may be represented by a mass $\frac{M}{2}$ at A and one $\frac{M}{2}$ at B for each cylinder, revolving in opposite directions. Now the two masses $\frac{M}{2}$ at AA_1 balance, whilst the two masses $\frac{M}{2}$ may be regarded as a total mass M always at the crank pin B.

This may be balanced by a mass M^1 at a radius R^1 upon the opposite side of the crankshaft, such that

$$M^1R^1 = MR.$$

The Secondary Harmonics.

A little consideration will show that when the crank is in the direction OC, the direct and reverse cranks for cylinder 1 will be at C, whilst those for cylinder 2, having rotated through twice the crank angle, will both be at D^1, and hence the octave forces will not be in balance, in fact the C.G. of the two secondary forces oscillates in a S.H.M. along AA^1. This can be easily understood if the positions of the direct and reverse cranks be taken for several main crank angles. The effect of these unbalanced octaves is to give to the engine a vibration of twice the frequency of the main harmonic.

The maximum value of the unbalanced secondary force will be the resultant of the two separate maximum forces $\frac{M}{n}.\omega^2R$ each at 90° to the other, that is, a resultant maximum force of $\frac{M}{n}\omega^2R.\sqrt{2}$ acting along AA^1.

The Two-Cylinder 180° Opposed Type.

This type of engine, which is illustrated in Fig. 198, has the cylinders placed upon opposite sides of the crankshaft, and the cranks are at 180° with each other. The firing strokes are quite evenly spaced, thus yielding a torque of regular character.

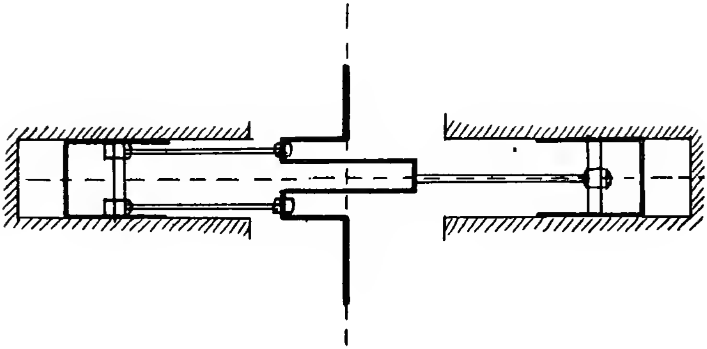


FIG. 198.—THE 180° TWO-CYLINDER OPPOSED ENGINE, CO-AXIAL TYPE.

By arranging the cylinders so that the two lines of stroke coincide, which can be accomplished in practice by employing a pair of connecting rods for one cylinder symmetrically placed upon either side of the other, then the fundamental and octave motions, etc., of the two cylinders balance each other, and no rocking moments occur.

The balance is, in fact, as nearly perfect as it is possible to get in any type of engine. The only unbalanced factors that occur are the variation in torque due to piston inertia and to the explosion impulse. With this type of engine, owing to its excellent balance, petrol-engine speeds of over 5,000 revolutions per minute have been attained at the time of writing.

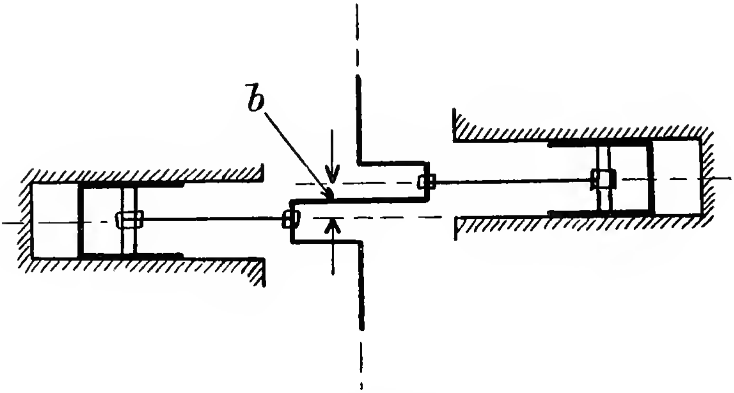


FIG. 199.—THE 180° TWO-CYLINDER OPPOSED ENGINE, OFFSET TYPE.

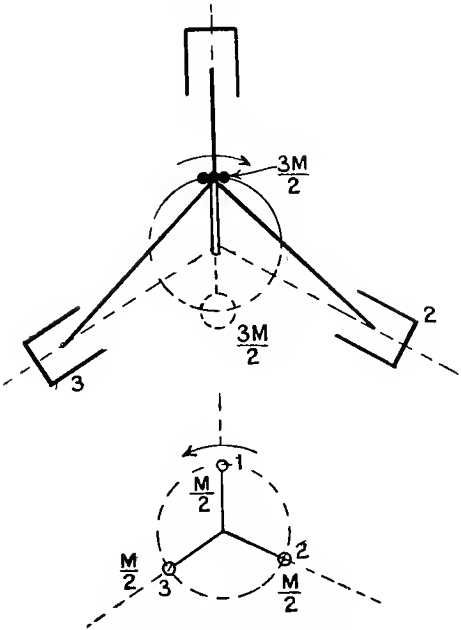
It is often impossible to arrange the cylinders co-axially for manufacturing reasons, so that the axis of each cylinder is situated opposite to its own crank, the two axes being out of line, as shown in Fig. 199.

In this case there will exist rocking couples of moment equal to the primary force multiplied by the axial separation, i.e., $M\omega^2 r.b$, and similarly for the secondary and higher harmonics.

The Three-Cylinder Engine, Cranks at 120°.

This type of engine, although not employed at all in motor-car practice, was fairly common in the case of low-powered aeroplane engines, such as the 35 H.P. Anzani **Y** type and like makes, and is still employed.

The principle of reverse cranks here provides a simple means of examining the engine balance.



Reverse Crank.

FIG. 200.—THE THREE-CYLINDER RADIAL ENGINE.

Referring to the diagram given in Fig. 200 it will be seen that the three fundamental direct cranks will all be in the position OA, whilst the three reverse cranks balance each other.

Hence the fundamental harmonic forces may be balanced by a mass $\frac{3M}{2}$ at an equal radius, but in direction opposite to the main crank OK.

The Secondary Harmonics.

Referring to Fig. 200A, it will be seen that the three direct octave cranks will be in positions OA, OB and OC respectively, and will therefore balance each other, whilst the reverse cranks will all be in the position OA, and can only be balanced by a mass $\frac{3}{2} \cdot \frac{M}{4n}$ placed opposite to OA and revolving counter-clockwise at twice engine speed.

The Fourth Harmonics.

Similarly it will be found that the three reverse cranks for the fourth harmonics all balance, whilst the direct cranks are all in position OA, and are only capable of balance by a mass $\frac{3}{2}\,\frac{M}{64_n{}^3}$ placed opposite, and rotating at four times engine speed.

The higher harmonics are of much less importance since they give rise to unbalanced forces of proportionately smaller magnitude, but higher frequency.

The same method of analysing the inertia forces in other unicrank multi-cylinder types of engine applies.

Radial Type Engines, Even Number of Cylinders.

If the preceding method be applied to the case of engines with four cylinders the axes of which are in one plane and at 90°, it will be found that the fundamental harmonics require balancing by a mass at crank radius of 2M. The direct and reverse cranks for the secondary or octave masses are each in perfect balance.

Similarly the fourth harmonics are also perfectly balanced in themselves.

Unfortunately this type of engine is not suitable for petrol or gas engines owing to the uneven firing periods, although two sets of four cylinders each could be made to give an explosion every quarter of a revolution and at the same time be balanced.

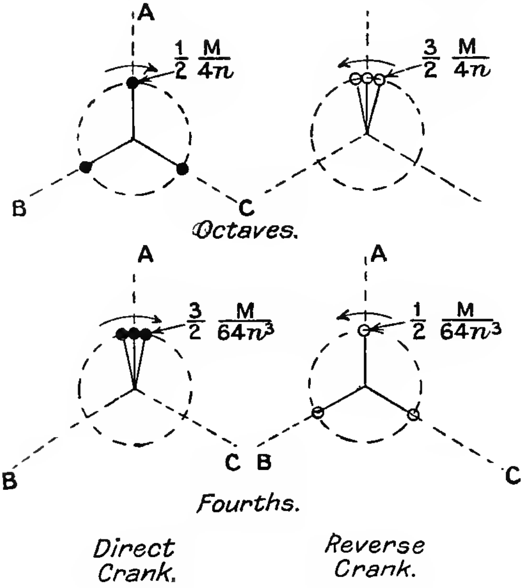

A
$\frac{1}{2}\frac{M}{4n}$
$\frac{3}{2}\frac{M}{4n}$
B
C
Octaves.
A
A
$\frac{3}{2}\frac{M}{64n^3}$
$\frac{1}{2}\frac{M}{64n^3}$
B
C B
C
Fourths.
Direct Crank.
Reverse Crank.

FIG. 200A.

By multiplying the number of cylinders in radial types of engine similar to the preceding examples the unbalanced harmonics can frequently be made to neutralize each other, but if the cylinder axes are not in the same plane (as in many multi-cylinder aeroplane engines), rocking couples are introduced even when the hammering actions are avoided.

It will generally be found that in cases of radial engines with an even number of cylinders, spaced at equal intervals, the harmonics are all in perfect balance and that the fundamental harmonics can all be balanced by means of a suitable balance weight placed opposite the single crank.

These even-cylindered engines, as mentioned before, are not always for explosion engines on account of the unequal firing intervals, unless the number of cylinders is at least six ; several six, ten and twenty cylinder engines of this type are employed in aeronautical practice.

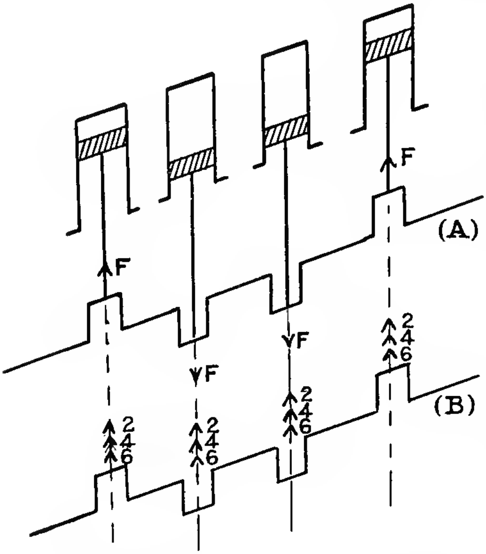
Radial Type Engines, Odd Number of Cylinders.

If the preceding method of analysis be applied to the case of the seven-cylinder radial type, such as the single-crank Gnome type, it will be found that the primary and secondary harmonics are in perfect balance and the fourth is out of balance, but being small in magnitude, is almost negligible, more especially at the comparatively low engine speeds of this type, in aeronautical work.

It can, further, be shown that for an even firing interval, an odd number of cylinders must be employed. The seven and nine cylinder Gnome engines are examples.

The Four-Cylinder Four-Crank Engine.

This is the ordinary type of motor-car engine with cranks arranged as shown in Fig. 201 and for which the torque curves are given.



(A). *Fundamental.* **(B).** *Harmonics.*

FIG. 201.

It will be seen later that the chief advantages of this type of engine consist in the regular firing intervals, and in the fundamental harmonics' forces and couples being perfectly in balance.

However, it should be remembered that all of the secondary harmonics synchronize, and therefore give rise to an unbalanced vibration of twice the frequency of the engine revolutions.

If the connecting rods of this four-cylinder type were of infinite length the engine would be in perfect balance as regards the whole of the harmonics. Since, however, on account of the angularity or obliquity of the connecting rod the inertia forces are greater for the top-centre crank positions than for the bottom positions, it follows that when two pistons are passing through the upper dead centre positions the two other pistons will be moving through the lower centre crank positions, with the net result that there will be an upward resultant force due to the difference between the inertias of the pistons at the top and bottom positions.

Further, this upward force will occur every half-revolution of the crankshaft, with similarly occurring downward forces at intermediate positions.

The net result is that the engine will experience a vibration of twice the frequency of the engine revolutions.

There is also another method of considering the balance of the four-cylinder engine, which is due to Lanchester.*

The effect of the angularity of the connecting rod is to cause each piston to reach its mid-position before the crank has moved through 90° from its top-centre position, or stated in another manner, when the crank pin is at 90° from the top or in-centre all four pistons are somewhat below their mid-stroke positions.

In the case of a 20 H.P. engine the position error *a* (Fig. 202), due to angularity, amounts to about a quarter of an inch.

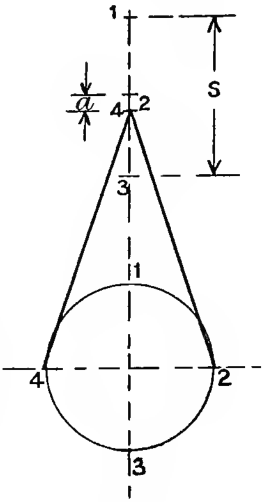


FIG. 202.

Further, the pistons will be symmetrical with regard to the mid-stroke positions twice every revolution at the dead centres, and unsymmetrical by the downward position error twice every revolution for the 90° crank positions. The effect of this is, that the displacement of the whole equivalent reciprocating mass occurs twice every revolution and gives rise to vibrations of the same periodicity, which are really the same thing as the octave vibrations.

Generally, the position error is approximately equal to the crank throw divided by the connecting rod length or $\frac{r}{2l}$, that is $\frac{1}{2n}$

It has further been shown that for a 20 H.P. engine with a connecting rod over crank ratio of $4\frac{1}{2}$, the octave amplitude is $\frac{1}{18}$ of the stroke, and since the disturbing force is proportional to the square of the periodic speed, then in the case of the octave harmonics the forces called into play will be $\frac{4}{18}$ of that due to the main component of each piston.

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* "Engine Balancing" (F. W. Lanchester), *Proc. I.A.E.*, 1914.

Hence, considering the whole four pistons, the unbalanced force due to the octave harmonic will be $\frac{16}{18}$ of the unbalanced fundamental piston force for a single cylinder engine, and of course gives rise to vibrations of twice the frequency of those occurring in the case of the single cylinder.

In the case of a four-cylinder engine of 4 inches bore and stroke, the weight of the piston being $4\frac{1}{4}$ lbs. and the connecting rod 4 lbs., and for a ratio of $n = 4$, the maximum value of the unbalanced vibrating forces at 1,000 revolutions per minute is about 320 lbs.

At higher speeds the forces will be very much greater since they vary as the square of the speed. It will therefore be evident that the secondary forces in a four-cylinder engine are quite appreciable; later on a method for absorbing these vibrations will be described.

The matter of the balance of this type of engine can also be dealt with by the same method as in the preceding examples. Examining first the fundamental harmonics of the piston's motion, it will be evident that at all times the two outer crank harmonic forces balance in direction and magnitude the two inner crank forces, and further that the moments of the primary couples for the two left-hand cranks always balance those due to the right-hand cranks, so that the primary balance is perfect.

Dealing next with the secondary or octave components of the piston's motion, a little consideration will show that the whole of the octave components for the crank position shown synchronize and act vertically upwards. When the main crank has turned through 90°, these octave components will have moved relatively through 180°, and will always be in synchronism. They will therefore give rise to vibrations along the line of stroke of twice the main harmonic's frequency. Further, the value of the maximum octave unbalanced force will be $F_2 = \frac{4M.\omega^2.r.}{n.}$

The Higher Harmonics.

As regards the fourth harmonics, it will be noted that these all synchronize and give rise to vibrations of four times the frequency of the primary harmonic; the maximum value of the fourth harmonic's unbalanced force will be

$$F_4 = \frac{M\omega^2 r}{n^3}$$

Similarly it can be shown that the sixth and higher harmonics each synchronize for the four pistons and give rise to vibrations of higher frequencies and with correspondingly smaller maximum forces.

Further, it should be mentioned that since each of the harmonics synchronize for all the pistons, and act in the same direction, there will be no resultant unbalanced couples for these harmonics.

In practice the effects of the unbalanced octave and fourth harmonics are noticeable—it is quite possible to observe in some car engines the vibrations due to the former, since it synchronizes in frequency with the explosions. In addition, when a four-cylinder engine of this type is raced under no-load conditions since the torque vibrations are practically negligible, the higher period vibrations are quite apparent. When it is remembered that in the case of a 25 H.P. engine the maximum value of thc unbalanced secondary force may be as much as half a ton alternately acting in an upward and downward direction, it will be seen that the effects must become of serious magnitude at high speeds. It is also a fact that at certain speeds of a motor-car engine the resonance, or sympathetic vibration effect between it and the body of the car itself, more especially limousines and landaulettes, accentuates this secondary vibration effect.

Method of Balancing Secondary Forces.

Lanchester has devised, and applied to car engines, an interesting piece of mechanism termed an "Anti-Vibrator," for the purpose of balancing the secondary harmonic effects. The principle of this arrangement consists in the introduction of an equivalent harmonic effect, but opposed in direction, to the secondary harmonic effect.

The mechanism employed, which is illustrated in Fig. 203, comprises two cranks, driven at twice crankshaft speed, and each rotating in opposite directions. In reality it is the practical application of reverse crank method, suitable balance weights being caused to give an exactly equal and opposite force at all times to the secondary forces due to the pistons' motion.

In the diagram the balance weights A are attached to shafts B_1, B_2, which are driven by means of helical gearing C_1, C_2, from the main crankshaft. The phase of these balance weights is the same, and it follows that a vertical harmonic reciprocating mass effect is obtained, opposed to the secondary piston reciprocating effect. The apparatus has been fitted successfully to the engines of the Lanchester car.

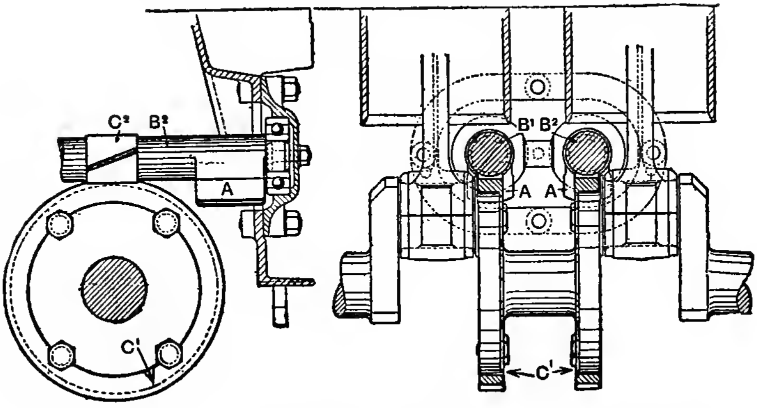

Fig. 203.—The Lanchester Method of Balancing Secondary Forces.

Other methods of attempting to balance the octave component have employed reciprocating masses of suitable frequency, but their non-success is attributable to the heavy stresses and consequent wear and tear involved.

Obviously in the method described the energy of the balancing mechanism system remains constant for constant engine speed, and no appreciable stresses are introduced, but the centrifugal component due to each revolving weight will naturally result in a corresponding load being thrown upon the balance shaft bearings, the direction of which is continually changing as the weight revolves.

The reduction in the weight of the reciprocating parts, such as the piston and connecting rod will tend to reduce the magnitude of the unbalanced harmonic forces.

The Six-Cylinder Engine Cranks at 120°.

The general arrangement of the six-cylinder engine, as used in car practice, is shown in Fig. 204.

It will be noticed that if the crankshaft be bisected by a plane perpendicular to the axis that the two halves of the crankshaft, each consisting of three cranks at 120°, will be symmetrically disposed with reference to this plane, and it is by viewing the matter in this light that it is proposed to investigate the problem.

Dealing first with the fundamental harmonics as represented by the components of rotating masses M upon the cranks, the three primary harmonics upon one side of the imaginary central

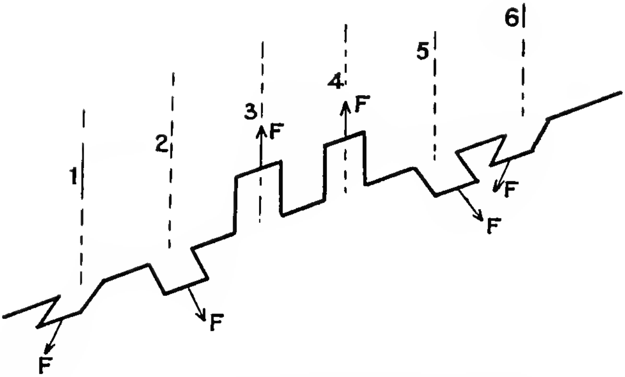

FIG. 204.—THE SIX-CYLINDER ENGINE (VERTICAL) CRANKS AT 120°.

plane will exactly balance the three primary harmonics upon the other side; or, stated in another manner, each set of three will be in balance so far as direction is concerned. And again, the moments of the primary harmonic forces about the central plane of the two sets of three cranks each, upon either side, will balance each other, so that the natural balance of the primary or fundamental harmonics is perfect.

Secondary Harmonics.

Reference to the diagram shown in Fig. 205, the upper circle of which represents the crank arrangement referred to a transverse perpendicular plane, will explain the position of the equivalent rotating masses, revolving at twice crankshaft speed for the octave harmonics indicated. These octave components will be seen to be in perfect balance. Similarly the fourth harmonics can be shown to be in perfect balance, both for magnitude and direction, and also moments. If the same method be applied to obtain the arrangement of the equivalent cranks for the sixth harmonics (that is, the 6θ term in the general expression for the acceleration force), it will be found that for the given crank position shown in Fig. 205, the arrangement of the sixth harmonic cranks will be as shown in the lower circle in Fig. 205, that is, the whole of these six harmonics of the piston's motion will synchronise and give rise to forces, the maximum value of which will be $6.\frac{36}{512n^5}M\omega^2r$ or $\frac{27}{64}.\frac{M\omega^2r}{n^5}$; this causes a vibration of frequency six times that of the primary, that is, occurring six times per revolution. There is, however, no resultant couple brought into play.

The practical effect of this rapidly occurring vibration is to all intents and purposes nil, as the magnitude of the maximum forces occurring due to this harmonic is very small compared with inherent defects due to workmanship and design.

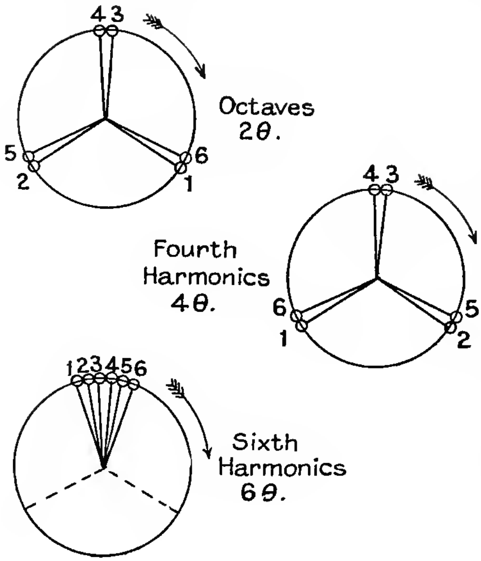

Fig. 205.

The Three-Cylinder Engine Cranks at 120°.

Resulting from this analysis of the balance of the forces occurring in the case of the six-cylinder engine, it will be apparent that if one-half of the crankshaft, as divided by the transverse central plane, be considered alone, then, so far as the primary, octave and fourth harmonics are concerned, the forces are in perfect balance, but that owing to their lines of action being separated, *unbalanced couples* will occur in each of these cases, giving rise to a " rocking " or " plunging " vibration.

The sixth harmonic is unbalanced both as regards the forces and the couples, due to axial separation of the cylinders.

The Eight-Cylinder V Engine.

This type of engine, which is represented in the car industry by the De Dion Company, and in the aeroplane world by the Renault, Wolseley, Sunbeam, R.A.F., etc., is of interest both as regards its torque and balance.

In passing, it is not out of place to note that it possesses marked advantages for motor-car use over other arrangements for the same number of cylinders. It enables one to get twice the power compared with a four-cylinder engine for a given longitudinal dimension, that is, for a definite available length of bonnet space. It is lighter than a vertical engine of the same number of cylinders and same size of bore and stroke; it is also lighter in proportion than a six-cylinder engine, and in addition, since the length of the crankshaft is less, the latter is relatively stiffer and less subject to whip.

One can also put to its credit account advantages in the matter of lubrication cooling, size, and manufacturing costs.

In respect to its application to aeronautical purposes, this type of engine conforms fairly well with the present type of aeroplane fuselage construction, and airship gondola design, and from the point of view of streamlining requirements, so that it is at present one of the most popular of the high-powered types, the horse-power of a single unit varying from 70 to 300. Moreover, in the case of air-cooled engines of this type, the cooling arrangements are simplified.

Reverting to the question of engine balance, the arrangement of this type of engine is exactly similar in transverse view to that of the 90° twin-cylinder type already discussed, and in longitudinal view to that of the four-cylinder vertical engine. For the sake of simplicity, then, the eight-cylinder type may be considered to be made up of four sets of twin engines, cylinders being set at 90°, as shown in Fig. 206.

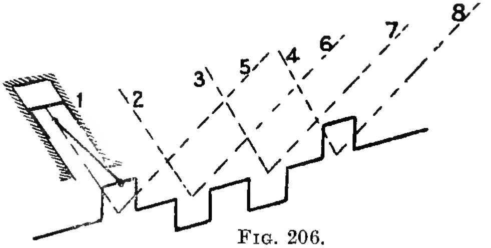

Fig. 206.
The Eight-Cylinder 90° V-Type Engine.

It has already been shown that in the case of a single 90° twin engine, the primary forces can be balanced by means of a counterbalance weight upon the opposite side of the crankshaft to the crank pin, but that the secondary forces are unbalanced, and give rise to a sidewise vibration. If now we consider the second set of twin engines with the crank at 180° to the first set, it will be seen that the unbalanced forces of the secondary harmonics synchronize with those of the first set. Hence for the whole set of four pairs of 90° twin cylinders with cranks arranged as in the four-cylinder vertical car engine, there will be four unbalanced secondary forces acting together or in synchronism.

The maximum value of the total unbalanced secondary forces will be $\cdot 707 \left(\frac{1}{\sqrt{2}}\right)$ times that of a four-cylinder engine having the same reciprocating mass.

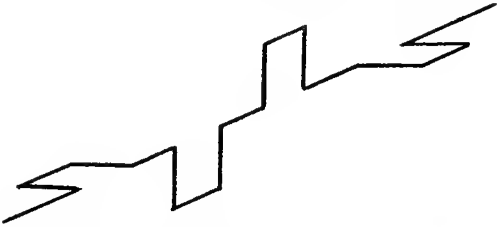

FIG. 207.—ALTERNATIVE ARRANGEMENT FOR EIGHT-CYLINDER V-TYPE ENGINE.

An alternative method of arranging the cranks of an eight-cylinder **V**-type engine is given in Fig. 207, the two centre cranks being at 180° with each other instead of together as before, and at 90° to the outer cranks.

In this case the unbalanced secondary forces of the second crank are twice 90°, or 180°, out of phase with those of the first crank, or opposed to them. Similarly for the third and fourth cranks the secondary forces will be opposed, the net result being that the four secondary forces are in perfect balance, and that there is no resultant couple. In respect to the fourth harmonics, these will be seen to be in synchronism, since they are at four times 90° to each other, or 360°, and hence will give rise to unbalanced forces of four times the frequency of the fundamental, but their magnitude being equal to $8.\left(\frac{M\omega^2R}{4n^3}\right)\sqrt{2}$ or $2\frac{M\omega^2R}{n^3}\sqrt{2}$ is so very small compared with the unbalanced secondary forces in the other arrangements of crank.

It has been estimated that the unbalanced forces of the fourth order in this case constitute only one-eightieth part of those of the unbalanced secondary forces in the ordinary type of eight-cylinder **V** engine.

Balancing of other Types.

It has only been possible to deal in the present Chapter with a few of the more important types of engine, in respect to balancing, but having become acquainted, however, with the methods employed in the foregoing considerations, there should be no difficulty in applying these to any type of engine.

A necessary course to pursue is to consider both the torque curves and engine balance for any particular type of engine under investigation, and then it is possible to form a fairly definite opinion as to the suitability or otherwise of this given type for the purposes required.

For more detailed treatment, with wider scope, of the subject of engine balance reference should be had to textbooks on the subject, such as " Engine Balancing," by Prof. A. Sharp, Dalby's " Balancing of Engines," etc., and to the papers referred to in the foot-notes.

Elimination of Rocking Moments in Engines.

As originally pointed out by Lanchester, the rocking moment which occurs and which is due to the axial separation of the cylinders in different types of engine may be eliminated in multi-cylinder types by the method of " optical or looking-glass " symmetry. The meaning of this phrase can be made quite clear by considering a particular case. It is known that in the two-cylinder vertical type with cranks at 180° there is a primary rocking couple. If now we imagine a transversely situated mirror at the end of crankshaft, then the reflected image of the crankshaft represents an arrangement of cranks, which with the original cranks will give no rocking couple—or in other words, the rocking couple of the original type exactly balances that of the reflected type. This combination of object and image is, of course, the ordinary four-cylinder arrangement and helps to prove the point.

Again, the six-cylinder 120° vertical engine can be divided by a central transverse plane, one half of the crankshaft being an exact reflection of the other half—and it is known that there is no resultant rocking couple in this type. Similarly the two-cylinder 180° opposed type represents a direct and reflected image of the single-cylinder type, and no rocking moment occurs.

Generally speaking, then, if there exists in any arrangement of cylinders an optical symmetry about a symmetrical plane, there will be no rocking couples.

Another way of putting it is that whatever rocking couple is set up by the direct or object half of the engine will be neutralized or balanced by the reflected image half.

If other arrangements of engine cylinders and cranks be examined, the above broad principle will be seen to apply.

The Effect of Cylinder Offset upon Engine Balance.

The practice of offsetting the crank in the direction of rotation has been adopted recently by several makers of car, cycle, and aeroplane engines, with a view to reducing the thrust upon the cylinder walls during the earlier part of the explosion stroke when such a thrust is a maximum. An engine so constructed is referred to as having a Desaxé disposition.

This arrangement will cause a modification in the torque diagram, and will also affect the balance of the engine.

It is not proposed here to deal with the question of offset in relation to engine balance in detail, as this would involve a tedious mathematical analysis out of proportion to the value resulting, but to briefly indicate the actual results of this practice, and to point out the lines along which to deal with problems relating to offsetting the crankshaft.

The matter has been treated in detail in articles by Prof. A. Sharp in *The Automobile Engineer*, November, 1910, and April, 1912, and F. W. Lanchester in a paper read before the Institution of Automobile Engineers, February, 1914, entitled "Engine Balancing," and it is to the latter that we owe the following treatment.

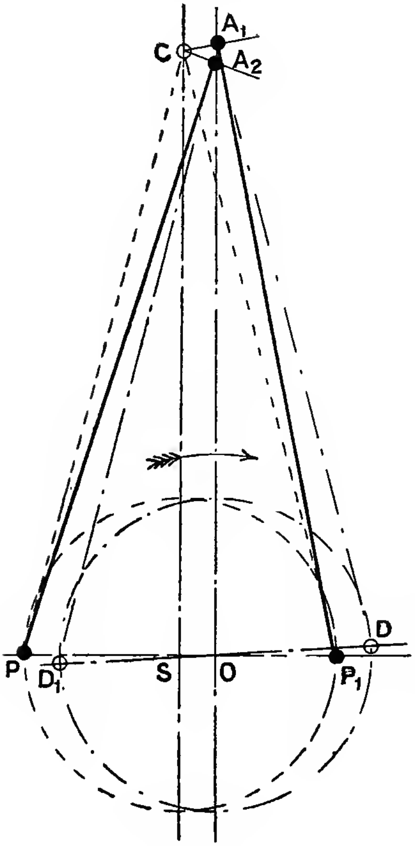

FIG. 208.

Referrng to Fig. 208, the line of stroke for an offset engine is represented by OA_1, and of a normal type by CS. When the crank is perpendicular to the line of stroke, the connecting rod is in the positions PA_2 and P_1A_1 respectively for the offset type and PC and P_1C for the normal type, so that in the former case the points A_1 and A_2 correspond to the point C in the latter case.

Further, since the angle A_1CA_2 is equal to the angle PCP_1, and this latter angle is a maximum for the crank positions shown, it will be evident that the distance A_1A_2 is a maximum as shown.

Hence the motion of the offset piston may be considered to be equivalent to the normal piston motion and an approximately harmonic motion superimposed of maximum amplitude AA^1, or alternatively the offset piston's motion may be approximately represented by the motion of the small end of the connecting rod driven by an imaginary crank OD with offset centre O, and it will be seen to be equivalent to an alteration in the phase of the normal type by an amount D_1OD.

The effect of offset then is equivalent to a retardation of the phase by a small angle, both for the finite and infinite connecting rods; but in the latter case the piston's motion will be strictly simple harmonic.

In the case of the finite connecting rod length, it can be shown that the secondary vibration is out of phase with the main vibration by the offset angle, and in such a direction as to oppose or counteract the effect of the offset angle, that is, in advance of the fundamental harmonic. Hence the secondary vibration will be in the same phase and in the same direction as in the normal type of engine.

The question of the influence of offset upon engine balance can be best considered analytically, and the general method adopted consists in obtaining an expression for the offset piston's position in terms of the crank angle, and by double differentiation deducing the acceleration expression.

The expression demonstrates that the offset piston's motion is due to a number of harmonic motions of different frequencies, the amplitudes and maximum values being dependent upon the connecting rod crank ratio and upon the degree of offset.

The expression for the acceleration force in the normal type engine is given by

$$\frac{M}{g}\frac{d^2x}{dt^2} = \overline{4}\,\frac{M}{g}\omega^2 r\left[\cos\theta + C_2\cos 2\theta + C_4\cos 4\theta + C_6\cos 6\theta + \ldots\right]$$

and in the offset type of engine by

$$\frac{M}{g}\frac{d^2x_1}{dt^2} = -\frac{M}{g}\omega^2 r\left[\cos\theta + A_2\cos 2\theta + A_4\cos 4\theta + A_6\cos 6\theta + \text{etc.}\right]$$
$$+\frac{M}{g}\omega^2 r\left[\sin\theta + B_3\sin 3\theta + B_5\sin 5\theta + \text{etc.}\right]$$

where C_2, C_4, C_6, etc., are constants depending upon the crank connecting rod ratio, and A_2, A_4, A_6, etc., B_3, B_5, B_7, etc., are constants depending upon both the crank connecting rod ratio and the amount of offset, which can generally each be expressed as an algebraic series in n, the crank connecting rod ratio, and a the offset.

The effect of offset is to introduce odd harmonics into the piston's motion, so that for symmetrical crank positions the piston's position is not the same (as in the normal type).

The quantitative effect of offset upon engine balance will be rendered more intelligible if a concrete example be given.

Taking the connecting rod crank ratio $n = 5$, and the amount of offset a = crank radius, the expression for the offset piston displacement is

$$x = r \cos\theta + l \ \{\cdot 9700 + \cdot 0106 \cos 2\theta - \cdot 00001 \cos 4\theta + \ldots \\ + \cdot 0415 \sin\theta - \cdot 0002 \sin 3\theta + \ldots\}$$

and the accelerating force by

$$\frac{M}{g}\frac{d^2x}{dt^2} = -\frac{M}{g}\omega^2 r \{\cos\theta + \cdot 0424 \cos 2\theta - \cdot 0001 \cos 4\theta + \ldots \\ + \cdot 0415 \sin\theta - \cdot 0018 \sin 3\theta + \ldots\}$$

The higher harmonics becoming of less importance.

It has been shown that in the case of a single cylinder with a connecting rod crank ratio of 5 and an offset equal to half the crank radius that the increase in the fundamental unbalanced force is only ·0002 of its normal value, and in the case of a four-cylinder offset engine that the secondary unbalanced force is about 1·5 per cent. greater than in the normal four-cylinder type. Generally speaking, with the degree of offset employed in practice the effect upon engine balance is negligible.

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